Equilibrium Curves and Growth Models to Deal with Forests in Transition to Uneven-Aged Structure – Application in Two Sample Stands

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Stem number distributions in uneven-aged forests are assumed to be stable, if they follow special functions, e.g. de Liocourt's reverse J-shaped breast height diameter distribution. These distributions therefore are frequently regarded as a target in all-aged forests. Intending to convert an even-aged forest or any other forest, not yet exhibiting this sort of equilibrium, towards a steady state forest, the question rises, how to choose an appropriate equilibrium curve and how to achieve this stem number distribution by an appropriate thinning and harvesting schedule. Two stands are investigated: One dominated by Norway spruce (*Picea abies*), having developed from a 120 year old even-aged stand 25 years ago, after several "target diameter thinnings". The other one is a mixed species stand of Norway spruce, white fir (*Abies alba*), larch (*Larix europea*), common beech (*Fagus silvatica*), and Scots pine (*Pinus sylvestris*), having lost its typical uneven-aged structure 20 years ago. These stands were used, together with the distance independent individual tree growth model PrognAus, to reveal that 1) there are more than only one equilibrium curve per stand, 2) not every hypothesised equilibrium can be reached with any stand, 3) an equilibrium in stem number does not necessarily mean a stable species distribution, and 4) growth models provide an excellent help to decide between several equilibrium curves and harvesting schedules to reach them.

Keywords uneven-aged forests, growth models, stem number distribution, transformation
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1 Introduction

Breast height diameter (dbh) distributions in uneven-aged forests are assumed to be stable, if they follow special functions, e.g. de Liocourt's (1898) semi logarithmic distribution, or if they fulfil certain conditions (Schütz 1975). Although most frequently uneven-aged forests are thought to be of mixed species composition, these equilibrium conditions usually are regarded as a target in all-aged forests, irrespectively of the species distribution. Although the form of the target distribution is well defined, its parameters have to be defined for a specific stand. Further on silvicultural strategy, i.e. a thinning and harvesting schedule, must be found, which finally leads to the selected target.

For both, the parameterisation of the equilibrium, and the definition of a thinning and harvesting schedule, individual tree growth models promise to be an appropriate tool. By the aid of such models in a trial and error procedure, parameters and thinning schedules can be assumed, applied, simulated and checked, if they result in the assumed equilibrium dbh-distribution.

Using the individual tree growth model Prognaus (Ledermann 2001), and two example stands, this study investigates whether there are only one or more than one possible equilibrium curves for these stands.

2 The Equilibrium Curves

Equilibrium curves are dbh-distributions, which under certain assumptions are stable over time, by keeping to a given harvesting strategy. They are sustainable in a sense that the same thinning and harvesting schedule can be kept on and on, always again resulting in the same dbh-distribution. There are mainly two types of equilibrium curves having been presented in the literature: First, and oldest, de Liocourt's (1898) semi-logarithmic dbh-distribution, with the extensions by Susmel (1956), who defined its parameters depending on a sort of site index (*Statur*, i.e. the highest trees of a stand), and recently the extension by Cancino and von Gadow (2002), who showed how the parameters and characteristics of this dbh-distribution depend on each other mathematically. The second equilibrium model by Schütz (1975) shows that a stable dbh-distribution needs not necessarily to have a semi-logarithmic form, but rather has to fulfil the condition, that for each dbh-class, the number of trees growing into it from the next class below, must equal the number of trees growing out from it to the next dbh-class above, plus the number of trees harvested (and/or died) in the subject dbh-class. If this condition is met, the number of trees in each dbh-class stays constant over time, and thus, the dbh-distribution is stable.

2.1 De Liocourt's dbh-Distribution and Its Interpretation by Cancino and von Gadow (2002)

De Liocourt (1898) assumed that the ratio, q, between the stem numbers of two neighbouring dbh-classes is constant over the whole range of diameters in an uneven-aged, stable forest. This kind of dbh-distribution can be depicted as

$$\ln N = a + b \cdot \mathrm{dbh} \qquad (1)$$

with N, the number of trees in the breast height diameter class, dbh; a and b are coefficients.

Cancino and von Gadow (2002) showed, that the ratio q depends on the chosen class width, dw, and that such an equilibrium curve is defined by 1) the maximum dbh, 2) the residual basal area, i.e. the basal are after thinning, 3) q, and 4) the number of trees in the dbh-class with the largest dbh, which can be harvested at every growth period. Three of these four characteristics need to be defined, the missing one resulting from them by simple mathematics.

2.2 The Equilibrium Model of Schütz (1975)

Schütz's (1975) equilibrium curve is defined by 1) a sub-model, describing the proportion of stem numbers, e, being removed per growth period as a function of the dbh, and 2) the increment in dbh, id, as a stable function of the dbh, and possibly the basal area of larger trees, Gcum. The latter parameter is used as a measure of competition, or social rank of a tree in the same way as it was later on used in the stand prognosis model by Wykoff (1990).

Using these two models, the stable stem number n_i in the i^{th} dbh-class is calculated from

$$n_i = n_{i-1} \frac{p_{i-1}}{p_i + e_i} \quad (2)$$

with p_i the ingrowth rate from the i^{th} dbh-class to the next one, calculated from

$$p_i = id_i / dw \quad (3)$$

Since the equations $e = f_1(dbh)$, and $id = f_2(dbh)$ are derived from former observations of the stand, the only parameter needed to be defined is n_0, the stem number in the first (smallest) dbh-class.

3 The Growth Model

While de Liocourt's equilibrium curve does not need any growth model, Schütz's equilibrium model is based at least on a stand specific diameter increment model, which is gained from former observations of the stand. For the question of this investigation, which concerns the transformation of stands, not being in an uneven-aged equilibrium, it must be doubted if equations derived from conditions, which do not at all contain any "stable" dbh-distribution, can describe the way to reach, and to keep just this equilibrium. Especially, if changes in species composition and in stand density are taken into account, the simple diameter increment equation of Schütz (1975) will probably not be stable over the conversion period.

On the other hand there are several individual tree growth simulators available, claiming to be appropriate for even-aged as well as for uneven-aged forest management, e.g. Hugin (Söderberg 1986), Silva (Pretzsch et al. 2002), Bwin (Nagel 1999), Moses (Hasenauer 1994), and Prognaus (Ledermann 2001). Growth models have already been used to compare even-aged versus uneven-aged management (Groot 2002, Hanewinkel 2002). Because of its representative, Austrian data base, and the fact that it does not need any information on age, site index and spatial stem distribution, in this study the simulator Prognaus was used.

This simulator is based on the data of the Austrian national forest inventory 1981–1990, and contains

1) A basal area increment model (Monserud and Sterba 1996), predicting the five year basal area increment from site descriptors, individual tree dimensions, stand density and competition;
2) A height increment model (Schieler 1997), predicting individual tree height increment from the dbh, the height and the dbh-increment of the tree;
3) A mortality model (Monserud and Sterba 1999), predicting the probability of a tree to die within the next 5 years, depending on its size and competition, and – especially important for this study;
4) An ingrowth model (Ledermann 2002), predicting the probability of a new tree entering the system by exceeding the minimum dbh of 5 cm.

Because PrognAus is an individual tree model, it is more appropriate to deal with forests in transition, than mean stem models (Groot 2002). Its well parameterised ingrowth-model (Ledermann 2002) facilitates its use in conversion forests better than models which do not have this component (Hanewinkel 2002). PrognAus does not explicitly distinguish between trees, growing in even- and uneven-aged stands. But the data base contains plots from even-aged and uneven-aged stands, pure and mixed stands as well. Schadauer and Büchsenmeister (2004) show from the data of this inventory, that 45% of the harvested volume comes from uneven-aged management. They as well show that this proportion is increasing, thus there must be also stands represented in the data, where the management is shifting from even-aged to uneven-aged management. The model recognises a tree growing under uneven-aged conditions from certain combinations of the independent variables in the model. Trees in uneven-aged stands of a given dbh, being subject to the same competition (measured in terms of stand density and basal area of larger trees), e.g. have larger crown ratios and smaller height diameter ratios if they are growing under uneven-aged conditions. Both, the crown ratio and the height diameter ratio, are predictor variables for growth in this simulator. That's why a similar simulator, Stage's (1973) stand prognosis model, has successfully been used to evaluate uneven-aged management (Haight and Monserud 1990a, b), and to create yield tables for even-aged stands (Stage et al. 1988).

The parameters of all the sub-models had been estimated species specific. Those of the basal area increment model and the mortality model have been estimated through simultaneous regression methods (Hasenauer 2000), those of the height increment model by linear regression, and the ingrowth model by logistic regression methods.

4 Material and Methods

4.1 The Sample Stands

The data for this study consists of two sample stands, describing two typical situations. One, the *Hirschlacke*-stand, is an older, even-aged, nearly pure Norway spruce stand, which was intended to be converted to an uneven-aged stand with a stable dbh-distribution, and the other one, the *Rosalia*-stand, is a mixed species stand, once having a typical Plenterwald-structure, the tending of which was neglected and now was intended to be regained. A short description of the main stand characteristics of both stands, at the time of the last observation, where the simulations were started, are given in Tables 1 and 2.

4.1.1 The Hirschlacke-Stand

The *Hirschlacke*-stand is situated in the northwestern corner of Austria, near the borders to the Czech Republic in the north and to Bavaria, Germany in the west. When it was established as a research stand in 1977 it was a 120 years old Norway spruce (*Picea abies* L. Karst.) dominated stand. Afterwards it was treated by target diameter harvesting (Reininger 1987), in order to become a more structured, uneven-aged stand. After 25 years of treatment and observation, at least the increase of stem number in the two smallest dbh-classes indicates the dbh-distribution approaching a more uneven-aged structure (Fig 1). The data of this stand, used in this study are breast height diameters and the heights of all trees with dbh ≥5cm.

Table 1. Characteristics of the investigated stands at the last observation in 1997 (*Hirschlacke*) and 2001 (*Rosalia*) respectively. iv is the periodic annual volume increment.

Stand	Area, ha	Dominant height, m	Volume, m^3ha^{-1}	Age, years	iv, $m^3a^{-1}ha^{-1}$
Hirschlacke	3.47	34.1	675	145	11.5
Rosalia	2.25	28.5	374	~	5.0

Table 2. Species proportions (% basal area) of the investigated stands at the last observation.

Stand	Species				
	Spruce	Fir	Larch	Pine	Beech
Hirschlacke	84.8	5.6	0.8	0.5	8.3
Rosalia	44.4	0.3	6.1	22.2	27.1

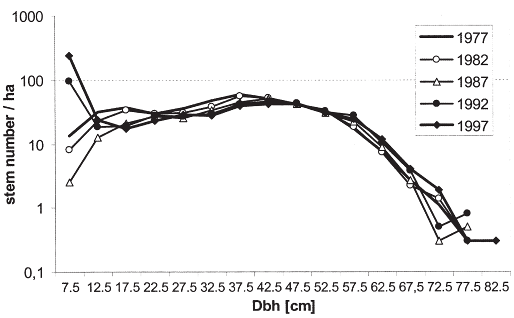

Fig. 1. The development of the dbh-distribution in the *Hirschlacke*-stand since 1977.

4.1.2 The Rosalia-Stand

The *Rosalia*-stand is situated in the Rosalia Mountains in the eastern part of Austria at the northern border between Lower-Austria and Burgenland. It has been a mixture of mainly Norway spruce, Scots pine (*Pinus sylvestris* L.), some larch (*Larix decidua* Mill.) and common beech (*Fagus silvatica* L.). It was – before 1980 – treated in a sort of uneven-aged management by its former owner, who was a farmer, owning only a few small woodlots, enclosed within a large forest area owned by the Austrian Federal Forests. Later on the ownership shifted to the Austrian Federal Forests, and the uneven-aged management was abandoned. Since its establishment in 1981, the management of the stand apparently did not improve its

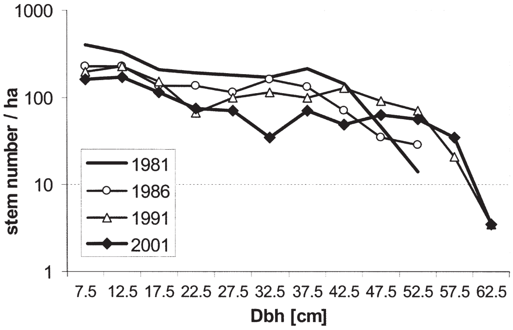

Fig. 2. The development of the dbh-distribution in the *Rosalia*-stand since 1981.

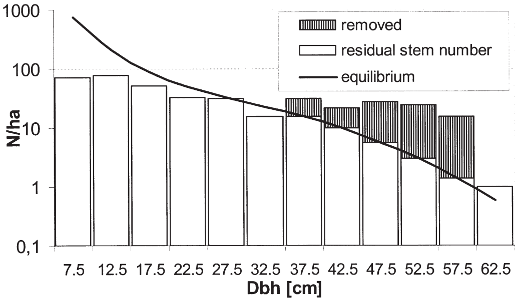

Fig. 3. The procedure by which the "equilibrium" stem number distribution is approached by removals in the respective dbh-classes.

structure towards a more uneven-aged structure (Fig. 2). The data of this stand, used in this study have been assessed in 9 circular plots with radius 15 m. This is only about half the amount of trees that should be observed according to Kerr et al. (2003), but nevertheless nearly 30% of the total stand area of 2.2 ha.

4.2 The Simulated Stand Treatments

For each of these two stands, several targeted equilibrium distributions were defined. Then the simulator PROGNAUS was used. In 10 year steps, the actual dbh-distribution (class width 5 cm) was compared with the target distribution, and if the stem number in a dbh-class exceeded the targeted stem number, it was reduced to the target stem number by thinning (Fig. 3).

After the removal of the trees, the remaining trees were grown by the simulator, the mortality model and the ingrowth model were applied in two 5-year periods, and then the whole procedure was repeated again until a simulation period of 100 years was reached.

As a measure for the deviation of an actual dbh-distribution from its target, the variance of the logarithmic stem number around the target was used according to Zingg and Duc (1998):

$$\mathrm{var} = \frac{\sum\left(\ln N_{actual} - \ln N_{target}\right)^2}{k} \tag{4}$$

with N_{actual}, the actual stem number of the respective dbh-class, N_{target}, the stem number of the target distribution in the respective dbh-class, and k, the number of dbh-classes. Zingg and Duc (1998) defined an actual dbh-distribution with $var < 0.5$ as stable, on with $0.5 < var < 1.0$ as critical, and one with $var > 1.0$ as instable.

For each of the two stands six equilibrium models were chosen. The Schütz-models, differing by the stem-number in the lowest dbh-class (Table 3a), and three semi-logarithmic models, each of them defined by different combination of residual basal area, q-value, and maximum breast height diameter (Table 3b).

Table 3a. Equilibrium models according to Schütz, selected for this study, defined by the stem number in the lowest dbh-class, n_0.

	Stand					
	Rosalia			*Hirschlacke*		
n_0	100	350	750	100	200	350

Table 3b. Equilibrium models according to de Liocourt, Cancino and Gadow

Stand	Steering parameter: Residual basal area, m²/ha	Maximum dbh, cm	q
Rosalia	25	60	1.2
	20	45	1.3
	40	60	1.1
Hirschlacke	20	45	1.3
	30	50	1.3
	40	60	1.1

5 Results

For interpretation, for each stand and for each equilibrium model the development of the periodic volume increment by tree species, the development of the stocking volume after thinning, the quadratic mean diameter of the removed trees,

$$dg = \sqrt{\frac{\sum dbh^2}{n}} \quad (5)$$

with *dbh*, the breast height diameters of the removed trees, and *n*, the number of removed trees, and the dbh-distribution after 100 years were evaluated. As an example, for the *Hirschlacke*-stand two targets can be compared (Figs. 4 and 5), where in the first case, the target was about achieved, while in the second case (Fig. 5), the targeted equilibrium could not be achieved. In another example (Figs. 6 and 7) for the *Rosalia*-stand it is shown, that two very different targets can be met.

In order to compare the results of all 6 simulation runs for every stand, the following characteristics were calculated and given in Table 4: For the final stand after 100 years, the variance *var* (Eq. 4), as an indicator for the stability of the dbh-distribution. For the last 30 years the proportions of fir, pine + larch and beech by basal area, the average periodic volume increment, the quadratic mean diameter of the removed trees, and the stocking volume. For these figures, additionally it is indicated, if they increased, decreased or were about stable during the last 30 years.

Table 4. Results of the simulated stand treatments after 100 years. *V* is stocking volume (m^3/ha), *iv* is periodic volume increment (m^3/a/ha), *var* is the variance around the target distribution at the end of the simulation period, BAres is the targeted residual basal area, *dg*, the quadratic mean diameter of the removed trees in cm. Species proportions (except spruce) are given in % of periodic increment. ↑ means that this figure is increasing during the last 30 years, ↓ means that it is decreasing, "–" means that this figure is about stable.

	Rosalia			n_0	*Hirschlacke*		
	100	350	750		100	200	350
Schütz							
var	0.001	0.613	0.299		0.137	0.425	1.909
iv	2.5–	6.0–	8.0–		6.0–	7.5–	7.5–
Fir%	12–	14–	20–		0–	0–	0–
Pine&Larch%	12–	4–	4–		0–	1–	0–
Beech%	3–	6–	11–		2↓	6↓	26↓
V	35–	100–	180–		200–	300–	400↓
dg	9–	13–	32–		40–	50–	62–
de Liocourts, Cancino & von Gadow							
BAres	25	20	40		20	30	40
Dbh_{max}	60	45	60		45	50	60
q	1.2	1.3	1.1		1.3	1.3	1.1
var	0.036	0.017	0.211		0.170	0.358	1.214
iv	9.5–	9.5–	10.0–		8.0–	9.0↑	6.0↓
Fir%	10↑	18↑	3↑		0–	0–	0–
Pine&Larch%	2–	2↑	4–		2↓	0–	0–
Beech%	27–	21↓	31		13↓	21↓	35–
V	350↑	250↑	450 –		250↑	300↑	200 –
dg	48–	40–	62–		40–	42–	62–

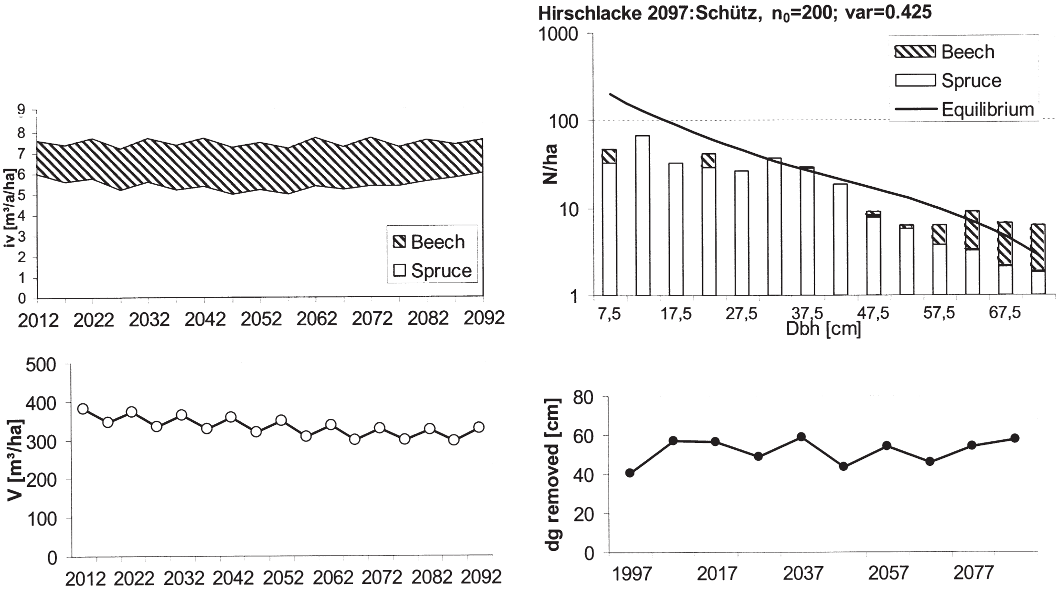

Fig. 4. The development of the *Hirschlacke*-stand after 100 years treatment targeting at a Schütz-equilibrium with $n_0 = 200$. The variance of the stem number distribution around the target, $var = 0.425$ indicates a stable distribution. iv is the current volume increment, V is the stocking volume, dg removed is the quadratic mean diameter of the removed trees.

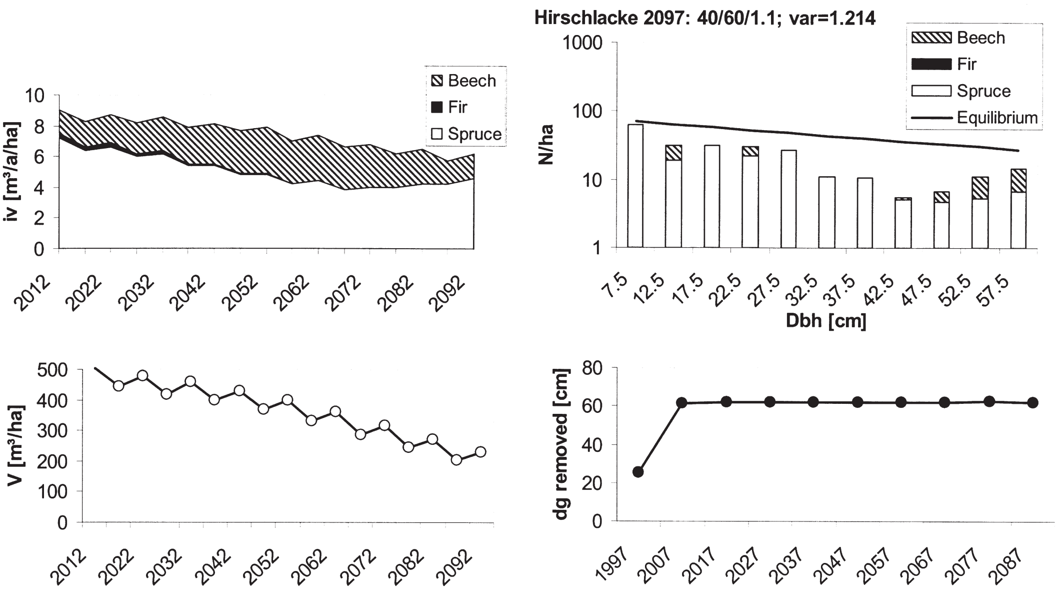

Fig. 5. The development of the *Hirschlacke*-stand after 100 years treatment targeting at a de Liocourt-equilibrium with a residual basal area of 40 m^2/ha, a maximum dbh of 60 cm, and $q = 1.1$. The variance of the stem number distribution around the target, $var = 1.214$ indicates an instable distribution. iv is the current volume increment, V is the stocking volume, dg removed is the quadratic mean diameter of the removed trees.

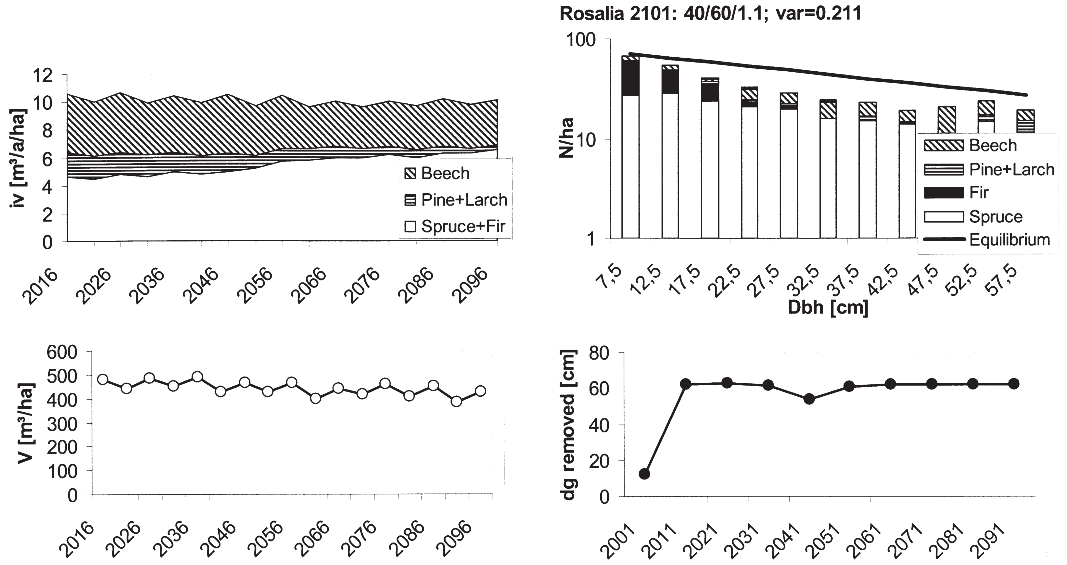

Fig. 6. The development of the *Rosalia*-stand after 100 years treatment targeting at a de Liocourt-equilibrium with a residual basal area of 40 m^2/ha, a maximum dbh of 60 cm, and $q = 1.1$. The variance of the stem number distribution around the target, $var = 0.211$ indicates a stable distribution. *iv* is the current volume increment, *V* is the stocking volume, *dg* removed is the quadratic mean diameter of the removed trees.

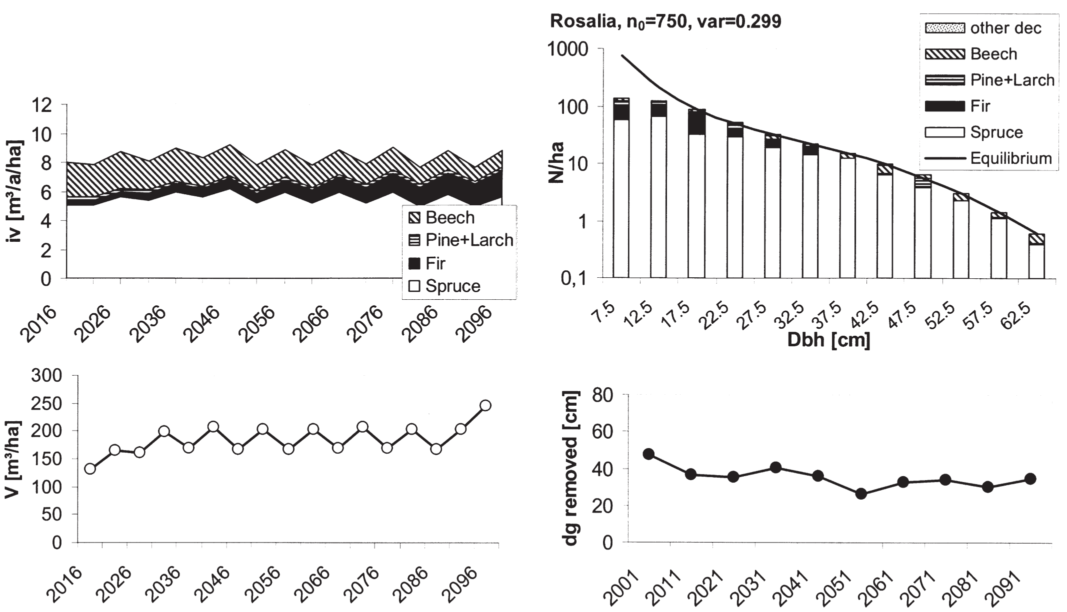

Fig. 7. The development of the *Rosalia*-stand after 100 years treatment targeting at a Schütz-equilibrium with $n_0 = 750$. The variance of the stem number distribution around the target, $var = 0.299$ indicates a stable distribution. *iv* is the current volume increment, *V* is the stocking volume, *dg* removed is the quadratic mean diameter of the removed trees.

6 Discussion

The following discussion is based on the assumption, that the underlying growth model is valid, and does, at least within reasonable limits, describe the real growth, mortality and ingrowth of the trees in the stands. This assumption is found to be justified, because of the underlying data, containing about 50% plots with some sort of uneven-aged management, and because PROGNAUS has been validated several times with very different and independent data sets (Sterba 1990, Sterba and Monserud 1996, 1997, Sterba et al. 2001, Sterba et al. 2002). One of this validation cases was the *Hirschlacke*-stand (Sterba et al. 2001), which since 25 years is treated to transform an even-aged stand towards an uneven-aged management. Of course, none of the plots and stands, used for validation, were uneven-aged stands observed for 100 years or more. Therefore the results of the simulations cannot replace long term experiments, but rather have the quality of scenario analyses. Nevertheless, simulations of different treatments do better meet the condition of identical site conditions, than any experiment may ever do.

A quick glance to the figures in Table 4 shows that, measured in terms of the variance, *var*, in both stands nearly all stated equilibrium curves were sufficiently reached after 100 years treatment. Only in the *Hirschlacke*-stand the Schütz's equilibrium with n_0 = 350, and the de Liocourt-model with a residual basal area of 40 m^2/ha, a maximum dbh of 60 cm and q = 1.1 were not achievable. There the final dbh-distribution was instable in terms of Zingg and Duc (1998). One more exception was the Schütz-equilibrium model with n_0 = 350 in the *Rosalia*-stand. There the final dbh-distribution is "critical" in terms of Zingg and Duc (1998), because the variance around the target distribution was *var* = 0.613 > 0.5. In two of these three examples also other figures, besides the variance indicate, that they did not result in an equilibrium after 100 years. In the *Hirschlacke*-stand with a targeted n_0 = 350 still the stocking volume and the proportion of beech is decreasing. This treatment (Table 4) restricts the harvest to the largest trees only ($dg_{removed}$ > 60 cm). Therefore, it takes a long time, until these large trees occur in the stand. During this time, the stocking volume increases, the ingrowth decreases, until there are enough large trees to harvest. If this is done, the volume decreases again, ingrowth increases, and this long term cycle starts again. In the same stand with the target residual basal area of 40 m^2/ha, a maximum dbh of 60 cm and q = 1.1, the current volume increment is not (yet) stable after 100 years. It still decreases during the last 30 years. There the site productivity seems too low to support a sufficient number of large trees and the high density, together with sufficient ingrowth.

On the other hand there are target-distributions, which are well met, measured in terms of the variance, *var*, but other figures indicate that they are not yet in an equilibrium. Especially the *Rosalia*-stand did, after 100 years, meet the required target of a de Liocourt-equilibrium with a residual basal area of 20 m^2/ha, a maximum dbh of 45 cm and q = 1.3, but during the last 30 years the proportion in beech is still decreasing in favour of fir, which maybe the reason why the stocking volume is still increasing. Anyhow the current volume increment seems to be stable at about 9.5 m^3/a/ha.

A comparison of the three Schütz-equilibriums in the *Rosalia*-stand interestingly shows, that all three of them are achievable within 100 years, but for sure not all of them are desirable. Only the one with n_0 = 750 delivers reasonable harvesting dimensions and a stable volume increment, probably near the site potential.

7 Conclusions

Provided the simulator PROGNAUS with all its submodels is valid, the six calculated scenarios in two stands, characterising very typical situations for stands to be converted to well structured uneven-aged stands, allow the following conclusions:

1) In a given stand several equilibrium dbh-distributions can be achieved,
2) stable dbh-distributions in terms of Zingg and Duc's (1998) equilibrium-criterion do not necessarily exhibit stable species distributions,
3) the growth-simulator provides an excellent tool to help deciding if a certain equilibrium can be achieved, and

4) to decide between different achievable equilibriums in terms of other important figures like volume increment, average harvesting dimensions and species distributions.

From only two examples, there should not be drawn too general conclusion on why a certain stated equilibrium cannot be reached in a certain stand. Nevertheless it seems that due to limited site productivity, targets which combine too high expectations on the size and number of large trees limit ingrowth, and then this equilibrium cannot be reached, because the treatment schedule desires to wait for large tree dimensions, and when they are reached, there is not enough ingrowth available. Appropriate growth simulators therefore have to include the fact, that ingrowth depends on site factors in another way than growth, and thus this equilibrium between growth, depending on density, and ingrowth is different on different sites. Only simulator that have this kind of different site dependence of ingrowth and tree growth can be used to answer the questions on which site, which equilibrium can be reached.

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