

# Multilevel Linear Mixed Model for Tree Diameter Increment in Stone Pine (*Pinus pinea*): a Calibrating Approach

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Diameter increment for stone pine (*Pinus pinea* L.) is described using a multilevel linear mixed model, where stochastic variability is broken down among period, plot, tree and within-tree components. Covariates acting at tree and stand level, as breast height diameter, density, dominant height or site index are included in the model as fixed effects in order to explain residual random variability. The effect of competition on diameter increment is expressed by including distance independent competition indices. The entrance of regional effects within the model is tested to determine whether a single model is sufficient to explain stone pine diameter increment in Spain, or if, on the contrary, regional models are needed.

Diameter increment model can be calibrated by predicting random components using data from past growth measurements taken in a complementary sample of trees. Calibration is carried out by using the best linear unbiased predictor (BLUP) theory. Both the fixed effects model and the calibrated model mean a substantial improvement when compared with the classical approach, widely used in forest management, of assuming constancy in diameter increment for a short projection period.

**Keywords** diameter increment, mixed model, calibration, stone pine

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## 1 Introduction

Tree diameter growth has usually been described using deterministic linear or non-linear equations. In these functions diameter increment is expressed as a function of measured or estimated variables related to tree size, stand attributes, competition, site quality and regional effects.

Ordinary linear (OLS) and non-linear (ONLS) least squares regression techniques have been widely used to fit these functions.

Tree diameter increment data are generally taken, for a single period or on repeated occasions, from trees growing in plots located in different stands or regions. This hierarchical structure results in a lack of independence

between observations, since data coming from the same sampling unit (tree, plot, region) tend to resemble each other more than average (West 1981, West et al. 1984, Fox et al. 2001). Lack of independence between observations results in biased estimates for the confidence interval of the parameters if ordinary least squares regression techniques are used (Searle et al. 1992). To deal with this problem, a multi-level linear mixed approach that includes random parameters has been widely proposed in forest research since the seminal works by Biging (1985), Lappi (1986) and Gregoire (1987). Mixed models are composed of a fixed functional part, common to the complete population, and random components acting at each sampling level. Mixed models take into account the correlation detected among observations coming from the same sampling unit by defining non-diagonal variance-covariance structure matrices, which are used to obtain best linear unbiased estimates for the fixed parameters of the model. Mixed models also permit the definition of a covariance structure for random effects and residual terms, and the best linear unbiased prediction for the random components specific to each unit.

The fixed effects model that includes explanatory variables explains in part, though not completely, total variation in tree growth. There is a residual variability component, probably representing differences in attributes acting at plot or tree level that are not easily observable, such as microsite, genetics, climate, or even measurement errors, defining a stochastic level of variability in tree growth. The use of mixed models allows us to identify the different sources of stochastic variability that are not explained by the fixed part of the model by dividing the residual variance into different components.

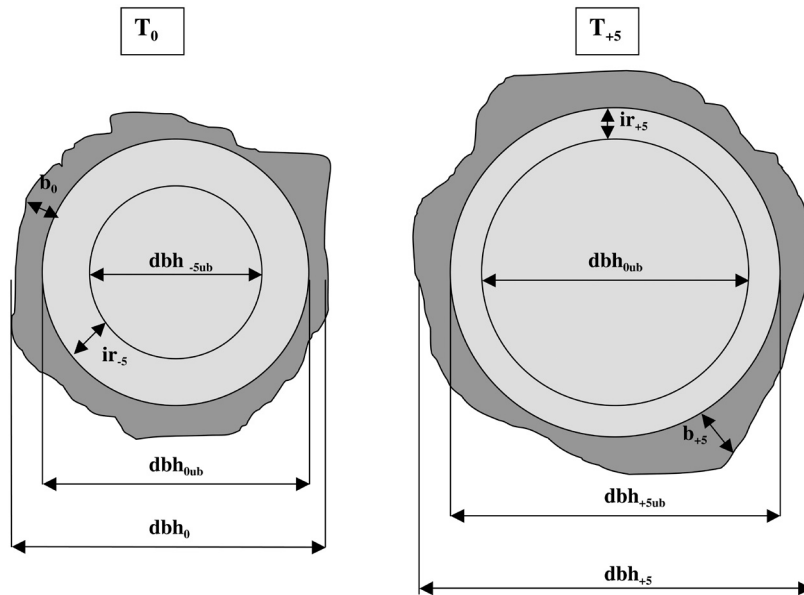
Mixed models allow calibration of growth models for a specific location and growth period. Prediction of the random components using best linear unbiased predictors (BLUP) can be carried out if a sample of complementary observations of the dependent variable is available. Since it is not possible to measure complementary future observations of growth, past growth data can be used to calibrate growth models.

Growth attained by a single tree in the past is a powerful predictor for the future increment of

that tree. Past increment has been used in growth models in different manners, starting with the traditional assumption of constancy in the increment for a given period. Pukkala (1989) and Pukkala and Kolstrom (1991) include the value of the past five year growth increment as a predictor covariate for the future increment, showing that this variable improves prediction more than spatial information. Henttonen (1990), Mabvurira and Miina (2002) and Palahí et al. (2003) included the ratio between breast height diameter and total tree age, a value indicating the mean diameter increment attained during the life of the tree, as a predictor in diameter increment models. Stage and Wykoff (1993) used the serial correlation between diameter increment in two successive periods of arbitrary length to calibrate the future increment of the trees. More recently, Trasobares and Pukkala (2004) developed an index for site evaluation based on the mean ratio between predicted and observed past diameter increment for the trees in a given stand. This index was then used as a predictor variable to model future diameter increment.

Mixed model calibration of diameter increment is based on the fact that the stochastic component of growth variability is a consequence of different factors acting simultaneously. If it is considered that the effect of some of these unobservable factors remains constant for a given period (Miina 1993), then it is possible to calibrate future increment by introducing into the model the stochastic effects predicted for a prior period.

Stone pine (*Pinus pinea* L.) is a typical Mediterranean species, occupying more than 450,000 ha. in Spain. This area represents more than 50 % of the total area covered by the species world-wide. The main productions of stone pine stands are pinyons (edible nut), timber, fuelwood, grazing, landscape and recreational use. Stone pine stands also play an important role as soil protectors, as they grow in sandy areas. Since the beginning of the 90's, a research line in the CIFOR-INIA has been devoted to the sustainable management and multiple use of the stands of stone pine in Spain. Different regional growth and yield models for the species (García Güemes 1999, Cañadas 2000, Piqué 2003) have recently been developed within this line. Of these, only the model by Cañadas (2000) is a single tree model, including a diameter



**Fig. 1.** Schedule representing the variables used to estimate diameter increment from the measurements taken on  $T_0$  and  $T_{+5}$ .

increment function for the trees of the species growing in Spain's Central Range. This model is deterministic, and parameters were estimated using ONLS regression techniques.

The main objective of the present work is to construct a model to predict diameter increment for stone pine trees growing in even-aged stands across Spain. The model would include explanatory tree, stand and regional variables. Since data come from a hierarchical structure, the model proposed would be a multilevel linear mixed model, including both fixed and random components. In order to improve predictive accuracy, the use of past growth data to calibrate the model was evaluated, and different calibration alternatives were compared.

## 2 Data

Between 1992 and 1999 a network of 470 sample plots was installed in even-aged stands of stone pine to develop growth and yield models for the species. These plots were placed in the four regions with the largest presence of the species in

Spain: The Northern Plateau, The Central Range, Catalonia and West Andalusia. Plots are circular, with variable size, and include 20 trees (except 50 ten-tree plots located in the Central Range). The design to select the plots attempted to include a balanced representation of the possible range of age, density, and site quality for each region. Only plots that had evidently not been thinned, pruned or harvested for a 10 year period were included. A buffer area with similar conditions was also defined around the plot.

For every tree, breast height diameter over bark ( $dbh_0$ ) and bark thickness ( $b_0$ ), both measured at 1.30 m above ground, total height of the tree, height to crown base, and crown diameter were measured. Position of each tree with respect to the centre of the plot was recorded. In the three to five trees nearest to the centre, total age and radial growth without bark for the last five years ( $ir_{-5}$ ) were also measured. Five years after plot installation, a second inventory was carried out, and radial growth without bark attained during that second period ( $ir_{+5}$ ) was then measured in the same trees (Fig. 1). In both inventories, radial increments were obtained as the average growth from two perpendicular measurements taken at

**Table 1.** Summary statistics for fitting and validation datasets.

	Fitting dataset (n = 1774 trees)				Validation dataset (n = 260 trees)			
	Mean	Min	Max	STD	Mean	Min	Max	STD
<i>dbh</i> (cm)	26.73	4.20	74.25	10.78	27.95	4	62.8	11.85
<i>H</i> (m)	9.08	1.75	22.50	3.61	10.15	2.40	24.00	4.22
<i>id</i> <sub>5</sub> (cm)	2.55	0.29	10.96	1.58	2.47	0.32	8.47	1.62
Age (years)	56	10	219	31	59	10	150	32
<i>H</i> <sub>0</sub> (m)	9.75	2.75	21.50	3.53	10.71	2.70	22.7	4.14
<i>N</i> (stem/ha)	341	27	1614	259	480	27	2800	592
<i>G</i> (m <sup>2</sup> /ha)	16.33	1.04	59.72	10.83	21.67	0.65	70.92	18.03
<i>SI</i> (m)	14.90	7.95	24.68	3.26	16.07	9.01	29.36	3.90

Where *dbh*: breast height diameter, *H*: total height, *id*<sub>5</sub>: diameter increment for five years, *H*<sub>0</sub>: dominant height, *N*: stand density, *G*: basal area, *SI*: site index. *STD*: standard deviation.

**Table 2.** Regional distribution for analysed trees, plots and studied periods.

Region	Fitting dataset		Validation data set		Installation year	1st growth period	2nd growth period
	N° plots	N° trees	N° plots	N° trees			
Central Range	59	197	6	21	1996	1992–1996	1997–2001
West Andalusia	164	746	26	117	1992	1988–1992	1993–1997
Northern Plateau	110	539	14	67	1995	1991–1995	1996–2000
Catalonia	60	292	11	55	1999	1990–1994	1995–1999

1.30 m above ground using a Pressler increment borer. Bark thickness was only measured at plot installation using a bark gauge.

When plot installation was carried out, radial growth for the last five years was recorded in 2252 trees from the 470 plots. A second inventory was only performed on those plots that had not been thinned, pruned, harvested or burned during the period. Since plots in Catalonia were installed in 1999 the second inventory was not available in the analysis. Consequently, data for the previous ten year increment period, taken at plot installation were used to define increment value for two successive five-year growth periods (*ir*<sub>-5</sub> and *ir*<sub>+5</sub>).

The total number of trees with two consecutive radial growth measurements was 2034 from 448 plots, including 4068 diameter increment observations. From this dataset, 57 plots including 260 trees were selected randomly within the four regions studied as validation dataset, while the rest were used as fitting dataset to develop the model. Summary statistics for both fitting and validation datasets at the moment of plot installation are shown in Table 1. Regional distribution for the analysed plots and trees, as well as the growth interval periods studied are shown in Table 2.

## 2.1 Calculation of *dbh*<sub>-5</sub> and *dbh*<sub>+5</sub>

The dependent variable in the growth model is future diameter increment over bark reached by the tree during a five year period. To obtain tree diameter over bark five years before (*dbh*<sub>-5</sub>) and after (*dbh*<sub>+5</sub>) plot installation from the increments measured in a core (*ir*<sub>-5</sub> and *ir*<sub>+5</sub>) and bark thickness (*b*<sub>0</sub>), adjustment for bark growth is needed. To compute *dbh*<sub>-5</sub> and *dbh*<sub>+5</sub>, the first step was to calculate the original breast height diameter under bark *dbh*<sub>0,ub</sub> by subtracting *b*<sub>0</sub> twice from *dbh*<sub>0</sub>. Next, breast height diameter under bark for five years before (*dbh*<sub>-5,ub</sub>) and after plot installation (*dbh*<sub>+5,ub</sub>) was computed by subtracting or adding the value of the diameter increment without bark (*2.ir*<sub>-5</sub> or *2.ir*<sub>+5</sub>), obtained from the cores. Bark thickness for these instances (*b*<sub>-5</sub> and *b*<sub>+5</sub>) was estimated assuming that the proportion of bark thickness with respect to total diameter remains constant during the studied 10 year period (Pukkala 1989, Hökkä and Groot 1999). Finally, *dbh*<sub>-5</sub> and *dbh*<sub>+5</sub> were computed by adding two times *b*<sub>-5</sub> or *b*<sub>+5</sub> to the corresponding value of breast height diameter under bark. Diameter increments for both periods, *id*<sub>-5</sub> and *id*<sub>+5</sub>, were computed by subtracting the estimated values for

breast height diameter over bark at the beginning and end of each period. Fig. 1 shows the whole calculation process in schematic form.

The model for diameter growth would consider diameter increment over bark for a future five year period,  $id_5$ , a single variable that joins both  $id_{-5}$  and  $id_{+5}$ , as the response variable. In the same way,  $dbh$  would be a single explanatory variable that considers both  $dbh_{-5}$  and  $dbh_0$ .

## 2.2 Reconstruction of Tree and Stand Characteristics Five Years Prior to and Five Years after Plot Establishment

Tree height was computed for each instance using the fixed effects part of the height diameter mixed model developed by Calama and Montero (2004). Difference in the predicted height between two successive inventories was added to or subtracted from the real height measured at plot installation to obtain the height in the instances  $\pm 5$  years, respectively. Crown dimensions were supposed to remain constant during the studied period.

Stand density, in terms of stem number per ha, was computed from plot data using the method proposed by Duplat and Perrotte (1981). Estimated stand density was considered constant throughout the studied period, since thinned or harvested plots were rejected during the second inventory. Plot age was computed as the average of those trees where it was measured, adding or subtracting five years, respectively. We knew the value for basal area at installation time,  $G_0$ , and the individual basal area increment (computed from  $dbh$ 's) for a subsample of three to five trees per plot. To obtain plot basal areas five years earlier ( $G_{-5}$ ) and later ( $G_{+5}$ ), the ratio between the basal area of the sampled trees and plot basal area was supposed to remain constant for the studied period. Mean squared diameters  $dg$ 's were computed directly from basal area estimates. Dominant height for the stand at different ages was computed using site index differential equations developed for the species by Calama et al. (2003).

## 2.3 Transformation of the Dependent Variable

To attain normal distribution for the residuals and reduce heteroscedasticity, a natural logarithmic transformation of the original dependent variable  $id_5$  was performed. A term  $+ 1$  was added to  $id_5$  before transformation, in order to stabilize variance behaviour and avoid high unstable values when diameter increment for the five year period was close to zero. The use of this transformation also resulted in a linear relationship between the dependent variable and those variables used as predictors.

# 3 Methods

## 3.1 Modelling Approach

Available data were based on a sample of repeated growth measurements taken from trees located in different plots. Plots were neither installed nor remeasured in the same year, indicating that there is also a level of variability associated with the different five-year calendar periods. This means that observations coming from the same tree, plot and period could be largely correlated.

To alleviate this, a multilevel linear mixed model, including both fixed and random components, was proposed. Between period, between plot, between tree and within tree differences were counted by including random parameters specific at those levels. General expression for the multilevel linear mixed model proposed was:

$$y = X\beta + Zb + \varepsilon \quad (1)$$

Where  $y$  is a  $n$ -dimensional vector including the  $n$  observations for the response variable  $\log(id_{5ijk} + 1)$  taken from  $n_v$  trees within  $n_u$  plots during  $n_w$  periods;  $X$  is a  $n \times p$  design matrix, including covariates and terms associated with fixed parameters of the model;  $\beta$  is a  $p$ -dimensional vector of fixed parameters of the model;  $Z$  is a  $n \times q$  design matrix for the random components of the model;  $b$  is a  $q$ -dimensional vector of random components acting at tree, plot and period levels;  $\varepsilon$  is a  $n$ -dimensional vector of conditional

residual terms (Gregoire et al. 1995).

Vector  $b$  can be divided into  $n$  subvectors  $b_{ijk}$ , each of them including random components at tree ( $v_{ij}$ ), plot ( $u_i$ ) and period ( $w_k$ ) level, specific to the observation taken from the  $j$ th tree within the  $i$ th plot during period  $k$ ;  $u_i$ ,  $v_{ij}$  and  $w_k$  are multivariate normally distributed vectors, with mean zero and variance matrices  $D_u$ ,  $D_v$  and  $D_w$ , respectively.  $D_u$ ,  $D_v$  and  $D_w$  explains between plot, between tree and between period random variability, and are considered common to every plot, tree and period analysed. Under these conditions,  $b$  is a multivariate normally distributed vector, with mean zero and variance matrix  $D$ . Assuming independence between random components corresponding to different hierarchical levels and independence between random components specific to different sampling units (plots, tree, periods),  $D$  is a  $q \times q$  block diagonal matrix, composed by  $n_u$  submatrices  $D_u$ ,  $n_v$  submatrices  $D_v$  and  $n_w$  submatrices  $D_w$ . Finally,  $\varepsilon$  includes  $n$  conditional residual terms, and it is assumed to be distributed with mean zero and residual within tree variance matrix  $R$ .

The usual aim in an analysis of this type is the estimation of the components of  $\beta$ ,  $D_u$ ,  $D_v$ ,  $D_w$  and  $R$ . To make inference over the fixed parameters vector  $\beta$ , generalized least squares techniques were used. A simultaneous estimation for the components of  $D_u$ ,  $D_v$ ,  $D_w$  and  $\sigma^2_\varepsilon$  was carried out following maximum likelihood (ML) and restricted maximum likelihood (REML) methods using MLwiN software (Rasbash et al. 2002). MLwiN allow us to obtain individual predictions of the best linear unbiased predictors (BLUP's) of the random parameters for each tree, plot and period.

### 3.2 Residual Variance

Vector  $\varepsilon$  includes  $n$  conditional within tree residuals  $\varepsilon_{ijk}$ , generally assumed to be i.i.d with mean zero and constant variance  $\sigma^2_\varepsilon$ . In that general case,  $R$  equals  $\sigma^2_\varepsilon I_n$ , where  $I_n$  is the identity matrix with range the number of observations. To detect a possible pattern of non-constant variance in the residuals (heteroskedasticity), mean variance for the conditional original and standardized residuals was plotted against categorized

explanatory variables. If residual variance is not constant, a variance function must be developed and included in the model, obtaining a specific value of the residual variance  $\sigma^2_{\varepsilon_{ijk}}$  for each observation.

### 3.3 Model Construction

A linear and additive functional form was used as a growth model. In a first step, a model including only a single-tree size fixed effect covariate and the intercept of the model varying randomly between units for the three hierarchical levels considered (tree, plot and period) was estimated from the fitting dataset. In subsequent steps, explanatory variables were included in order to explain systematic variability in the mean response, and the rest of the coefficients in the model were tested to vary randomly at different hierarchical levels. Order of inclusion was: single-tree size variables, stand variables, competition indices, and regional effects.

Criteria for including explanatory variables were the level of significance for the parameters, reduction in the values of the components of the variance-covariance matrices, and significant decrease for the statistic  $-2$  times logarithm of the likelihood function ( $-2LL$ ). Models were fitted and compared using maximum likelihood methods, and the final model was then fitted using the REML method. In order to avoid multicollinearity, evaluated models were fitted using OLS techniques and the value for the variance inflation factor was computed. Those alternatives where this factor was larger than 5 were then rejected.

Evaluated explanatory variables were:

*Single tree size variables:*

Breast height diameter  $dbh$  (cm); total height  $h$  (m); crown diameter  $cw$  (m); height to crown base  $hcb$  (m); crown ratio  $CR$ . Logarithmic and inverse transformations of these variables were also tested.

*Stand variables:*

Density  $N$  (stems/ha); dominant height  $H_0$  (m), computed as the average value of the 20% thickest trees within the plot; basal area  $G$  for the plot

(m<sup>2</sup>/ha); mean squared diameter  $dg$  (cm); site index  $SI$  (m) (Calama et al. 2003). Logarithmic and inverse transformations of these variables were also tested.

#### Competition indices:

Distance independent indices used were ratio between subject tree breast height diameter and mean squared diameter  $didg$ ; ratio between subject tree basal area and plot basal area  $gigm$ ; basal area of the trees larger than the subject tree  $BAL$ . Distance dependent competition indices were not evaluated since we did not have enough data to reconstruct the state of all the trees in the plot five years before (necessary to develop spatial models).

#### Regional variables:

A higher level of variability, not assumed to be random, can be explained including categorical regional parameters as fixed effects. Several alternatives grouping different regions under a single regional effect were compared on the basis of the log likelihood ratio test.

### 3.4 Prediction

The fixed effects part of a mixed model can be used to predict the value for the response variable if all covariates required are measured or estimated. In this case, if no additional information is available, expected value of the BLUP's for the random components are zeroes, and we would obtain the fixed effects marginal prediction  $E(y) = X\beta$ . A main advantage of mixed models when used to predict the value for a response variable is that the value for the random parameters vector  $b$ , specific for a given unit, can be predicted if a complementary sample of observations taken from that sampling unit is available. In this case we obtain the calibrated conditional prediction  $E(y | b) = X\beta + Zb$ .

It is possible to predict the random components associated with between-plot, between-tree and between-period variability using past increment data, which is easily obtained using an increment borer or consecutive measurements of the tree diameter taken on permanent plots. Random components obtained in this way can be used

to calibrate tree diameter increment under the hypothesis that both plot and tree random components are constant over time, at least for a five year period. To make predictions of the BLUP's for the random components using past increments the following expression was used:

$$\hat{b} = \hat{D} \hat{Z}^T (\hat{R} + \hat{Z} \hat{D} \hat{Z}^T)^{-1} \hat{e} \quad (2)$$

Where

$\hat{b}$  is a vector of BLUP's for the random components, acting either at tree, plot or period level.

$\hat{D}$  is a block diagonal matrix whose dimension is given by the number of random effects to be predicted. Blocks of  $\hat{D}$  are the matrices  $D_u, D_v, D_w$ , repeated as many times as the number of different units detected at each level.

$\hat{Z}$  is the design matrix for the random components specific to the additional observations.

$\hat{R}$  is the estimated matrix for the residual variance.

Finally,  $\hat{e}$  is a vector whose dimension is the number of observations, and whose components are the values for the marginal unconditional residuals of the model (difference between the observed increment and the predicted increment using the fixed effects marginal model).

To solve  $\hat{b}$  from Eq. 2, a SAS program was developed using IML language. Since BLUP for the period random component is not constant over time, to carry out future predictions with the model, expected value for this effect, which is zero, should be included.

To evaluate the accuracy of the calibration, two alternatives were compared. In the *single tree calibration*, increment measurements corresponding to the first growth period of the trees included in validation dataset (260 trees from 57 plots) were used to predict random tree and plot components, obtaining the calibrated conditional diameter increment attained for those trees during the second period. Results were compared with the measured increments for that second five year period. Comparison statistics were modelling efficiency (EF, defined as 1 minus the ratio between the sum of squares of the error and the corrected sum of squares for the variable), mean residual error and root mean square error (RMSE), for both residual error ( $id_5 - \hat{id}_5$ ) and relative error ( $(id_5 - \hat{id}_5) / \hat{id}_5$ , with reference to the original

non transformed variable. Calibrated predictions were also compared with the marginal predictions obtained using the model including only fixed effects, and with the classical approach assuming tree diameter increment to remain constant for a ten year period ( $id_{-5} = id_{+5}$ ).

Another possibility tested was the *standwise calibration* (Lappi 1986), where random plot components predicted from the prior measurement of a small sample of trees per plot were used to predict the increment of the trees within the plot not utilized in calibration. In this case the 45 plots from validation dataset containing five trees with increment measurements available for the two growth periods were utilized (225 trees). Random plot components were predicted using prior measurements from 1, 2 or 3 trees per plot, and these components were then used to predict future increment for the remaining 4, 3 or 2 trees. For each option and plot, 500 random realizations were performed, including a different sub-sample of trees at each one.

Errors computed within each realization were divided into a mean value, a between plot component (with variance  $Sd_B^2$ ) and a within plot component (with variance  $Sd_w^2$ ), including residual, tree and period components. Average values for each component were computed over the 500 realizations. Root mean square error was then computed as the root of the sum of the squared mean error and the variance components of the error averaged for each option (Lappi 1986). Modelling efficiency for the different options was computed as the average over the realizations. When analysing modelling efficiency, it is necessary to consider that the number of trees involved in each calibration option is different. Statistics were computed for both the residual and the relative errors.

## 4 Results

### 4.1 Diameter Increment Modelling

After testing different single tree size variables, the basic increment model included the logarithm of the breast height diameter,  $\log d$ , as a single explanatory variable. The intercept for the model

was then divided into a fixed part, specific to the population, and random between-tree, between-plot and between-period components, specific to each unit. The coefficient associated with  $\log d$  also varied randomly between plots. The expression for the basic increment model is therefore:

$$\log(id_{5\ jik} + 1) = \mu + u_{1i} + v_{ij} + w_k + (\eta + u_{2i}) \log d_{ijk} + \varepsilon_{ijk} \quad (3)$$

Where  $\mu$  and  $\eta$  are fixed parameters, common to the population;  $v_{ij}$  is a random tree parameter, specific to the  $j$ th tree within the  $i$ th plot, with mean zero and variance  $D_v = \sigma_{ij}^2$ ;  $w_k$  is a random period parameter, specific to the observations taken during the  $k$ th period, with mean zero and variance  $D_w = \sigma_k^2$ ;  $(u_{1i}, u_{2i})^T$  is a vector of random plot parameters, specific to the  $i$ th plot, with mean  $(0\ 0)^T$  and variance matrix

$$D_u = \begin{pmatrix} \sigma_{1i}^2 & \sigma_{12i} \\ \sigma_{12i} & \sigma_{2i}^2 \end{pmatrix};$$

$\varepsilon_{ijk}$  are within tree residual error terms. Simpler structures for  $D_u$  were tested, but they did not allow convergence of the model. Results obtained after fitting model in Eq. 3 are included in Table 3.

Although independence between random components specific to different sampling units has been assumed previously, independence between random period components is a matter for discussion. As is shown in Table 2, there are several years overlapping between periods corresponding to different regions, and we should evaluate whether there is any level of dependence associated with common growth years.

To do this, in a first approach, random period components were not considered independent between regions. In this case, components in  $D$  expressing covariance between random components associated with overlapping periods were supposed to be proportional to the number of overlapping years. Results from fitting the basic model (3) under this variance structure were compared to those obtained by solving the simpler structure assuming independence between period effects (Table 3). Since covariation terms were identified as non-significant, and model fit was not improved, the simpler structure was selected.



**Table 3.** Comparison of fitting statistics and estimated variance components of the models with different alternatives of covariates inclusion, residual variance function and variance components estimation method.

Model	1	2	3	4	5	6	7
Intercept	$\mu$	$\mu$	$\mu$	$\mu$	$\mu$	$\mu$	$\mu$
Single tree	$\log d$	$\log d$	$\log d$	$\log d$	$\log d$	$\log d$	$\log d$
Stand	–	–	–	$H_0, SI, \log(N)$	$H_0, SI, \log(N)$	$H_0, SI, \log(N)$	$H_0, SI, \log(N)$
Competition	–	–	–	–	didg	didg	didg
Region	–	–	–	–	–	cat, cat*SI	cat, cat*SI
Residual	$\lambda_1$	$\lambda_1$	$\lambda_1 + \lambda_2 \log d$	$\lambda_1 + \lambda_2 \log d$	$\lambda_1 + \lambda_2 \log d$	$\lambda_1 + \lambda_2 \log d$	$\lambda_1 + \lambda_2 \log d$
–2LL	419.50	428.92	283.392	20.0489	5.8058	–10.4184	–10.1283
Method	ML	ML	ML	ML	ML	ML	REML
$\sigma_{ij}^2$ (tree)	0.0141	0.0141	0.0143	0.0135	0.0131	0.0132	0.0132
$\sigma_{1i}^2$ (plot)	0.4689	0.4713	0.3920	0.2101	0.1861	0.1607	0.1681
$\sigma_{2i}^2$ (plot)	0.0336	0.0337	0.0235	0.0161	0.0142	0.0123	0.0131
$\sigma_{12i}$ (plot)	–0.1181	–0.1187	–0.0904	–0.0550	–0.0483	–0.0415	–0.0437
$\sigma_k^2$ (period)	0.0081	0.0522	0.0083	0.0072	0.0073	0.0052	0.0068
$\sigma_{12k}$ (period)	–	NS	–	–	–	–	–
$\lambda_1$ (residual)	0.0379	0.0379	0.1140	0.1140	0.1154	0.1156	0.1153
$\lambda_2$ (residual)	–	–	–0.0244	–0.0242	–0.0246	–0.0246	–0.0244

Where  $\log d$ : log breast height diameter,  $H_0$ : dominant height,  $SI$ : site index,  $N$ : stand density, cat: dummy variable which values 1 if the tree comes from Catalonia, and 0 if not, didg: ratio between breast height diameter and plot mean square diameter, ML: maximum likelihood, REML: restricted maximum likelihood, –2LL: –2 times logarithm of likelihood,  $\sigma^2$ = variance terms,  $\sigma_{12}$  = covariance parameters, NS: all covariance terms associated with random period parameters are non-significant, – = not included

## 4.2 Residual Variance Function

Mean value for the variance of the  $\varepsilon_{ijk}$  conditional residual terms obtained after fitting model (3) was computed per different classes of dbh and  $\log d$  and plotted against them. Despite logarithmic transformation, a clear pattern of decreasing variance associated with increasing diameters was still detected. This is an expected behaviour probably explained because the growth of small trees is still influenced by establishment and initial conditions not considered in the model. Several functions of model variance were tested, and the structure finally selected for the variance function was:

$$\sigma_{\varepsilon_{ijk}}^2 = \lambda_1 + \lambda_2 \log d_{ijk} \quad (4)$$

Where  $\lambda_1$  and  $\lambda_2$  are fixed parameters jointly estimated with the rest of the components of the model. A large decrease in –2LL (Table 3) compared to the model with constant residual variance indicated that the inclusion of the variance function improved model fit.

## 4.3 Covariates Inclusion

Several models, including different subsets of explanatory variables, were evaluated. Table 3 only shows fitting statistics for those models showing the best performance for each proposal of variables type inclusion. Parameter estimates for the diameter increment models have biological sense and are significant at 0.05 level.

Inclusion of a specific effect for each region was not considered necessary, since the parameters for these effects were non-significant. Better results were obtained when including a single regional effect specific to Catalonia, influencing the intercept and site index.

The definite diameter increment model for stone pine trees growing in the four analysed regions was:

$$\begin{aligned} \log(id_5 + 1)_{ijk} = & 2.2451 - (0.2615 + u_{2i}) \log d_{ij} - 0.0369 H_{0i} - \\ & 0.1368 \log(N_i) + 0.0448 SI_i + 0.1984 \text{didg}_{ij} - \\ & 0.5542 \text{cat} + 0.0277 \text{cat} SI_i + u_{1i} + v_{ij} + w_k + \varepsilon_{ijk} \end{aligned} \quad (5)$$

Units:  $id_5$ ,  $d$  (cm);  $H_0$ ,  $SI$  (m);  $N$  (stems/ha).

Rest of variables are non-dimensional.

Where

cat = 1 if the observation comes from Catalonia;

cat = 0 if not

$$v_{ij} \sim N(0, 0.0132); w_k \sim N(0, 0.0068);$$

$$\varepsilon_{ijk} \sim N(0, \sigma^2_{e\ ijk} = 0.1153 - 0.0244 \log d_{ijk});$$

$$(u_{1i}, u_{2i})^T \sim N \left[ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.1681 & -0.0437 \\ -0.0437 & 0.0131 \end{pmatrix} \right]$$

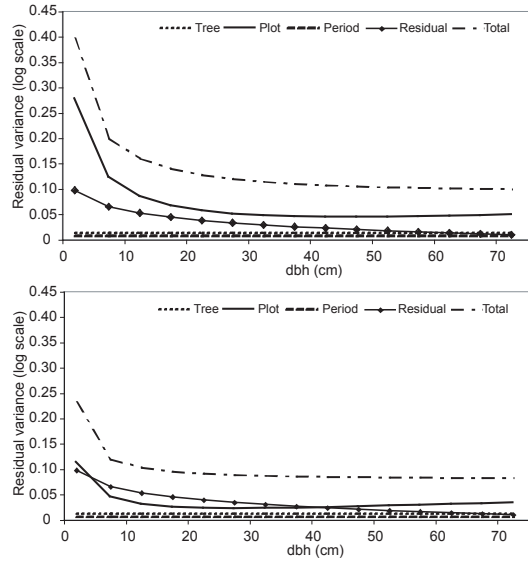
Fig. 2 shows how between-plot, between-tree, between-period and residual within-tree effects account for total variation in both the basic model including only  $\log d$  as an explanatory variable and the final model including the rest of covariates. In both cases, variance is expressed as a function of tree diameter. In the basic model, between-plot random effect accounted for most of the total variation. After inclusion of explanatory variables, total variability decreased, associated with a large decrease in between plot variability. A higher level of variability is therefore associated with within-tree residual variance. Variances associated with tree and period effects were smaller than those estimated for between plot and within tree residual random effects in both cases.

The bias of the model was analysed by plotting the mean value and the standard error for the residuals of the fixed part of the model (Hynynen 1995, Hökkä et al. 1997, Mabvurira and Miina 2002), in logarithmic scale and real scale, as a function of the predicted and the predictor variables included in the model (Figs. 3a and 3b).

No noticeable trend between the residuals and the explanatory variables was detected. Significant bias was only shown for those trees with the largest predicted diameter increment, i.e. those trees with the smallest values of original breast height diameter and dominant height.

#### 4.4 Anti-Logarithmic Transformation for the Dependent Variable

To revert the logarithm of the predicted increment into the arithmetic scale, it is necessary to include the bias correction multiplicative factor  $[\exp(s^2/2)]$  proposed by Flewelling and Pienar (1981), where  $s^2$  is the total variance for the prediction. If we are transforming a fixed effects marginal prediction, total variance is the residual



**Fig. 2.** Effect of between-plot, between-trees, between-period and residual within-tree variability over total variation for the basic model (Eq. 3) (above) and the final model (Eq. 5) (below).

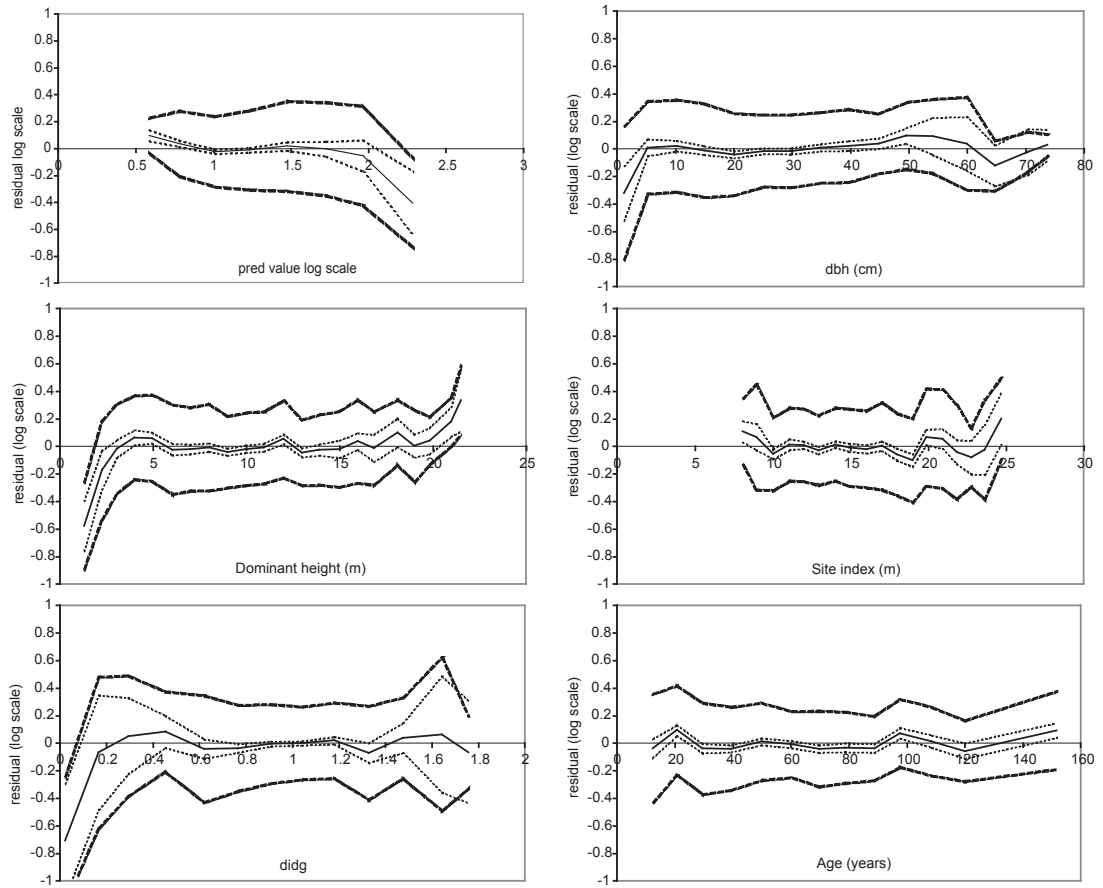
variance plus the variance components associated with the random effects:

$$s^2_{ijk} = \sigma^2_{ij} + \sigma^2_{1i} + \sigma^2_{2i} (\log d_{ijk})^2 + 2 \sigma_{12i} \log d_{ijk} + \sigma^2_k + \lambda_1 + \lambda_2 \log d_{ijk} \quad (6)$$

If values to transform are calibrated conditional predictions, prediction variance is computed using the following expression:

$$s^2_{ijk} = \text{var}(b - \hat{b}) + \sigma^2_{e\ ijk} = z_{ijk} [\hat{Z}^T \hat{R}^{-1} \hat{Z} + \hat{D}^{-1}]^{-1} z_{ijk}^T + \lambda_1 + \lambda_2 \log d_{ijk} \quad (7)$$

where the first term on the right hand side expresses the prediction variance for the random components, and  $z_{ijk}$  is the row of  $\hat{Z}$  associated with the observation. On several occasions, some of the predicted random components are excluded from the model, e.g. random period effects predicted from past measurements should not be included in future predictions. In this case, prediction variance  $s^2_{ijk}$  is computed using Eq. 7 again, but removing from  $z$ ,  $\hat{Z}$  and  $\hat{D}$  all the components and terms associated with the excluded random effects, and adding to the equation the values for the variance components associated with these omitted effects.



**Fig. 3a.** Mean residual in log scale (solid line) for the fixed effects model as a function of predicted value and explanatory stand and tree level variables. Dotted lines indicate standard error for the mean, and dashed lines indicate standard deviation.

## 4.5 Prediction

### 4.5.1 A Case Study of Calibration

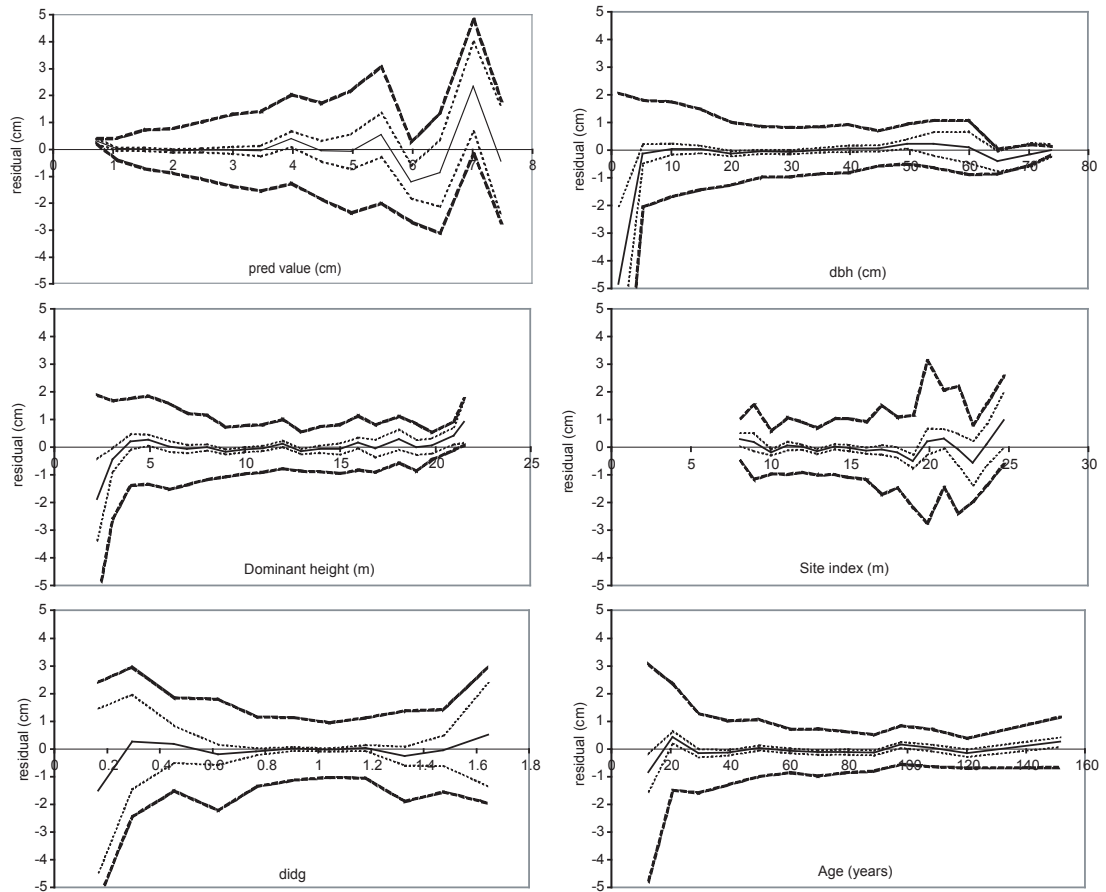
To demonstrate the calibration task we will describe the simplest case, where the previous five year increment for a tree is available, and it is used to calibrate the future increment. Let us use tree number 12 from plot 21116, located in West Andalusia. Tree and plot characteristics, measured on two occasions, are shown in Table 4. In this example we want to obtain the predicted values of  $dbh_{+5}$ ,  $id_5$  and  $\log(id_5+1)$  for the second growth period, using the rest of variables included in the table.

#### *Fixed effects marginal prediction:*

In this case, measurements from the first period are not considered. Since all the variables included in the model are known for the second period, the value for  $\log(id_5+1)$  was computed from Eq. 5, taking as values for random components their expected mean values, i.e., zero. Predicted value for  $\log(id_5+1)$  is 1.3954. Standard error for this individual prediction is computed as the square root of the total variance, given by Eq. 6:

$$\text{var}(y_{ijk} - \hat{y}_{ijk}) = s^2_{ijk} = 0.07378 \tag{8}$$

Where  $y$  and  $\hat{y}$  are observed and predicted value for  $\log(id_5+1)$  respectively. The value for standard



**Fig. 3b.** Mean residual in real scale (solid line) for the fixed effects model as a function of predicted value and explanatory stand and tree level variables. Dotted lines indicate standard error for the mean, and dashed lines indicate standard deviation.

**Table 4.** Main characteristics for tree 12 from plot 21116, used in the case study of calibration, measured at the beginning of the growth periods.

Period	$dbh_0$ (cm)	$dbh_{+5}$ (cm)	$id_5$ (cm)	$\log(id_5+1)$	Age (years)	Density (stem/ha)	$SI$ (m)	$H_0$ (m)	$didg$
1988–1992	24.990	31.250	6.259	1.9823	37	48	17.63	9.58	0.7746
1993–1997	31.250	35.162	3.912	1.5916	42	48	17.63	10.50	0.9011

Where  $dbh_0$  and  $dbh_{+5}$ : breast height diameter at the beginning and the end of the period,  $id_5$ : diameter increment attained during the period,  $SI$ : site index,  $H_0$ : dominant height,  $didg$ : ratio between breast height diameter at the beginning of the period and mean square diameter for the plot.

error is 0.2716, and the 95% confidence interval for prediction is [0.8521, 1.9386]. To transform this prediction into real predicted diameter increment  $\hat{id}_5$ :

$$\hat{id}_5 = [\exp(0.5s^2_{ijk}) \cdot \exp(\hat{y}_{ijk})] - 1 \tag{9}$$

In real scale, predicted value is 3.1882 cm, with confidence interval [1.4327, 6.2105].

**Table 5.** Comparison between calibrated model, fixed effects model and classical approach of constancy in increment, for the calibration approach including random plot and tree components over the 260 trees in validation data set (*single tree calibration*).

Variable	Statistic	Fixed effects	Calibrated response: tree and plot effects	Constancy in increment ( $id_{.5} = id_{+.5}$ )
$id_5 - \hat{id}_5$	SSE	500.03	352.49	565.55
	EF %	27.26	48.72	17.73
	RMSE (cm)	1.389	1.166	1.477
	Mean error	0.085	0.208	0.177
$(id_5 - \hat{id}_5) / \hat{id}_5$	SSE	73.57	50.89	99.23
	RMSE (%)	53.2	44.3	61.9
	Mean error	0.044	0.061	0.188

Where SSE: sum of squares of the error; EF: modelling efficiency; RMSE: root mean square error

### Calibration using prior measurements

#### – Conditional prediction:

Using data from a single prior measurement it is possible to obtain predictions for the vector  $\hat{b}$  containing random plot, tree and period components specific to this tree. To do this Eq. 2 was used, where in this case:

$$\hat{Z} = [1, 1, \log(24.990), 1] \quad (10)$$

$$\hat{D} = \begin{pmatrix} 0.0132 & 0 & 0 & 0 \\ 0 & 0.1681 & -0.0437 & 0 \\ 0 & -0.0437 & 0.0131 & 0 \\ 0 & 0 & 0 & 0.0068 \end{pmatrix} \quad (11)$$

$$\hat{R} = 0.1153 - 0.0244 \log(24.990) = 0.0367 \quad (12)$$

$$\hat{e} = 1.9823 - 1.4627 = 0.5196 \quad (13)$$

The BLUP for the vector of random parameters so obtained is [ $u_{1i} = 0.1799$ ,  $u_{2i} = -0.0101$ ,  $v_{ij} = 0.0865$ ,  $w_k = 0.04457$ ]. To predict calibrated increment for the future, random period component  $w_k$  was not included in Eq. 5. Predicted value for  $\log(id_5 + 1)$  is therefore 1.6272. Total variance for the prediction is given by Eq. 7:

$$s^2_{ijk} = 0.05479$$

Standard error for the prediction is 0.2340, with a 95% confidence interval [1.1590, 2.0953]. Real value for predicted diameter increment is 4.2309 cm, with a 95% confidence interval [2.2754, 7.3539].

As shown, calibration has decreased both the absolute value for prediction error (0.7235 cm in fixed effects response against 0.3190 cm in calibrated) and the standard error (0.2716 to 0.2340), narrowing the confidence interval for the prediction. Mean predicted values are not centred within its confidence interval as a consequence of anti-logarithmic transformation.

### 4.5.2 Calibration Alternatives

#### Single Tree Calibration

Table 5 compares the results from the marginal fixed effects prediction with the calibrated predictions including both tree and plot random components, and with the method that assumes the past five year increment equalling future five year increment.

Results from calibrated conditional predictions are significantly better than those attained with the fixed effects model, with modelling efficiency being increased from 27.2 to 48.7%, and RMSE being reduced from 1.39 to 1.16 cm. On the other hand, calibration results in a slight bias in mean error when compared with the fixed effects model, probably derived from omitting random period components. In any case, the calibrated and fixed effects predictions lead to a substantial improvement in predictive accuracy when compared to assumed constancy of the diameter increment during the two five-year periods analysed.

**Table 6.** Comparison between fixed effect model, and calibrated model using *standwise calibration* with different number of trees per plot. Each result is obtained as the average from 500 random realizations, including different trees in the calibration data set. Data from 45 plots in validation data set with 225 trees.

Variable	Statistic n	Fixed effects 225	Number of trees used in calibration		
			1 180	2 135	3 90
$id_5 - \hat{id}_5$	<i>EF (%)</i>	26.50	38.79	42.11	43.48
	<i>RMSE (cm)</i>	1.425	1.329	1.293	1.279
	<i>Sd<sub>B</sub></i>	1.052	0.887	0.831	0.812
	<i>Sd<sub>w</sub></i>	0.967	0.973	0.973	0.970
	Mean error	0.087	0.177	0.181	0.179
$(id_5 - \hat{id}_5) / \hat{id}_5$	<i>RMSE (%)</i>	54.5	52.1	50.1	49.1
	<i>Sd<sub>B</sub></i>	0.407	0.362	0.336	0.316
	<i>Sd<sub>w</sub></i>	0.364	0.370	0.369	0.371
	Mean error	0.034	0.055	0.055	0.053

Where n indicates the number of remaining tally trees used to estimate the statistics, *RMSE*: root mean square error, *EF*: modelling efficiency, *Sd<sub>B</sub>* and *Sd<sub>w</sub>* between plot and within plot standard deviation.

## Standwise Calibration

Table 6 shows the results of the second calibration approach. In this case, prior measurements of a sample of trees from each plot were used to predict random plot component and calibrate future predictions for the rest of the trees within the plot. As Fig. 4 shows, standwise calibration reduces RMSE when compared with the fixed effects model, even if only a single tree per plot is used for calibration. Standwise calibration reduces between stand variability, while the error associated with the rest of the random levels (tree, period and residual) remain practically constant. Mean error value for calibrated prediction is slightly increased if compared with the fixed effect model, as a consequence of excluding predicted random period and tree components.

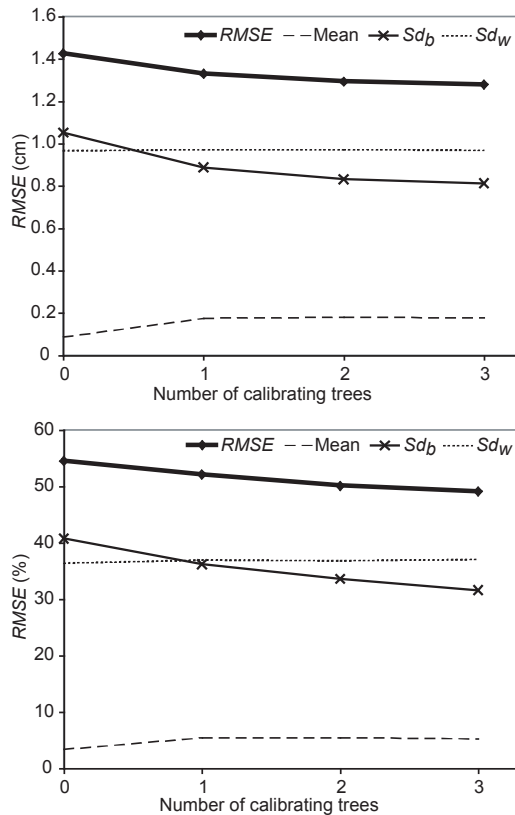
## 5 Discussion and Conclusions

In the present work, diameter increment for stone pine trees in Spain is described as a stochastic process, where a fixed, deterministic model, explains the mean value for the increment, while unexplained residual variability is described and modelled by including random parameters acting

at period, plot, tree and residual within-tree levels. Modelling diameter increment as a stochastic process considering random variability has already been proposed, among others, by Stage (1973), Wykoff et al. (1982), Hilt (1983), Henttonen (1990), Johnson et al. (1991), Miina (1993), Vettenranta (1999) and Palahí et al. (2003).

In the fixed part of the model the primary explanatory variable was size of the tree, defined by the logarithm of breast height diameter. Current tree size is a good indicator of future growth, since it includes both past competitive interactions and genotypic differences in response to environmental variability (Perry 1985). In the proposed model it was shown that diameter increment of a tree decreased as diameter increased.

Together with tree size, the fixed part of the model also included variables commonly used as predictors in single tree diameter growth models, such as the number of stems per ha. (West 1981), site index (Wykoff 1990, Palahí et al. 2003, Soares and Tomé 2003) and dominant height (Hynynen 1995, Mabvurira and Miina 2002). As a result of competition, larger stand densities are associated with smaller diameter increments. Negative value for the parameter associated with dominant height is related to the fact that in two stands with the same site quality, larger dominant height indicates older trees, and, consequently, smaller diameter



**Fig. 4.** RMSE, mean, between stand ( $Sd_b$ ) and within stand ( $Sd_w$ ) standard deviation, as a function of the number of calibrating trees, for residual error [ $id_5 - \hat{id}_5$ ] (above) and relative error [ $id_5 - \hat{id}_5$ ] /  $\hat{id}_5$  (below).

increments. Site quality is positively related to increment, indicating that in better sites trees are attaining larger diameter increments. Finally, a competition index denoting the social status of the tree within the stand is positively correlated with increment, which means that trees from the dominant stratum are expected to reach larger increments than those from other strata (Holdaway 1984, Pukkala 1989, Miina 1993).

In comparing the present model with the diameter increment model for stone pine growing on the Central Range proposed by Cañadas (2000), it is possible to see that the latter one also includes variables indicating stand density, such as crown competition factor (Krajicek et al. 1961); site quality (dominant height at 100 years); tree size

(crown ratio) and social status of the tree (BAL). As shown, the basic structure of the models is similar, although the covariates used to explain variability acting at different levels are not the same.

Smaller expected diameter increment was detected for the stands from Catalonia. In this region the presence of an understory layer of cork oak (*Quercus suber* L.) and other Mediterranean shrubs is quite common. Inter-specific competition from this layer, not considered in the model, could be a reason for these smaller increments. Another possible reason could be the fact that Catalonian increment data were obtained in a different manner than those from other regions.

The stochastic part of the model includes both nested hierarchical random effects (trees within plots), and crossed random effects (periods). The structure for the stochastic variability is therefore similar to that proposed by Henttonen (1990), Hökkä and Groot (1999) and Miina (2000). The largest components of residual stochastic variability, as stated in Fig. 2, were associated with both between-plot and within-tree residual effects. Between-plot variability can be due either to not considering interactions between plot variables and period effects (Henttonen 1990, Hökkä and Groot 1999), to ignoring ecological variables characterising site quality more adequately than site index or to not taking into account silvicultural treatments applied in the past (Hynynen 1995, Hökkä et al. 1997). Large within-tree residual variability can be due to human error in diameter increment measurement (Pukkala 1989, Palahí and Grau 2003), since increment cores cannot be taken on repeated occasions at the same height and orientation within the stem.

Inclusion of random period components within the model indicated differences in the growing pattern of the trees between the five-year calendar periods (Henttonen 1990, Miina 2000, Yeh and Wensel 2000). Independence between overlapping period effects indicated that growth period effects are different for different regions, meaning that climatic conditions for a single five year period cannot be considered common for different regions. It was not possible to analyse the dependence between overlapping observations coming from the same region, since this type of data were not available.

Finally, significant random variability among trees suggested that some tree variables were ignored in the model, e.g. factors related to genetics or microsite. The relatively small level of variability detected between trees growing in the same plot is in line with the results obtained by Mabvurira and Miina (2002), although Henttonen (1990), Hökkä and Groot (1999) and Palahí et al. (2003) detected a pattern of larger variability between trees than between plots. The main reason for this behaviour can be related to limitations from the dataset, since only two consecutive growth observations are available for the same tree, and part of the tree level variability was then included in the residual within-tree variance. In any case, a small tree level variability of growth is in line with the results also obtained by Calama (2004) for stem form, indicating that trees within the same stand have more similarities with each other than with trees growing in other stands.

Application of the fixed part of the model results in unbiased estimates of the increment, except for those trees whose initial breast height diameter is less than 5 cm. The results after applying the fixed part of the model to the 260 trees in the validation dataset indicated that 27% of the total variability was explained. We can therefore conclude that the fixed effects model results in a significant improvement when compared with the approach of assuming past increment as a good estimate for the future period, and to apply it, measurements from the past are not required.

Calibration results in significant improvement when compared with the results attained using only the fixed part of the model. Since variability explained by tree random components is quite small, results from single tree calibration are not much better than those from standwise calibration, although in the latter approach the number of trees used in calibration is smaller. With standwise calibration using past increment data from only one tree per plot (57 additional increment measurements in validation dataset), modelling efficiency reaches about 39% with RMSE 1.329 cm. Using single tree calibration, 260 additional measurements are required to obtain an efficiency value of 48.7% and RMSE 1.166 cm. Given the high costs of measuring past increments, it seems reasonable to select standwise calibration, although this decision should be based on a util-

ity model that considers both the cost of taking additional measurements and the benefits derived from improving accuracy.

The model developed in the present work can be considered a useful tool to simulate the growth of the trees. The ability of the calibrated model to be used in long term projections is based on the assumption of constancy in time for both random plot and tree components. Besides this, the possible effect of a reduction in stand density, as a consequence of natural mortality or silvicultural treatments, has not been properly addressed. On account of these two considerations, simulations with a projection length longer than 10 years should be considered cautiously.

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