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A UTILITY MODEL FOR TIMBER PRODUCTION BASED ON DIFFERENT INTEREST RATES FOR LOANS AND SAVINGS

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Seloste

SÄÄSTÖ- JA LAINAKORKOON PERUSTUVA PUUNTUOTANNON HYÖTYMALLI

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We discuss the evaluation of timber production policies with different income (timber drain) schedules. Special attention is given to the temporal smoothness of the income flow. A utility model is formulated in which the objective is to maximize a fixed consumption pattern, and money can be saved and borrowed at different interest rates. We thus have smoothness requirements only for consumption, the capital market then determines the smoothness of the optimal income flow. Present discounted value and maximization of even income flow criteria are special cases of the utility model. Consumption can be maximized by linear programming. A sample problem is presented.

INTRODUCTION

Long-term planning of timber production is possible if there are available:

1) a forest model that describes the biological and technical possibilities,
2) a price model that describes the price development for the inputs and outputs of timber production (excluding capital),
3) a capital model for the price of the capital, and
4) a utility function which quantifies the preferences of the decision maker, and
5) a suitable optimization method.

In this paper the capital model and utility function are treated simultaneously and are called a utility model. We assume, that the utility can be measured with money. The model is based on the differentation between income and consumption and on two different interest rates, one for saving and a different one for borrowing. We impose constraints for the temporal smoothness of consumption, which is the only variable in the utility function. More specifically, for each time period we have a minimum consumption requirement and we have fixed the proportions of the additional consumptions. The interest rates for savings and loans determine, how the capital market can transfer money from the time of earning to the time of consumption. If the interest rate for savings is very low and the interest rate for loans is very high, the utility model is a simple generalization of the maximization of even flow of incomes. Even flow of income is a natural economic interpretation of the 'sustained yield' concept, which seems to have a special appeal among foresters. If the interest rates for savings and loans are equal, then the utility model is equivalent to the maximization of the present discounted value.

Dual interest rates for lending and borrowing are commonly used in investment analysis. We try to apply these notions to give a specific interpretation to the temporal smoothness of timber production. In this paper we refer to investment theory only as it relates to our specific problem formulation. For more general discussion see, e.g., Baumol (1977), or Hirshleifer (1970). This paper gives a simpler economic explication for the ideas of Kilkki et al. (1984), who used a multiplicative utility function to study the temporal smoothness of the timber production.

In order to make the utility model operational, we must have a forest model and a price model, i.e., a production model that describes stand treatment alternatives in economic terms. First, therefore we briefly describe a system that yields these alternatives (see also Siitonen 1983). Alternatives can also be simulated by any available system (reviewed by Hann and Brodie 1980). Thereafter we describe the utility model more precisely. It will be seen that consumption can be maximized using linear programming. Then we discuss the connection between the interest rates and the shadow prices derived from the LP-solution. The behavior of the utility model is also demonstrated with a sample problem. Furthermore, we indicate briefly how optimization can be speeded up using linear programming iteratively.

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PRODUCTION MODEL

In our forest model a forestry unit is described using forest stands and their production possibilities. Each stand is described by its area and site variables and by the measurements on the sample trees. To reduce the size of the model and to facilitate the organization of information concerning the forestry unit, stands are grouped into calculation units. Stands belonging to one calculation unit have similar stand characteristics.

Feasible production processes for each calculation unit are described by simulating several management alternatives over the planning period, e.g. a hundred years. Each alternative describes a sequence of silvicultural measures, e.g., thinnings, clear cuttings, regeneration, tending of young stands and fertilization. Development of the growing stock is simulated by models describing the ingrowth, growth, and mortality of the trees.

Reliable prediction of future prices is a difficult (or even impossible) problem in long-term planning. This holds true also in timber production, even if the end product is a flexible raw material of several industries, and the total demand for timber does not change as rapidly as production technologies and the markets of specific products. In the sample problem we use constant long-term average prices to maintain a one-to-one correspondence between harvest level and income level. The changes in relative prices may, anyhow, make the forestry projects seem more profitable in comparison with non-forestry investments, as suggested by Nautiyal and Rezende (1982).

The planning period is divided into subperiods; let p denote the number of subperiods and d the length of each subperiod. In the example we have used five subperiods of twenty years each. The subperiod sums of all variables of interest are calculated for each management alternative of every calculation unit. These variables include volumes by tree species, timber assortments (sawlogs, pulpwood, etc.), volume increment, drain, income, cost, etc. In this paper we utilize only net incomes during the subperiods and the total timber volume of the calculation units in the final state. In economic calculations, we assume that incomes occur at the midpoints of the subperiods.

UTILITY MODEL

Money available at a given point in time can either be expended or saved. Money is available either as an output of the production process or from the capital market. In this paper we assume that the decision maker wants to maximize his consumption level. Given a desired consumption pattern, an optimal production schedule is derived from the production model and from the capital model that describes how money can be saved and borrowed outside the production unit.

The appropriateness of the present discounted value criterion depends on the assumption that money can be saved and borrowed in unlimited quantities at a common interest rate. If this assumption holds and money is always valuable, then the utility of the income pattern is a strictly increasing function of the present discounted value for all time preferences of consumption. Thus, every decision maker can use present discounted value as the objective function.

The assumptions for present discounted value seldom hold completely. For relatively short-term investments, however, this criterion can lead to sound conclusions, if the capital market works properly. As the planning horizon increases, the validity of the present discounted value criterion decreases. On the other hand, the maximization of a constant income flow is based on the assumption that the decision maker wants an even level of consumption coupled with the assumption that money cannot be saved or borrowed. Present discounted value criterion neglects all smoothness requirements, and maximization of even flow of incomes neglects all other aspects, e.g., the interest rate.

Here, we consider the case where interest rates for savings and loans differ from each other. For any (sub)period we can expend any combination of our income during that period, our earlier savings or borrowed money. All excess income is saved. We assume that at the period t money can be saved at interest rate i_{tS} and borrowed at interest rate i_{tL}. We further assume that borrowing is not intrinsically valuable, i.e., i_{tL} is positive and greater than or equal to i_{tS}.

We can simulate production alternatives only for a finite planning period. The better we can evaluate the final state, the shorter the simulation time that needs to be applied (for formal treatment of trade-offs see, e.g., Grinold 1983). We first suppose that the incomes after the planning period are concentrated at the midpoints of periods of d years, as during the planning period. These periods are denoted by p+1, p+2, etc. To adjust the planning period and the final state, we reason as follows:

i) We have a specific saving interest rate i_{pS} and loan interest rate i_{pL} for the final state. A saving of S_p in the final state will return an interest income $((1+i_{pS})^d - 1)S_p$ after every d years. In the same way, a loan L_p in the final state will cost $((1+i_{pL})^d - 1)L_p$ after every d years. The amount of the loan or saving is assumed to remain constant forever.

ii) After the planning period, we have even incomes and even consumption. Let x_t and z_t denote the timber income and consumption, respectively, in period t. Thus we assume:

[1] $x_{p+1} = x_{p+2} = \ldots$, and

[2] $z_{p+1} = z_{p+2} = \ldots$.

iii) The annual timber income after the planning period is estimated using the total volume in the final state. First, the even flow of income is temporarily maximized for the planning period by linear programming, subject to the constraint that the total volume of the growing stock at the end of the planning period, V_p, is equal to the total volume in the beginning of the planning period, V_0. This volume constraint guarantees that we can approximately have the same even flow of income after the planning period as during the planning period (this constraint is not a part of our utility model). Then we assume that the ratio $f = x/V_0$, obtained between the income level x and the total volume, generally will continue to hold true in the same forestry unit. So we get a rough additive estimate for the income level after the planning period, x_{p+1}:

[3] $x_{p+1} = V_p/f$

The first crucial consequence of having different interest rates for loans and savings is that the evaluation of different consumption patterns can no longer be avoided. The starting point for our utility function is the intuitive idea that the decision maker wants to have an even flow of consumption. However, he may have different minimum consumption requirements for different periods. Also the incomes from other sources than the forestry unit must be taken into account. Furthermore, the decision maker may want the consumptions above the minimum requirements to be in any fixed predetermined proportions, and not necessarily equal. These assumptions lead to the following maxi-min utility function:

[4] $u = u(z_1, \ldots, z_{p+1}) = \text{Min}\{(z_t - b_t)/a_t, t=1, \ldots, p+1\}$,

where
z_t is consumption in period t,
b_t is the minimum consumption requirement in period t subtracted by the incomes from sources other than the forestry unit ($z_t - b_t \geq 0$), and
a_t is a parameter which determines the desired allocation of the consumption $z_t - b_t$; a_t-parameters are scaled by defining $a_1 = 1$.

Suppose we want to keep consumption even all the time and we have no minimum consumption requirements or incomes from other sources. In that case:

$b_t = 0$ and $a_t = 1$ for $t=1, \ldots, p+1$.

According to the postulated capital model we can transfer money between all periods. Hence, it necessarily holds true in the optimum (recall that $a_1 = 1$) that:

[5] $(z_1 - b_1) = (z_2 - b_2)/a_2 = \ldots = (z_{p+1} - b_{p+1})/a_{p+1}$.

Thus the utility function u is maximized when $z_1 - b_1$ is maximized subject to the constraint given in Eq. (5).

Selection of the optimal production plan can now be formulated as the following linear programming problem:

Max y_1

subject to:

1) $-y_1 + x_1 - S_1 + L_1 = b_1$

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t+1) $-a_{t+1}y_1 + x_{t+1} + r_{tS}S_t - S_{t+1} - r_{tL}L_t + L_{t+1} = b_{t+1}$

p+1) $-a_{p+1}y_1 + x_{p+1} + r_{pS}S_p - r_{pL}L_p = b_{p+1}$

$x_t - \sum_i \sum_j x_t^{ij} w_{ij} \leq 0$ for $t=1, \ldots, p+1$

$\sum_j w_{ij} = 1$ for every i

$w_{ij} \geq 0$ for all i, j

$S_t \geq 0$, $L_t \geq 0$ for $t=1, \ldots, p$

where

y_1 = $z_1 - b_1$, z_1 and b_1 are defined after Eq. (4)
x_t = net incomes during subperiod t, $t=1, \ldots, p$
x_{p+1} = estimate for the even level of income after the planning period obtained using Eg. (3)
x_t^{ij} = net incomes during subperiod t from calculation unit i if management alternative j is applied.
w_{ij} = proportion of calculation unit i treated according to alternative j.
S_t = savings from subperiod t
L_t = loans from subperiod t
r_{tS} = $(1+i_{tS})^d$ where i_{tS} is the annual interest rate for savings
r_{tL} = $(1+i_{tL})^d$ where i_{tL} is the interest rate of the loan
r_{ps} = $(1+i_{pS})^d - 1$ where i_{pS} is the annual interest rate for savings in the final state and thereafter
r_{pL} = $(1+i_{pL})^d - 1$ where i_{pL} is the interest rate for loans in the final state and thereafter
d = length of the subperiods.

The first constraint states that the consumption in the first period is:

[6] $z_1 = y_1 + b_1 = x_1 - S_1 + L_1$.

Note that any current dividends can be included in b_1. In the constraint row for period t+1 we take into account that we can have dividends from the period t, and that the consumption is (see Eq. 5):

[7] $z_{t+1} = (z_{t+1} - b_{t+1}) + b_{t+1} = a_{t+1}y_1 + b_{t+1}$.

The linear programming formulation guarantees that either S_t or L_t or both are zero for each period t. Recall that incomes, consumption, and the amount of savings or loans remain constant for periods p+1, p+2, etc., thus the constraint row p+1 will take care of the time after the planning period up to infinity.

If the interest rate for savings is very low and the interest rate for loans is very high, there will be no loans or savings in the optimal solution, and thus the formulation leads to the same fixed income pattern as required for the consumption. An indication that the production model has not provided enough alternatives is when the interest rate for savings is low and the interest rate for loans is high and there still exist savings or loans in the solution. In practice, the forest incomes can be moved smoothly from one time point to another in any forest area having initially marketable timber. If savings and loans have a common interest rate, the formulation leads to the maximization of the present discounted value. Thus, by changing the interest rates, we get a continuum of utility models between the present discounted value and the even flow of incomes (or any other fixed income pattern).

For small forestry units, real bank interest rates for loans and savings may be used directly as the interest rates in the model. Using b_t-parameters, any known variation in consumption or incomes from other sources can be taken into account. The pattern of the free part of the consumption is specified by a_t-parameters.

If the size of the forestry unit is very large (e.g. the entire forestry sector of a country), the financial system can no longer filter fluctuations in incomes so well, and production factors other than the biological production capacity of the forest (e.g., labor and timber market) also make the production less flexible. The decrease in the flexibility as a function of the size of the planning unit can be handled with the given utility model by decreasing the interest rate for savings and increasing the interest rate for loans. We are not offering any new solutions in the controversy concerning the appropriate discount rate for public (forestry) projects (see, e.g., Manning 1977, Foster 1979); rather, we want to bring a new aspect into the picture.

SHADOW PRICES AS INTEREST RATES

Shadow prices derived from the solution of the linear program express the marginal properties of the solution. A one to one correspondence exists between interest rates and shadow prices of the incomes at a specific time (see, e.g., Kilkki 1968).

Let v_t be the shadow price of x_t, $t=1, \ldots, p+1$, then define

[8] $r_t = v_t/v_{t+1}$ for $t=1, \ldots, p$.

Let i_t be the annual interest rate from the middle of subperiod t to the middle of subperiod t+1. According to principles of interest calculation, the utility of money in the middle of subperiod t is $(1+i_t)^d$ times the utility of money in the middle of subperiod t+1. Factors r_t, $t=1, \ldots, p-1$ can therefore be converted to interest rates simply by

[9] $i_t = r_t^{1/d} - 1$

The interpretation of r_p is slightly more complicated because it is not the marginal rate of substitution between two time points, but rather between the last subperiod and the estimated level of even income after the planning period. As noted, it is assumed that an even income means that the income is concentrated at the midpoints of periods of d years, as during the planning horizon.

Suppose the rate of interest in the middle of the last subperiod and thereafter is i_p. Therefore, capital C in the middle of the last subperiod will be multiplied in d years by a factor $(1+i_p)^d$ and thus will produce a rent of

$$[10]\quad ((1+1_p)^d - 1)\, C$$

after every d years. Thus:

$$[11]\quad r_p = v_p/v_{p+1} = (1+i_p)^d - 1 \text{ or}$$

$$[12]\quad i_p = (1+r_p)^{1/d} - 1.$$

Conversely, if the interest rates, i_t, and thus the r_t-coefficients are known, the shadow prices v_t can be derived from them. First, from the equation

$$[13]\quad r_t = v_t/v_{t+1},\ t = 1, \ldots, p$$

it follows that

$$[14]\quad v_t = c\, r_p\, r_{p-1} \ldots r_t,$$

where c is a scaling constant. If each x_t, $t=1, \ldots, p+1$ is increased by a_t units, then the value of the objective function y_1 is increased by one unit. In the neighbourhood of the LP-solution, the changes in the objective function can be expressed in terms of shadow prices. Thus for infinitesimal units:

$$[15]\quad \sum_{t=1}^{p+1} a_t v_t = 1.$$

From Eq. (14) and (15) we obtain:

$$[16]\quad c = 1/(r_p \ldots r_1 a_1 + r_p \ldots r_t a_t + \ldots + r_p a_p + a_{p+1}).$$

The growing trees can have economic value either as output or as a production factor. If trees are changed into money (i.e., they are cut), their value depends on the marginal value of money at that time. If trees are allowed to grow further, their value depends on how much they grow and what is the marginal value of the money at the time of cutting. The interest rate gives the marginal value of money.

If the capital market is perfect, it determines the interest rates. If the capital market is not perfect (e.g., if the interest rate for savings is different from the interest rate for loans), the proper interest rate is determined by interaction of the capital market and the forestry unit. If the forestry unit has savings (loans) the interest rate for savings (loans) is the appropriate interest rate.

In the case where the LP-solution indicates neither saving nor loan, the appropriate interest rate lies between the interest rate for savings and that for loans. In this case, the forest is marginally the optimal bank for the decision maker. If he wants to increase his consumption, he should cut more; and conversely, it is more profitable to invest in growing stock than in the capital market.

The optimal management alternatives can be found separately for each calculation unit by maximizing the present value discounted by the variable interest rate derived from the LP-solution. However, if at the optimum two different alternatives are applied in different parts of the same calculation unit, then the proportions for these alternatives cannot be derived from the present discounted values.

Suppose that the timber production plan has been made for the entire forestry sector of a country. Then the interest rates derived from the shadow prices will state how the government (or market forces) should regulate the interest rate in order to make the nationally optimal policy optimal also for individual forest owners, who are interested only in the present value of their forests. This presentation is, of course, oversimplified because in the imperfect actual capital market the forest owners are not concerned with present value. But this formulation may give some insight into the problem of how to make national economic plans profitable also for individual forest owners.

If any LP-solvable utility model includes incomes for different time points among its variables, the respective shadow prices can be expressed in terms of interest rates. The decision maker can then decide if he can accept these interest rates or the marginal rates of substitution implied by the shadow prices. The advantage of the utility model presented is that the decision maker can fix beforehand the range within which the interest rates (marginal rates of substitution) are allowed to vary. Thus it would be easier to find an acceptable solution by using the utility model presented than by just trying to modify the income constraints of the LP-problem by trial and error.

A SAMPLE PROBLEM

Our sample problem refers to a forest area in southern Finland. The forest data are based upon the national forest inventory in 1978. A total of 2929 sample trees on 436 sample plots are used. Each sample plot represents a forest stand. The sample plots are grouped into 107 calculation units. For all calculation units a total of 9781 management alternatives over a 100-year planning period have been simulated.

To apply the presented utility model the parameters of the model must be determined. Without any further information the most natural consumption pattern may be an even consumption with no incomes from other sources, i.e., $b_t=0$ and $a_t = 1$ for $t=1, \ldots p+1$. We tested several interest rates for loans and savings. To decrease the number of combinations the interest rates were held constant over time.

In the sample problem we required that there be neither loans nor savings at the end of the planning period ($i_{pS}=-0.30$ and $i_{pL}=0.30$). Thus no permanent transfer of capital from or into the forestry unit is allowed. The time pattern of consumption, incomes and savings (negative savings are loans) are presented in Fig. 1 for different combinations of interest rates. Recall how we estimated the income level after the planning period using the total volume in the final state (see Eq. 3). Thus, the consumption level is, in our example, directly proportional to the total volume in the final state, which is also equal to the initial total volume in case where income is constant over time.

When the interest rate for savings is above 2 % (subfigures $i_S=i_L$ = 2,3,4,5 %), it does not pay to borrow money at any rate greater than or equal to the interest rate for savings. If the interest rate for savings is below 0.7 % and the interest rate for loans is above 1.8 % (as in subfigure i_S=0 %, i_L=2 %), the income is constant over time, i.e., the sustained yield forestry is practiced. If interest rates for both savings and loans are zero (subfigure i_S=0 %, i_L=0 %), it is profitable to invest heavily in the growing stock and borrow for consumption. If the interest rate for loans is 1 % (subfigures i_S=0 %, i_L=1 % and i_S=1 %, i_L=1 %), one should first balance consumption and incomes, and borrow money later (in subfigure i_S=1, i_L=1 the initial balance is only approximative).

In the case where the interest rate for savings (and loans) is 2 % or more (subfigures $i_S=i_L$ = 2,3,4,5 %), one should first cut more than is needed for consumption, and later a part of the consumption can be paid by the savings. Interestingly, high interest rates therefore lead to an increase in the growing stock during the planning period. If savings are allowed in the final state, high interest rates lead to a permanently reduced growing stock. The concentration of the incomes at the midpoints of twenty-year subperiods may cause some bias in the results.

The problem formulation presented here defines a normal linear programming problem. Thus any computer program for linear programming can be used for its solution. We used a VAX-version of MINOS (see Murtagh and Saunders 1978) to solve the sample problems.

Computation time was decreased to approximately one tenth (from 10 minutes to one minute of VAX-11/780 CPU-time per problem), if the selection of the activities, i.e., management alternatives was done iteratively. The first set of alternatives was selected for the LP-problem using a heuristic algorithm based on the correspondence between interest rates and shadow prices. Thereafter, shadow prices derived from the LP-solutions were used to select new sets of alternatives until the optimal basis was found.

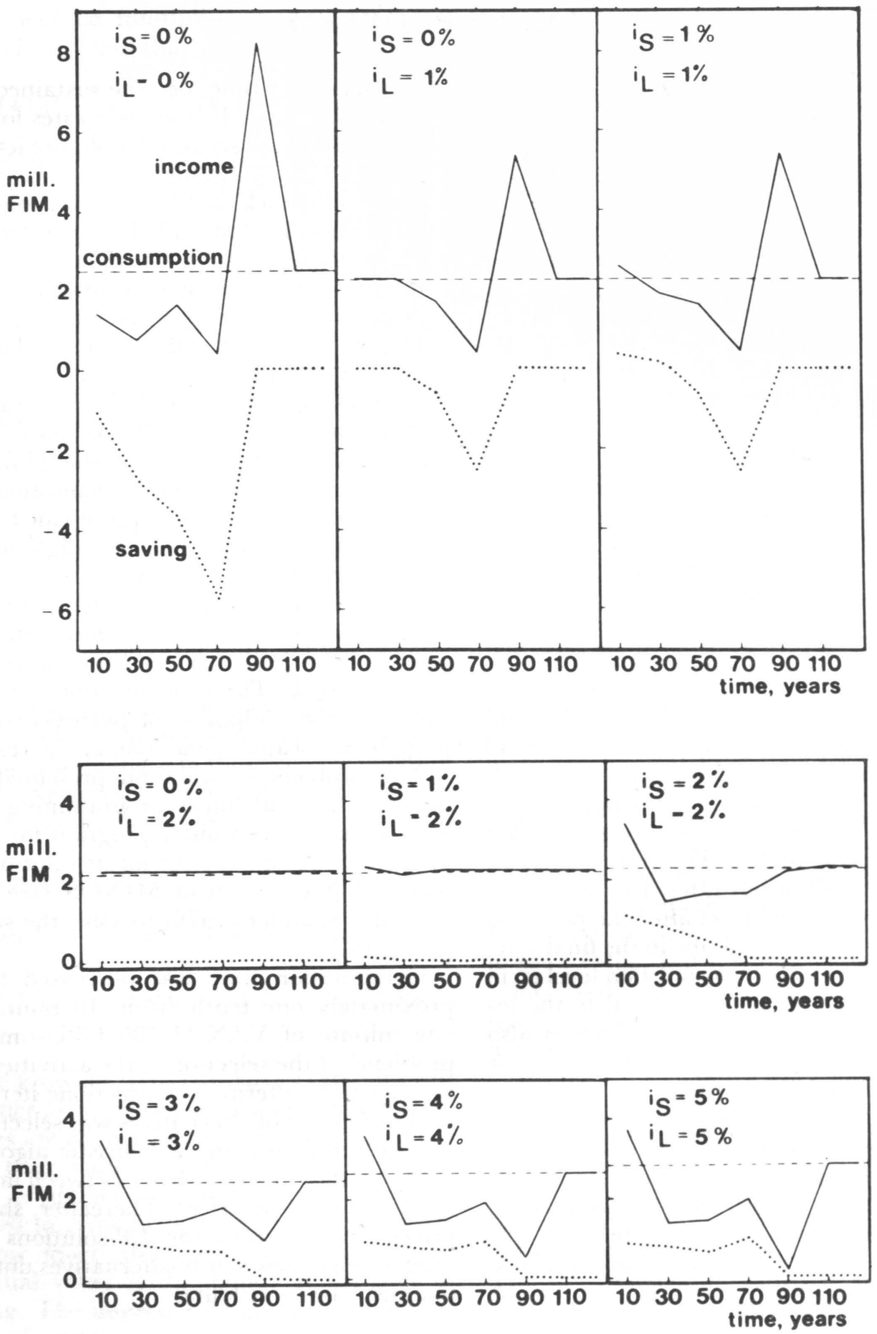

Fig. 1. In each subfigure the twenty-year income (solid line), consumption (broken line), and amount of savings (dotted line, negative savings are loans) are given as a function of time. Subfigures correspond to different constant interest rates for savings (i_S) and loans (i_L); units are Finnish marks (FIM).

DISCUSSION AND CONCLUSIONS

This paper is motivated by the conviction that proper evaluation of the traditional principles of forest management (e.g., sustained yield) calls for the application of quantitative economic principles. If the standard economic investment calculations imply unacceptable conclusions in the forestry context, then foresters should adjust the theory properly rather than just select their own ad-hoc criteria for decisions (see Samuelson 1976 for the implications of a perfect capital market).

We attempted to modify the assumptions about a perfect capital market in order to give a specific interpretation to the temporal smoothness requirements of timber production. The model presented can be modified further in several ways. We can also take interest rates (or prices of inputs and outputs) as a function of the amount of savings or loans (or of the amount of inputs and outputs). We could also use a less restrictive utility function, e.g., there are multiplicative utility functions that have our maximin criterion as a limiting case. However, after these modifications, the optimization can no longer be made using standard linear programming.

The smoothness problem of timber production becomes more complex if it is required that the production behaves smoothly both in the whole forestry unit and in each subregion, and there are also smoothness requirements for different timber products (assortments). Perhaps the ideas presented in this paper can also be extended to this more general case.

The presented utility model can describe also after these extensions only a part of any realistic decision situation. For instance, the utility function may depend also on other variables than money. Furthermore, the uncertainty of the future prices, interest rates and preferences makes the decision situation more complex. The long-term plans of timber production can only summarize the strategic context of the decision making, and they cannot provide any direct decisions.

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Total of 12 references

SELOSTE

SÄÄSTÖ- JA LAINAKORKOON PERUSTUVA PUUNTUOTANNON HYÖTYMALLI

Kirjoituksessa tarkastellaan puuntuotanto-ohjelmien vertailua tulovirtojen avulla. Erityistä huomiota kiinnitetään tulojen (hakkuiden) ajalliseen tasaisuuteen. Esiteltävässä hyötymallissa maksimoidaan kiinteän muotoista kulutusvirtaa, ja rahaa voidaan säästää ja lainata erisuurella korolla. Ainoastaan kulutuksella on siten tasaisuusvaatimuksia; pääomamarkkinat sitten määräävät tulojen (hakkuiden) tasaisuuden. Nykyarvon ja tasaisen tulovirran maksimointi ovat hyötymallin erikoistapauksia. Kulutus voidaan maksimoida lineaarisen ohjelmoinnin avulla. Hyötymallia havainnollistetaan esimerkkilaskelmien avulla.