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A NONLINEAR SIMULTANEOUS EQUATION MODEL TO DETERMINE TAPER CURVE

PEKKA KILKKI and MARTTI VARMOLA

SELOSTE:

RUNKOKÄYRÄN MÄÄRITTÄMINEN EPÄLINEAARISEN SIMULTAANISEN MONIYHTÄLÖMALLIN AVULLA

Saapunut toimitukselle 1979-01-15

In the original set of equations derived by regression analysis, 10 relative-height diameters (endogenous variables) are presented as nonlinear functions of the other relative-height diameters and of the height of the tree (an exogenous variable). Any of the original equations can be replaced by an interpolation formula which links a measured diameter to the four closest relative-height diameters. The solution of the simultaneous equation model yields 10 relative-height diameters. Intermediate values are obtained by the interpolation formula. A Taylor's series correction is applied to avoid biases due to the nonlinearity of the simultaneous model equations.

1. INTRODUCTION

In our previous paper (Kilkki *et al.* 1978) a simultaneous equation model was introduced to determine taper curve for Scots Pine. The model consisted of 11 equations in which each relative-height diameter ($d_{.0h}$, $d_{.05h}$, $d_{.1h}$, $d_{.2h}$, $d_{.3h}$, $d_{.4h}$, $d_{.5h}$, $d_{.6h}$, $d_{.7h}$, $d_{.8h}$, and $d_{.9h}$) was predicted by the other diameters (endogenous variables), and by the height of the tree (exogenous variable).

The coefficients of the 11 equations were estimated by the ordinary least squares method. Unlike with many econometric applications of simultaneous equations (cf. e.g. Theil 1971) the use of the OLS method does not lead here to biased estimates. This is due to the fact that the measurement error in the diameters is practically zero. Consequently, the OLS estimates of the regression coefficients are unbiased.

In order to calculate intermediate values between the prefixed relative-height diameters an interpolation formula which uses 4 successive diameters was developed. This formula is applicable also when the diameter measurements are made at absolute heights of the tree. Then, the equation(s) representing the diameter(s) closest to the measured diameter(s) is (are) replaced in an iterative process by estimate(s) derived from the measured diameter(s). A computer program to facilitate this procedure was developed.

In the model described above, the relationships between the prefixed relative-height diameters were linear. In this paper we try to answer the question of whether the simultaneous equation model is applicable also in a situation where the relationships are nonlinear. The nonlinearity may be due either to the specific taper form of the tree species, or to the limited number of relative height diameters available in the analysis.

Acknowledgement. The authors are greatly indebted to Prof. Yrjö Vuokila and Mr. Jouko Laasasenaho who gave us the data, and to Mr. Risto Sievänen who provided us with the variance formula (18).

2. DATA

Data comprised 2 000 Scots Pines measured from artificially regenerated young pine stands in Finland. The measurements were made in the years 1968 ... 71 in connection with a growth and yield study. Altogether 100 plots with sizes varying from 225 to 2 000 m² were located in representative parts of the stands. The age of the trees on the measured plots varied from 12 to 35 years, dominant height from 3.7 to 10.1 meters, and number of trees per hectare from 1 327 to 24 067. 71 stands were established by seeding and 29 by planting.

From each plot 20 sample trees were selected subjectively. The aim was to cover the whole range of the breast height diameters on the plot. The dbh-height distribution of all sample trees is given in Table 1. Dbh and diameters $d_{.01h}$, $d_{.1h}$, $d_{.2h}$, $d_{.3h}$, $d_{.4h}$, $d_{.5h}$, $d_{.6h}$, $d_{.7h}$, $d_{.8h}$, and $d_{.9h}$ were measured at one millimeter's accuracy in two directions and the average of these measurements were used. The height and lower limit of the live crown of the tree were measured at the accuracy of 0.1 meter. Furthermore, the age and the breast height age of all trees were measured.

Table 1. Dbh-h distribution of the sample trees.
Taulukko 1. Koepuiden jakautuminen rinnankorkeusläpimitta-pituusluokkiin.

d, mm	h, m								
	2	3	4	5	6	7	8	9	10
10	9								
20	27	8							
30	5	63	19						
40		66	69	12	3				
50		16	118	51	14	1			
60		6	83	96	56	7			
70		1	50	101	82	37	4		
80			9	75	99	57	9	1	
90			1	40	90	65	22	2	
100			1	23	58	63	28	7	
110				4	28	52	30	6	
120				1	11	46	27	8	
130					10	19	24	7	5
140					3	10	8	11	1
150					1	2	11	5	
160						2	4	3	
170						1	2	2	1
180									1

3. REGRESSION MODELS

Preliminary analyses indicated that the residual variance of diameters was heavily dependent upon the size of the tree. Therefore, each diameter was divided by the height of the tree, and these new variables ($d_{.xh}/h$) (cf. CAJANUS 1911) were used in the regression analysis. This transformation resulted in satisfactorily homogeneous residual variances.

Unlike the previous study (KILKKI *et al.* 1978), the relationships between the diameter-height ratios did not appear to be linear. A sufficient transformation to linearize the regression models was to add the squares of the variables into the equations as predicting variables. After this was done no bias could be traced in the examination of the residuals.

The analysis of the residuals showed that in addition to the diameter-height ratios only the height of the tree was useful as a predicting variable in the regression models. Dbh might also have been useful but it was omitted in order to be able to apply the models when dbh is not measured. The regression coefficients, coefficents of determination and standard errors of estimates are given in Table 2.

4. INTERPOLATION

In order to use the simultaneous equation model in practice an algorithm estimating the intermediate diameters between the prefixed relative heights has to be developed. First, the intermediate diameter estimates are needed if the simultaneous model is to be applied to trees for which diameters other than the prefixed relative-height diameters are known. Then, these extra diameters have to be linked with the original set of diameters by an interpolation formula. Secondly, interpolation is needed to derive the continuous taper curve after the prefixed relative-height diameters are determined.

In the following, three interpolation formulas are suggested. In all of these formulas the basic diameter is derived from the straight line connecting two known diameters just above and below the required diameter. Other known diameters are employed to adjust this rough estimate.

The first interpolation formula based upon four successive diameters corresponds to »Intervallweise Hermite-Interpolation» (KIESEWETTER and MAESS 1974, p 148) if the distances between the diameters are equally long.

$$d_h = f3 + 1/2\ (f2 - f3)\ \left(\frac{h4 - h}{h4 - h3}\right)^2 + 1/2\ (f4 - f3)\ \left(\frac{h - h3}{h4 - h3}\right)^2 \qquad (11)$$

where $h3 \leq h \leq h4$

The explanation the variables is given in Figure 1.

The formula (11) ensures that there is no change in the first derivative of the

$$f1 = d1 - \frac{d2-d1}{h2-h1}\, h1 + \frac{d2-d1}{h2-h1}\, h = a1 + b1\, h$$

$$f2 = d2 - \frac{d3-d2}{h3-h2}\, h2 + \frac{d3-d2}{h3-h2}\, h = a2 + b2\, h$$

$$f3 = d3 - \frac{d4-d3}{h4-h3}\, h3 + \frac{d4-d3}{h4-h3}\, h = a3 + b3\, h$$

$$f4 = d4 - \frac{d5-d4}{h5-h4}\, h4 + \frac{d5-d4}{h5-h4}\, h = a4 + b4\, h$$

$$f5 = d5 - \frac{d6-d5}{h6-h5}\, h5 + \frac{d6-d5}{h6-h5}\, h = a5 + b5\, h$$

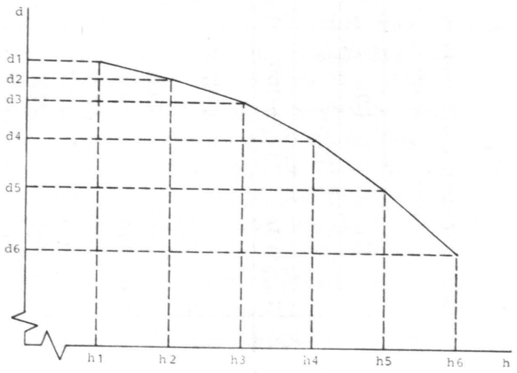

Figure 1. Calculation of straight lines connecting successive diameters.
Kuva 1. Peräkkäisiä läpimittoja yhdistävien suorien laskenta.

Table 2. Regression coefficients, coefficients of determination, and standard errors of estimates for the regression equations.
Taulukko 2. Regressioyhtälöiden regressiokertoimet, yhteiskorrelaatiokertoimet ja keskivirheet.

Independent variables *Selittävät muuttujat*	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Dependent variable — *Selitettävä muuttuja*									
	$d_{.01h}/h/100$	$d_{.1h}/h/100$	$d_{.2h}/h/100$	$d_{.3h}/h/100$	$d_{.4h}/h/100$	$d_{.5h}/h100$	$d_{.6h}/h/100$	$d_{.7h}/h/100$	$d_{.8h}/h/100$	$d_{.9h}/h/100$
$d_{.01h}/h/100$		−.1107	.1383	−.0706	−.0740	.1074	−.0727	−.0074	.0602	−.0216
$d_{.1h}/h/100$	.8806		.2850	.1606	.1179	−.2076	−.0290	.0901	−.0269	.0460
$d_{.2h}/h/100$	1.0855	.9082		.4782	−.0890	.3837	−.0619	−.1878	−.0815	.0120
$d_{.3h}/h/100$	−.4742	.5039	.6002		.6931	−.0790	.4108	.0583	−.0507	−.0717
$d_{.4h}/h/100$	−.2842	.0832	−.0767	.3311		.2885	.2711	−.1667	−.0164	.0919
$d_{.5h}/h/100$	.6858	−.7007	.4061	−.0032	.1183		−.1862	.3143	.0345	.1630
$d_{.6h}/h/100$	−.4606	.2795	−.3127	.3115	.1727	.2197		.7337	.1894	−.1262
$d_{.7h}/h/100$	−.0170	.1057	−.1364	.0133	−.1042	.3419	.4604		.2783	.1918
$d_{.8h}/h/100$	.6146	−.1600	.0846	−.1787	.1333	−.0354	.1341	−.1927		.2315
$d_{.9h}/h/100$	.6024	−.2277	.0182	.2131	−.0926	−.0703	−.1500	.1598	.2786	
$(d_{.01h}/h/100)^2$		.7758	−.3265	.2035	.1919	−.3581	.1891	−.0131	−.1682	.1354
$(d_{.1h}/h/100)^2$	.4279		.5477	−.3179	−.3936	.6250	.0507	−.3272	.0568	−.0977
$(d_{.2h}/h/100)^2$	−3.1503	−.3874		−.0964	.6209	−1.2196	.0870	.6437	.2634	−.0166
$(d_{.3h}/h/100)^2$	1.9264	−1.4185	−.4806		−.9246	.8751	−1.4319	−.3294	.1208	.1404
$(d_{.4h}/h/100)^2$	1.1982	−.4366	.7030	.1746		.3341	−.5539	.8656	.0218	−.5181
$(d_{.5h}/h/100)^2$	−4.3448	3.2720	−1.9096	.7045	1.0591		2.7350	−.9438	−.1370	−.8006
$(d_{.6h}/h/100)^2$	2.6368	−1.6090	1.5581	−1.7570	−.3196	.9294		−1.9391	−.3240	.5240
$(d_{.7h}/h/100)^2$	−.5150	−.9936	1.1096	−.3230	1.1089	−1.7103	−.5450		.4782	−1.1340
$(d_{.8h}/h/100)^2$	−7.1847	1.5940	−.8638	1.7650	−1.5429	.4311	.1650	6.2320		2.0111
$(d_{.9h}/h/100)^2$	−4.8413	4.3349	−.1799	−4.2782	.6985	.9374	1.8441	−1.9298	4.2294	
const.	−.0375	.0121	−.0053	−.0046	.0073	−.0018	.0051	.0010	.0102	.0143
h/100	.2460	−.0542	.0894	−.0565	−.0579	.0462	.0595	.0447	−.1102	−.3736
$(h/100)^2$	−1.5017	.6512	−1.1929	.3712	.6655	−.1828	−.3351	−.1212	1.1637	2.1427
R^2	.8863	.9709	.9782	.9722	.9569	.9326	.8877	.8020	.6716	.5236
$^{s}d/h/100$	.01275	.00563	.00439	.00434	.00458	.00474	.00487	.00483	.00454	.00406

taper curve in both ends of the segment (h3h4). Furthermore, the second derivative of the formula remains constant if the angle differences b2−b3 and b4−b3 are equal.

However, formula (11) may not use all the a priori information of the tree taper. If, for instance, there is no tapering between two successive diameters the use of formula (11) may yield a sigmoid curve between these diameters. Without specific studies it is impossible to predict the probability of this phenomenon.

Interpolation formula (12) is based upon the assumption that whenever there is no tapering between two successive diameters the taper curve must equal a straight line (see Kilkki *et al.* 1978). This interpolation formula also employs four successive diameters.

$$d_h = f3 + \frac{|b3|}{|b2| + |b3|} (f2 - f3) \left(\frac{h4 - h}{h4 - h3}\right)^m + \frac{|b3|}{|b3| + |b4|} (f4 - f3) \left(\frac{h - h3}{h4 - h3}\right)^n \qquad (12)$$

where $h3 \leqq h \leqq h4$

The explanation of the variables is given in Figure 1.

The values for parameters m and n should be determined from proper data. In our earlier paper the value 2.4 was applied in both cases. In this paper value 2 was used. When formula (12) was applied to derive dbh from the measured relative-height diameters an average overestimate of .0093 mm was found. This bias may be considered negligible. However, it might be argued that parameters m and n should be changed in accordance with the coefficient $\frac{|b3|}{|b2| + |b3|}$ and $\frac{|b3|}{|b3| + |b4|}$ in order to improve the symmetry of the model.

It may also be required that whenever three successive diameters fall on a straight line the tree taper must equal a straight line between these diameters. The interpolation formula then employs six successive diameters and is as follows:

$$d_h = f3 + \frac{|b3 - b4|}{|b1 - b2| + |b3 - b4|} (f2 - f3) \left(\frac{h4 - h}{h4 - h3}\right)^2 + \frac{|b2 - b3|}{|b2 - b3| + |b4 - b5|} (f4 - f3) \left(\frac{h - h3}{h4 - h3}\right)^2 \qquad (13)$$

where $h3 \leqq h \leqq h4$

Explanation of the variables is given in Figure 1.

The foregoing interpolation formulas are applicable to trees where 10 or more diameters are known. The relative efficacy of the models could not be ranked in this study. It may depend upon the tree species and upon the number of known diameters. The applicability of the models may also be influenced by whether the basic diameters are based upon measurements, or have already smoothed values.

In both ends of the tree stem the information needed for the interpolation formulas is inadequate. In this study only formulas (11) and (12) were tested and it was assumed that the slope of the imaginary segment below the first segment of the tree stem is twice that of the first segment, and that the slope of the imaginary segment above the last segment equals that of the last segment. These rough assumptions do not probably lead to any marked deviations from the true taper curve. Results from formula (11) were slightly better than those from formula (12). Consequently, formula (11) was applied in the final calculations.

5. SIMULTANEOUS EQUATION MODEL

51. Combination of the regression equations and the interpolation formula

The basic simultaneous equation model consists of the regression equations (1), ... , (10). Any diameter measured at the prefixed relative heights of the tree replaces the respective regression equation in the simultaneous equation model. The accuracy of the model is improved by the addition of each new diameter.

When diameters measured at other heights than the prefixed relative heights are employed, the interpolation formulas corresponding to these diameters must be incorporated into the model. It was decided to use interpolation formula (11) for this task. In our earlier paper (Kilkki *et al.* 1978) where the simultaneous equation model was based upon linear equations, the incorporation of the measured diameters was done by an iterative procedure developed by the authors. Since the regression models in this study are not linear, and consequently no simple solution of the system of equations is available, it was decided to combine both the regression equations and the interpolation formulas in the same simultaneous equation model.

In combining the regression equations and the interpolation formulas the number of endogenous variables and the number of equations must be kept equal. If, for instance, the breast height diameter is measured, the interpolation formula which links dbh to the neighbour diameters has to be added to the model. Consequently, one of the old equations must be dropped, most naturally the one that represents the relative-height diameter closest to the breast height.

52. Elimination of the biases of the system

If the equations of the simultaneous equation system are linear in respect to the endogenous variables and the coefficients of the equations are unbiased, the system yields unbiased estimates for the endogenous variables. Unfortunately, this does not hold true if the equations are nonlinear. Biases may occur since each endogenous variable is estimated as a function of the other endogenous variables, and the values of these variables are only estimates with certain error variances. Taylor's series gives a method to eliminate the bias. If, for instance, the following relationship exists between y and x:

$$y = x^2 \qquad (14)$$

and the expected value of x and its variance are $\hat{x}$ and $s^2_{\hat{x}}$, respectively, the unbiased estimate for y is:

$$\hat{y} = \hat{x}^2 + s^2_{\hat{x}} \qquad (15)$$

If y is the second or third degree function of x, the unbiased estimate for y is:

$$\hat{y} = f(\hat{x}) + 1/2 \frac{\delta^2 y}{\delta x^2} s^2_{\hat{x}} \qquad (16)$$

Formula (16) is a good approximation for a wide variety of functions if the variance $s^2_{\hat{x}}$ is reasonably small. The respective correction formula for a function with several right hand side variables is:

$$\hat{y} = f(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n) + 1/2 \sum_{i=1}^{n} \frac{\delta^2 y}{\delta \hat{x}_i^2} s^2_{\hat{x}_i} \qquad (17)$$

The regression equations which describe the relationships between the prefixed relative-height diameters are of the second degree. Consequently, formula (17) is directly applicable.

Since the relationship between the interpolated diameter and the relative-height diameters is linear in formula (11), no correction was needed for the interpolation formula.

53. Estimation of the error variance

In order to use formula (17) the error variance of each endogenous variable has to be known. If the equations were linear the variance of the i^{th} variable could be calculated by the following formula:

$$V_i = \sum_{j=1}^{n} \sum_{k=1}^{n} b_{ij} b_{ik} s_j s_k r_{jk} = \sum_{j=1}^{n} \sum_{k=1}^{n} b_{ij} b_{ik} cov_{jk} \qquad (18)$$

where

V_i = error variance of variable i

b_{ij} = ijth element of the inverse matrix B of the coefficient matrix A of the endogenous variables in the simultaneous equation model

s_j = standard error of the jth regression equation

r_{jk} = correlation coefficient between the residuals of the jth and kth regression equations

cov_{jk} = covariance between the residuals of the jth and kth regression equations

n = number of endogenous variables

Since the equations in our simultaneous model are nonlinear, the Jacobian matrix of the partial derivatives has to be employed as matrix A.

The use of formula (18) is simple if the diameters are measured at prefixed relative heights. Each regression equation corresponding to a measured diameter is replaced by the equation giving the equality between the diameter and its measured value. After these replacements the inversion of the new Jacobian matrix gives new elements b_{ij}. The standard error of the original regression equation is replaced by the standard error of the diameter measurement; this error frequently assumes the zero value.

If a diameter is measured at some other height than the prefixed relative heights, the nearest prefixed relative-height diameter is replaced by interpolation formula (11). Jacobian matrix is calculated from this new set of equations. Since the interpolation formula (11) is »exact», its standard error should equal zero, in theory. The calculations, however, showed that better variance estimates were obtained if formula (11) assumes the same standard error as the regression equation it replaces.

Standard errors of the regression equations are given in Table 2. To keep these figures comparable to those obtained from the simultaneous equation model, no corrections due to the loss of the degrees of freedom were made. The correlation matrix between the residuals of the regression equations is given in Table 3. The correlation coefficients indicate that there is a strong negative correlation between the residuals of the neighbour diameters. This phenomenon markedly decreases the error variance (cf. formula 18).

Table 3. Correlation coefficients between the residuals of the regression equations.
Taulukko 3. Regressioyhtälöiden residuaalien väliset korrelaatiokertoimet.

	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(1)	–.454	–.021	–.033	–.008	.098	–.009	.034	.020	–.103
(2)		–.610	–.064	.024	–.012	.013	.028	.009	–.017
(3)			–.452	–.108	.012	.030	–.019	–.004	–.007
(4)				–.397	–.164	.012	.035	.014	.032
(5)					–.362	–.121	–.053	.011	.046
(6)						–.401	–.106	–.005	.014
(7)							–.381	–.139	.036
(8)								–.367	–.041
(9)									–.469

6. SOLUTION OF THE SIMULTANEOUS EQUATION MODEL

Since the equations in our simultaneous model are nonlinear, standard matrix operations do not give the solution of the system. Instead the subroutine ZSYSTM of the IMSL program library (IMSL Library 2, 1977) was applied. This subroutine solves a system of n simultaneous nonlinear equations in n unknowns and uses Brown's method (BROWN 1969; BROWN and DENNIS 1971).

Subroutine ZSYSTM is called by a Fortran program written by the authors. The whole program contains approximately 500 lines, excluding ZSYSTM. The standard input information for the system includes the height and one diameter of the tree. The diameter may be measured at any height of the tree. The program may easily be modified to include any number of measured diameters. The basic data of the system contains the coefficients and the standard errors of the regression equations, and the correlation coefficients of the residuals of the regression equations (Tables 2 and 3).

The output of the system comprises the estimates of the relative-height diameters, and the stem volume calculated as a numerical integral.

It seemed appropriate to accept the result whenever the solution was either closer than .00001 of the exact solution, or whenever two successive approximations agreed in their first 4 digits.

The solution of the model took less than one second of the UNIVAC 1108 time. To make the program faster, average diameter-height ratios scaled to the measured diameter were given as original values for the subroutine ZSYSTM. Also the variance estimation (formula 18) was limited to the first round of iteration.

Occasionally, the system failed to converge. The most probable reason was the fact that the starting values for the diameter-height ratios were not quite suitable for the shortest trees.

7. RELIABILITY OF THE MODEL

The original material of the 2 000 sample trees were used to test the reliability of the simultaneous equation model. Because the system failed to converge in some small trees, all trees with height below 2 meters were deleted. This reduced the total number of trees to 1992.

Initially it was assumed that $d_{.2h}$ and h are known. The absolute and relative standard errors and biases of the diameter estimates are given in Table 4. In the same table the standard errors calculated by formula (18) are also given. The results show that formula (18) gives slight overestimates. This may be due to the fact that the Jacobian matrix was calculated only during the first round of iteration and this matrix was used for the other rounds of iteration. The variance overestimation causes slight negative biases in the upper diameters. In the lowest diameter the effect is opposite.

In this connection it should be noted that the negative biases in our earlier paper (KILKKI *et al.* 1978, Table 3) were due to the erroneous use of the measured diameters, instead of the estimated diameters, as denominators in calculating the relative biases.

The relative standard error of the volumes was 5.6 percent and the bias +.04 percent. The positive bias in volume, in spite of the mainly negative biases in the diameters, is due to the overestimation of the diameter variances (formula 18) which are needed in the basal area calculations (cf. KILKKI *et al.* 1978, p 124).

In another test of the simultaneous equation model it was assumed that dbh and h are known. The results of this test are given in Table 5. The biases are somewhat greater than in the previous case, but they can still be attributed to the erroneous variance estimates from formula (18). The relative standard error of the volume was 6.8 percent and the bias −.16 percent.

Table 4. Standard errors (s) and biases (b) of the diameters when $d_{.2h}$ and h are known.
Taulukko 4. Läpimittojen keskivirheet (s) ja harhat (b) kun $d_{.2h}$ ja h tunnetaan.

Relative height *Suhteellinen korkeus*	True values — *Todelliset arvot*				Standard errors estimated by formula (18) — *Kaavalla (18) lasketut keskivirheet*
	mm		percent, %		
	s	b	s	b	percent, %
0.01	8.4	+.06	7.7	+.00	7.8
0.1	3.5	+.00	4.0	−.03	4.3
0.2	0.0	.00	0.0	.00	0.0
0.3	3.0	−.02	3.9	−.04	4.7
0.4	3.9	−.05	5.7	−.04	6.3
0.5	4.6	−.06	7.5	−.05	7.8
0.6	5.0	−.07	9.5	−.08	10.7
0.7	4.8	−.06	11.6	−.10	12.7
0.8	4.2	−.03	14.8	−.07	15.0
0.9	3.0	−.01	18.0	−.05	18.0

Table 5. Standard errors (s) and biases (b) of the diameters when dbh and h are known.
Taulukko 5. Läpimittojen keskivirheet (s) ja harhat (b) kun $d_{1.3}$ ja h tunnetaan.

Relative height *Suhteellinen korkeus*	mm		percent, %		Standard errors estimated by formula (18) — *Kaavalla (18) lasketut keskivirheet*
	s	b	s	b	percent, %
0.01	8.7	−.04	8.3	−.10	8.7
0.1	4.0	−.10	5.2	−.14	5.5
0.2	2.6	−.11	4.0	−.16	3.0
0.3	3.0	−.10	4.1	−.18	4.1
0.4	3.8	−.11	5.3	−.15	5.7
0.5	4.5	−.11	6.9	−.15	7.3
0.6	5.0	−.10	9.1	−.16	10.1
0.7	4.7	−.08	11.4	−.17	12.3
0.8	4.2	−.05	14.7	−.14	14.8
0.9	3.0	−.01	18.0	−.08	18.0

8. DISCUSSION

This paper is a continuation of our previous study (KILKKI *et al.* 1978) in which the possibility of using a simultaneous equation model in the determination of the taper curve was suggested. In this paper the approach was studied using new and more comprehensive data. Also the reliability of the present data are of a higher degree. In one respect the new data are inferior to the previous data: There are no measurements of the diameters $d_{.05h}$ and d_6.

The present data clearly indicates that the residual variance of the diameters is greatly influenced by the size of the tree. Standardization of the variance was done by dividing the relative-height diameters by the height of the tree. Furthermore, it appeared that the relationships between the diameter-height ratios were not always linear but quite often of second degree.

The nonlinearity of the equations in the simultaneous equation model greatly complicated the solution of the system. The computer time required increased dramatically and slight biases due to the inaccurate estimation of the residual variance emerged. It might be possible to reduce the biases by replacing the estimated variances by the exact variances. The exact variances, however, change with the change of the diameter measurement height(s), and to calculate all of them in advance is only practicable when one diameter is measured at a prefixed relative height. In this case, a practical way to calculate better variance estimates is to calculate the regression equation in which each diameter-height ratio is predicted by the measured diameter-height ratio and by the height of the tree. For instance, the regression equation which predicts $d_{.5h}$ as function of $d_{.2h}$ and h might be:

$$d_{.5h}/h/100 = .0111 + .6683\ d_{.2h}/h/100 - .3841\ (d_{.2h}/h/100)^2 - .1116\ h/100 + 2.3693\ (h/100)^2 \quad (19)$$

The residual standard deviation of $d_{.5h}$ derived from this formula is almost equal to the one produced by the simultaneous equation model. If the relationships between the diameter-height ratios were linear in the simultaneous model (cf. Kilkki *et al.* 1978), the linear single equation models and the simultaneous model would give exactly the same results with the same known diameters.

It is possible that the nonlinearity of the simultaneous model equations originates from the small number of relative-height diameters. The nonlinearities were most profound in the lower part of the stem. Since no nonlinearities were found in our previous study, it is possible that even addition of diameter $d_{.05h}$ might significantly linearize the model. There is a new study under way in which a great number of additional relative-height diameters will be available.

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SELOSTE:

RUNKOKÄYRÄN MÄÄRITTÄMINEN EPÄLINEAARISEN SIMULTAANISEN MONIYHTÄLÖMALLIN AVULLA

Tutkimus on jatkoa Silva Fennicassa aiemmin ilmestyneeseen tutkimukseen (12.2: 120–125). Aineistona on ollut 2 000 viljelytaimistoista mitattua mäntyä. Simultaanisen moniyhtälömallin 10 perusyhtälöä on laadittu regressioanalyysia käyttäen. Yhtälöissä ovat selitettävinä ja selittävinä muuttujina kymmeneltä suhteelliselta osakorkeudelta mitattujen läpimittojen ja puun koko pituuden suhteet. Lisäselittäjänä on puun pituus. Kaikista selittävistä muuttujista ovat mukana sekä ensimmäisen että toisen asteen termit.

Mallia sovellettaessa on tunnettava puun pituus ja haluttaessa vähänkin luotettavia tuloksia myös yksi läpimitta. Mitattu pituus sijoitetaan yhtälöihin ja mitattua läpimittaa lähinnä vastaava regressioyhtälö korvataan interpolointikaavalla, joka liittää mitatun läpimitan neljään lähimpään perusläpimittaan. Näin saatu lopullinen epälineaarinen simultaaninen moniyhtälömalli ratkaistaan iteroimalla. Ratkaisusta saatujen 10 suhteellisen osakorkeusläpimitan väliarvot saadaan interpolointikaavalla. Mallia on mahdollista soveltaa myös silloin, kun mitattuja läpimittoja on useita.