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A theory of stumpage appraisal

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TIIVISTELMÄ: TEORIA KANTOHINNAN MÄÄRITTÄMISESTÄ

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This paper is a theoretical study of what is considered to constitute the proper perception of time in forest economics and management. A stumpage appraisal model that recognizes the influence of time is developed within the framework of a national aggregate economy. To demonstrate how a socially optimal land for timber production may be determined in a given nation, a stock-supply model is derived. The stumpage appraisal rule developed determines the market stumpage price that maintains a state of balance between timber production and other land use activities.

Kirjoituksessa selvitetään teoreettisesti, miten aikatekijä tulisi ymmärtää metsäekonomiassa. Ajan vaikutuksen huomioon ottava optimaalisen kantohinnan määrittävä malli on kehitetty kansantalouden näkökulmasta. Johdettu varantotarjontamalli (stock-supply model) kehitettiin osoittamaan, kuinka yhteiskunnan kannalta optimaalinen metsäala voidaan määrittää. Kehitetty kantohintamalli määrittää tasapainohinnan puuntuotannon ja muiden maankäyttömuotojen välille.

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1. Introduction

Classical theory of economic rent forms the basis of princing timber. We have stumpage appraisal models that treat the market value of timber as price-determined (Matthews 1935, pp. 275–336; Steer 1938, p. 8; Rothery 1945; Chapman and Meyer 1947, pp. 361–471). Such models assume forests to be a natural endowment of a given piece of land that has got no alternative use. The basic formulation of these models is

$$p = v - (K + M) \tag{1-1}$$

where

p = stumpage price per cubic metre
v = forest product price
K = wood processing firm costs of production plus logging and transportation costs
M = wood processing firm margin for risk and profit.

To account for observed differences in stumpage prices based on Eq. (1–1) in a given nation, appraisal models based on von Thunen (1875) theory of agricultural rent and land use have been proposed (Heikinheimo and Lehikoinen 1981). Stumpage prices vary due to cost of availability as follows

$$(1-2) \quad p - T \geqq 0$$

where T is the cost of availability of rawwood. This formulation defines the forest working circle with the marginal forest owner located at that point where p = T.

Gray (1983, pp. 33–38) asserts that Eq. (1–1) determines the maximum willingness to pay for standing timber by a wood processing firm. This assertion could be more convincing if we had an unequivocal rule for setting the level of, M, in this appraisal rule. The rationale of derived demand upon which this rule is based assumes the pricing of capital goods to be a gray area by presupposing that producers of capital goods are essentially different from producers of consumer goods. According to this hypothesis, the functioning of the rest of the national economy affects the prices of forest products and such influences are transmitted to stumpage prices via derived demand for rawwood (Haynes 1977). Forest owners are treated as agents independent of the rest of the national economy.

If we simply choose a sign convention to distinguish inputs from outputs, and designate consumer inputs, e.g., paper, as positive; outputs, e.g., labour, negative and producer inputs, e.g., silviculture as negative; outputs, e.g., stumpage as positive, we immediately notice that there is no real justification for assuming that the product prices and factor prices are two different things. On the one hand, there exists a connection between consumer goods prices and wages plus cost of rawmaterials as a result of an adjustment process in the market. On the other hand, there exists also a connection between capital goods and the price of capital used in producing such goods, which is more direct as compared to the less direct connection between consumer goods price and wage-cost levels.

Therefore, in a general equilibrium setting there is no distinction between producers of capital goods and producers of consumer goods. Hence, the pricing of stumpage should not be treated as a gray area. In this study, we shall show how a forest owner's decision to supply rawwood at any time depends on: a) opportunity cost of waiting, and b) opportunity cost of capital invested in silviculture. As long as stumpage market value is not lower than the opportunity cost of waiting and opportunity cost of silvicultural investments, there is no problem about the existence of forest owners.

Besides the differential rent stumpage appraisal models, we also have price-determining appraisal models (Marquis 1939, pp. 27–37; Hanson and Leslie 1965). This approach to stumpage appraisal recognizes the fact that not all forest resource is purely a natural endowment. Whenever silviculture is introduced in forestry, the growing stock becomes a capital good. Silviculture is defined to include investment expenditures in: a) Land, b) infrastructure improvements, c) administration, d) stand establishment and improvement, e) protection. Hence, the minimum stumpage price paid to a forest owner should at least cover all silvicultural expenditures incurred by a forest owner. A popular name for such models is: cost-recovery method, and Gray (1983, pp. 49–54) asserts that this approach to stumpage appraisal determines the minimum willingness to sell standing timber by a given forest owner.

Market stumpage value determination according to cost-recovery policy is based on maximization of the difference between expected stumpage income and silvicultural expenditures with stand age as an input factor. The justification of optimum rotation is very popular in forest economics and management literature (see, e.g., Goundrey 1960; Anderson 1976; Comolli 1981; Kemp and Van Long 1983). One of the daunting problems this approach has to encounter is that of identifying a cash flow in an enterprise where the bulk of the revenue does not materialize until sometimes in the distant future.

Cost-recovery policy may be couched *ex-ante* in terms of regeneration costs or *ex-post* in terms of establishment costs. The latter is more usually preferred to the former with the basic formulation as

$$(1-3) \quad p = \frac{I}{Q}$$

where

p = stumpage price per cubic metre
I = total silvicultural establishment costs
Q = timber yield in cubic metres.

Strictly speaking, when optimizing Faustmann formula, cost-recovery policy *ex-ante* or *ex-post* arguments are essentially the same, since it is assumed that all silvicultural expenditures are automatically incurred once timber in harvested.

When managing a natural resource like forest growing stock, it is wrong to maximize the difference between total revenue and total costs since this tends to imply that forest owners do not discount future benefits. Silvicultural investments, whether referred to as regeneration costs or establishment costs are in the main sunk costs that no rational owner of a wood processing firm would be willing to pay to a forest owner in order to cut stumpage. Finally, cost-recovery policy, like residual stumpage methods, assumes that a forest owner is operating in an environment independent from the rest of the national economy.

In this study we shall endeavour to solve the issue of stumpage price determination, i.e., the market value of timber as it stands uncut in a forest owner's forest growing stock. We shall assume a setting of a closed national economy in which an economic agent referred to simply as a forest owner may be: a) An individual farmer, b) a private company, or c) a government agency. To model the behaviour of stumpage price determination of a forest owner in an aggregate economy, we derive two interrelated but still distinct models namely: a) Stock-Supply Model, and b) Stumpage Appraisal Model. This approach to determination of market stumpage value is inevitable because a forest owner's income is made up of: 1) sales of raw wood (realized income), and 2) natural wood increment (unrealized income).

To circumscribe mistakes made by previous related studies, special attention is paid to complexity due to stumpage longevity and the related aspect of a forest owner's growing stock being the capital asset as well as the product. The complexity due to stumpage longevity calls for an understanding of the meaning of time in economics. There exists no correspondence between Marshallian concepts of short-run and long-run that describe the adjustment behaviour of economic agents and the chronological time scale as we know it. The two concepts are basically static in nature, with short-run referring to an equilibrium where an agent has a profit incentive to change certain inputs but he is unable to do so; and the longrun referring to an equilibrium where an agent has got no profit incentive to change the level of his input factors under the prevailing prices and output conditions.

Consideration of how future influences the behaviour of forest owners is more relevant to our study than the two Marshallian concepts of short-run and long-run. This is accomplished by introducing expectations with respect to: a) pure role of time, and b) the role of uncertainty. Even if we imagine that a forest owner operates in an environment of certainty, we still must have his current rational economic behaviour reflect the existence of a future that will generally differ from the present. This proposition exposes the weakness behind modelling forest owners behaviour on an infinite time horizon as if they are not beings with finite life time (Vehkamäki 1986, p. 52). If on top of the pure role of time a forest owner faces a future with unknown events, time introduces another complexity referred to as the role of uncertainty.

The dual nature of forest growing stock implies that stumpage appraisal is an exchange problem and not a production problem to be solved on the basis of regeneration costs, or its equivalent counterpart establishment costs. There is no short-run linkage between establishment costs and the level of stumpage prices. Precisely, the Marshallian concept of short-run is non-existence in forest management. With respect to growing stock regulation, a forest owner is always operating in the long-run equilibrium equipped with two decision variables: a) cuttings, and b) silvicultural investments. Stumpage price determines the quantity of cuttings in a given period. It does not determine the quantity of merchantable timber produced in a given period because forest resource is essentially a stock resource in the short and medium terms.

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2. The nature of stumpage market

21. Assumptions

We seek to establish exchange value for stumpage in a barter economy. To suppress the influence of economic agents other than forest owners and owners of wood processing firms, we assume, F, types of participants in the national economy with, r, member so that F = 2 and

$$r_i = \{r_1, \dots, r_n\} \quad i = 1,2$$

where

1 = forest owners
2 = owners of wood processing firms.

This assumption will be dropped when we move to section 3 where the market value for stumpage is derived. Members of category 1 are assumed to be of the same type and so are members of category 2.

Since r = 2 consists of more than one industry, we shall further simplify the analysis by taking into account only owners of sawmills. Moreover, we shall assume that one cubic metre of stumpage when processed produces one cubic metre of sawnwood; hence, the quantity of sawnwood sold is equal to the quantity of stumpage cut. This is a technical assumption, dropping it complicates the analysis but leads to the same results. Finally, we invoke the behavioural rule of profit maximization so that

(2–1) $\pi_1 = \pi_1(p,q)$
(2–2) $\pi_2 = \pi_2(p,q)$

where π_1 and π_2 are profit functions of a forest owner and a sawmill owner respectively that depend only on cuttings, q, and stumpage price, p.

22. Derivation of the core economy for the stumpage market

The profit functions take on a specific form

(2–3) $\pi_1(p,q) = pq - S(q)$
(2–4) $\pi_2(p,q) = vq - pq - K(q) = (v-p)q - K(q)$

where

q = stumpage price
q = cuttings
S = forest owner's opportunity cost of waiting plus opportunity cost of capital invested in silviculture
v = sawnwood price (assumed to be competitive)
K = sawmill costs of production less stumpage.

From Eqs. (2–3) and (2–4) we obtain the following isoprofit functions that specify all combinations of stumpage price and cuttings for which the profit of each owner is a given constant

(2–5) $p - \frac{S(q)}{q} + \frac{\pi_1}{q}$

(2–6) $p = v - (\frac{K(q)}{q} + \frac{\pi_2}{q})$,

where Eqs. (2–5) are assumed to be strictly convex and increasing. The shape of the two functions is implied by the first derivatives of Eqs. (2–1) and (2–2): $\pi_1'(p) > 0$ and $\pi_2'(p) < 0$. An increase in stumpage price leads to an increase of a forest owner's profit and a decrease in a sawmiller's profits, and the converse being true.

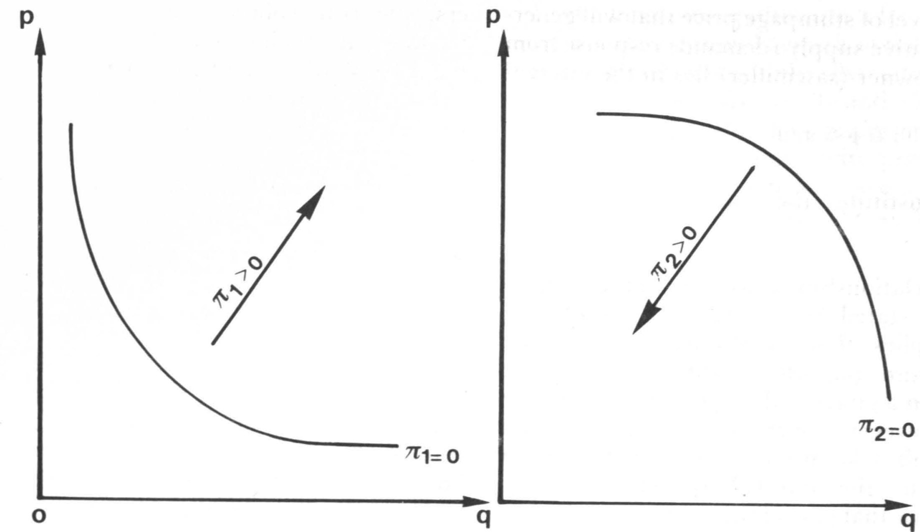

Figure 1. Forest owner profit curve (left). Sawmill owner profit curve (right).

If a forest owner aims at too high stumpage price, he will eliminate the demand for stumpage. Similarly, if a sawmiller aims at too low stumpage price, a forest owner will be unwilling to supply timber. Hence a precondition for stumpage trade to take place; maximize either π_1 or π_2 subject to $q \neq 0$. A forest owner faces a constraint of maximizing π_1(p, q) subject to $\pi_2 = 0$ Figure 1(b), while a sawmiller has to maximize π_2(p, q) subject to $\pi_1 = 0$ Figure 1(a). Therefore, $q \neq 0$ is defined over a closed interval

(2–7) $q\ [\pi_1(0), \pi_2(0)]$.

We postulate that whenever $q \neq 0$, there exists a number μ such that the derivatives of Eqs. (2–3) and (2–4) with respect to p, and q, are equal to zero. Thus,

(2–8) $pq - S(q) + \mu[(v - p)q - K(q)] = 0$.

The derivative with respect to p, is zero exactly when μ = 1; and with respect to q we get

(2–9) $\frac{dp}{dq} = -\frac{p - S'(q)}{q}$

(2–10) $\frac{dp}{dq} = \frac{v - p - K'(q)}{q}$

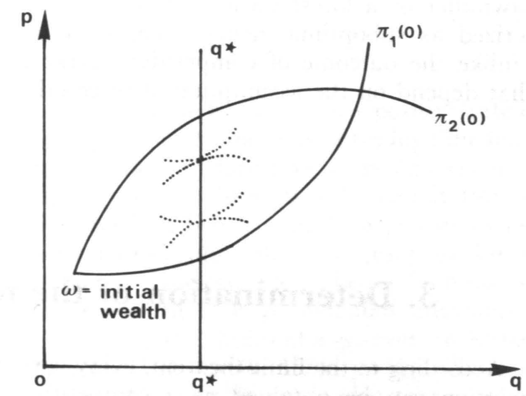

Figure 2. The core of the stumpage market.

and by setting to zero implies

(2–11) $\frac{v - p - K'(q)}{q} = \frac{S'(q) - p}{q}$,

or

(2–12) $v = K'(q) + S'(q)$.

Eq. (2–12) defines the contract curve labelled q*–q* in Figure 2.

The level of stumpage price that will generate a positive supply (demand) response from a forest owner (sawmiller) lies in the interval

$$(2-13)\quad \pi_1(0) \leqq p \leqq \pi_2(0).$$

This constitutes the core of the stumpage market.

The relationship between p and q is undefined as stated by Eq. (2–12). Intuitively, this implies that in stumpage trade, the equilibrium quantity of stumpage ready for cutting in a given trading period is essentially fixed; however, the stumpage price at which trade will take place is restricted only to belong to the interval specified by Eq. (2–13), or that part of the contract curve that falls within the area enclosed by the two isoprofit curves in Figure 2.

In stumpage trade, any trading behaviour that will lead to the establishment of a stumpage value outside the core by either the sawmiller or a forest owner will be characterized as suboptimal hence, unacceptable. Unlike the outcome of competitive markets that depend on the assumption of price takers, the core solution set depends on the trading rule (behaviour). However, both of them emphasize equal treatment.

By Edgeworth–Debreu–Scarf limit theorem (Takayama 1985, pp. 204–215);

$$(2-14)\quad c - \lim_{r\to\infty} c(r) = 0,$$

where c, is the core solution set and r, is the total number of forest owners and sawmillers participating in the stumpage trade. The limit theorem Eq. (2–14) states that, as the number of forest owners and sawmillers increases without bound, the core solution set becomes unique and coincides with the outcome of a competitive market economy. The limit theorem implies that there is no difference between an equilibrium stumpage price based on a given trading rule and an equilibrium stumpage price in the market that requires agents to be price takers. The implication of this theorem may be true with respect to manufactured commodities but it may not hold in its entirety when it comes to trading in roundwood standing in the forest where its location is expected to influence the equilibrium value.

3. Determination of the market value of stumpage

According to the limit theorem, every core solution can be obtained as a competitive equilibrium price. We are now faced with the problem of demonstrating how this is possible. To do this, we have to introduce the notion of prices in our core economy. Eklund – Kirjasniemi (1969) and Keipi (1978) have proposed models based on modern mathematical techniques of optimization.

The scope of optimization of Eklund – Kirjasniemi model includes: procurement of rawmaterials, manufacturing and distribution of forest products by the wood processing firms. Except for raw wood, all other input factors are available at known prices and in quantities large enough not to act as constrains. The equilibrium stumpage price is determined according to

$$(3-1)\quad p = \frac{R-K}{Q}$$

where

p = maximun allowable stumpage price
R = wood processing firm revenue
K = costs of production less stumpage
Q = total rawwood consumption requirements.

The undesirable features of the model are: a) it assumes both forest owners and owners of wood processing firms to have the same preferences, b) it does not take into account the fact that forest owners may be spatially distributed. By definition stumpage in different locations cannot be treated as the same commodity. This makes it difficult to comprehend the idea of trading different commodities at a unique calculated equilibrium price.

Keipi (1978) model deals with the issue of transfer pricing of raw wood in an integrated forest products firm. To apply this model, we have to assume that forest owners and owners of wood processing firms are participants in a large integrated forest products firm. The model exhorts virtues of Walras tâtonnement process. For it to function, a Czar should be present to announce an arbitrary stumpage price at which stumpage should be traded. If at such a price there is either positive or negative excess demand, the appointed Czar makes the necessary price adjustments until excess demand is zero and trade takes place. This model is a good candidate for pricing stumpage although it falls short of being explicit on our economic agents' preferences and behaviuor.

3.1 Derivation of a forest owner's stock-supply model

A positive interaction between natural forest growing stock and silviculture is given by a general macro forest growth function

$$(3-2)\quad G(t) = G(V(t), S(t))$$

where

$G(t)$ = growth in cubic metres
$V(t)$ = growing stock in cubic metres
$S(t)$ = silviculture in monetary units
t = time

moreover, $G' > O$, $G'' < 0$.

A forest owner's enterprise is said to be in the long-run equilibrium when

$$(3-3)\quad \dot{V} = G(V(t), S(t)) - q = 0$$

$$(3-4)\quad \dot{S} = -sS + I = 0$$

where

$\dot{V}$ = time derivative of the growing stock
q = rawwood cuttings
$\dot{S}$ = time derivative of silviculture
s = depreciation rate of silviculture
I = silvicultural investments.

In the long-run equilibrium a forest owner cuts only the marginal growth of the growing stock and undertakes only silvicultural reinvestments. From Eqs. (3–3) and (3–4) it is possible to derive the stock-supply model by the usual marginal productivity principle.

We assume that a forest owner aims at maximizing the following linear objective functional

$$(3-5)\quad \max_{q(t)} \int_0^T e^{-\delta t}\, pq(t)\, dt$$

subject to

$$(3-6)\quad \dot{V} = G(V(t), S(t)) - q$$

$$(3-7)\quad \dot{S} = -sS + I$$

$$(3-8)\quad V(0) = V_0,\ V(T) = V_T$$

$$(3-9)\quad S(0) = S_0,\ S(T) = S_T$$

$$(3-10)\quad 0 \leqq q \leqq q_{max}$$

$$(3-11)\quad 0 \leqq I \leqq I_{max}$$

where δ, is the forest owner's positive rate of time preference, and T denotes his time horizon. In case the forest owner is a government agency, we should replace T, with an infinite time horizon because unlike a private owner with a finite life time, it seems plausible to consider society as being immortal. Moreover, there is no essential information concerning the choice of a government's time horizon (Vehkamäki 1986, p. 51). The control set Eq. (3=10) must also take into account ecological and amelioriation externalities.

By Pontryagin's maximun principle theorem (Pontryagin et al. 1962), a forest owner's realized and unrealized income is maximized as follows

$$(3-12)\quad H = e^{-\delta t} R + \lambda_1(sS+I) + \lambda_2(D(V(t), S(t)) - q$$

where

R = $pq(t)$
λ_1 = opportunity cost of capital invested in silviculture
λ_2 = shadow price of the growing stock (i.e., opportunity cost of waiting).

The first part of the Hamiltonia function is the forest owner's stumpage income given by the objective functional Eq. (3–5). The last two parts give his income postponed to the future.

Since the Hamiltonian function is concave in the control and state variables and the admissible set of the control variables is a convex set, by Mangasarian's theorem (Mangasarian 1966), the necessary conditions are also sufficient for a global optimun. Moreover, because the function is linear in the control variables, which take extreme values, the solution is a combination of singular and bang-bang solutions (see, e.g., Clark 1976, p. 92).

With the following conditions specified:

a) The state equations
b) the initial and terminal state conditions
c) the costate equations
d) the maximum principle,

we have the right number of conditions to determine the unknown functions; V(t), S(t), I(t), $\lambda_1(t)$, and $\lambda_2(t)$.

Optimum growing stock, V(t) and silvicultural investments, S(t), are determined by solutions to the state equations (3–3) and (3–4). The shadow prices of silvicultural investments and the growing stock satisfy the equations:

(3–13) $\dot{\lambda}_1 = \lambda_1 s - \lambda_2 G_s$

(3–14) $\dot{\lambda}_1 = -\lambda_2 G_v$

where

G_s = derivative of the growth function with respect to silviculture

G_v = derivative of the growth function with respect to the growing stock.

Solutions to the control variables are specified by the following equations:

(3–15a) $q = q_{max}$ if $\lambda_2 < e^{-\delta t}p$

(3–15b) $q = q^*$ if $\lambda_2 = e^{-\delta t}p$

(3–15c) $q = 0$ if $\lambda_2 > e^{-\delta t}p$

(3–16a) $I = I_{max}$ if $\lambda_2 > e^{-\delta t}$

(3–16b) $I = I^*$ if $\lambda_2 = e^{-\delta t}$

(3–16c) $I = I$ if $\lambda_2 < e^{-\delta t}$

311. Model

The switching function Eqs. (3–15b) and (3–16b) give the values of the control variables q and I when the forest owner's enterprise is in the long-run steady state equilibrium Eqs. (3–3) and (3–4). From the two switching functions, we derive the stock-supply model with respect to the growing stock and silviculture as follows:

By setting Eq. (3–14) equal to the time derivative of (3–15b) we get

(3–17) $G_v = \delta$.

If we substitute Eq. (3–15b) into Eq. (3–13) and then set Eq. (3–13) equal to the time derivative of Eq. (3–16b) we get

(3–18) $G_s = \dfrac{\delta + s}{p}$

or

(3=19) $pG_s = \delta + s$.

A forest owner's stock-supply model Eq. (3–17) with respect to the growing stock states that, a forest owner has to balance the optimum marginal growth of his growing stock to his time preference rate. Since in the longrun equilibrium quantity of cuttings equal the marginal growth of the forests, we may conclude that the smaller the absolute value of marginal growth calculated with respect to the growing stock, the greater the cuttings equalling growth.

The discount rate affects the optimum equilibrium growing stock V* as follows, *ceteris paribus*

(3–20) $0 = \delta \leqq V^* \leqq \delta = \infty$

(3–21) $V^* = \lim_{\delta \to \infty} V^*(\delta) = 0$

(3–22) $V^* = \lim_{\delta \to 0} V^*(\delta) = \bar{V}$

where $\bar{V}$ is the growing stock corresponding to the maximum growth. Eqs. (3–21) and (3–22) represent the rent dissipating level and rent maximizing level respectively. The interior solution, V*, Eq. (3–20) reflects the inevitable compromise between the desire for current versus future stumpage income.

The interpretation of the other component of stock-supply model Eq. (3–19) is simple; in the long-run equilibrium a forest owner will balance the value of the forest's marginal growth with respect to silviculture to capital costs arising from the rate of time preference and depreciation rate of silviculture.

Notice that Eqs. (3–17) and (3–18) are independent of time. To introduce dynamic behaviour, i.e., disequilibrium behaviour in the stock-supply model, we postulate a separable demand function

(3–23) $q(p(t),t) = z(p(t)\eta(t))$

which implies that $(\partial q/\partial p)\ (p/q) = (dz/dp)\ (p/z)$ and so the elasticity of demand, ε, and hence $MR = p(1+1/\varepsilon)$ are functions of p(t) only. Based on Eq. (3=23), we specify the changes in stumpage price as follows

(3–24) $p = p(0)e^{\eta(t)}$

where

$\eta \gtreqless 0$.

An alert reader must have noticed that in deriving our stock-supply model, we did not seek solutions to differential Eqs. (3–13) and (3–14). This would have resulted in a stock-supply model that could be used to study predictive dymanic behaviour of forest owners. On the contrary, the derived stock-supply model can only be used to study descriptive dynamic behaviour of a forest owner. In its dynamic form it tells us what, with given expectations, a forest owner will at a given moment decide to do. Arrow (1968, pp. 3–4) has characterized such behaviour as being myopic.

With the introduction of Eq. (3–24), the stock-supply model Eq. (3–17) becomes

(3–25) $G_v = \delta - \eta$.

The other component Eq. (3–18) remains the same; however, in light of Eq. (3–24), it becomes a dynamic model because p, is a function of time. We are now ready to study disequilibrium cutting and silvicultural investment behaviour due to short-period variations in demand for raw wood based on the stock-supply model.

In the long-run equilibrium, $\eta = 0$ therefore, cuttings and investments will be determined by

(3–26) $G_v = \delta$

(3–27) $G_s = \dfrac{\delta + s}{p}$

(3–28) $\dot{V} = G(V(t), S(t)) - q = 0$

(3–29) $\dot{S} = -sS + I = 0$.

The quantity of cuttings will be equal to the growth increment of the growing stock and only silvicultural reinvestments will take place.

An increase in demand for rawwood implies $\eta > 0$ which leads to the disequilibrium

(3–30) $G_v > \delta - \eta$

(3–31) $G_s > \dfrac{\delta + s}{p}$

This state of disequilibrium causes

(3–32) $\lambda_2 > e^{-\delta t}p$

(3–33) $\lambda_1 > e^{-\delta t}$

hence, a forest owner will drastically reduce the quantity of rawwood cuttings and at the same time undertake maximum silvicultural investments. This mode of behaviour will continue until that point in time when the increase in the growing stock and silviculture is in balance with the magnitude of increase in stumpage price. Thereafter, a new equilibrium

(3–34) $G_v = \delta - \eta$

(3–35) $G_s = \dfrac{\delta + s}{p}$

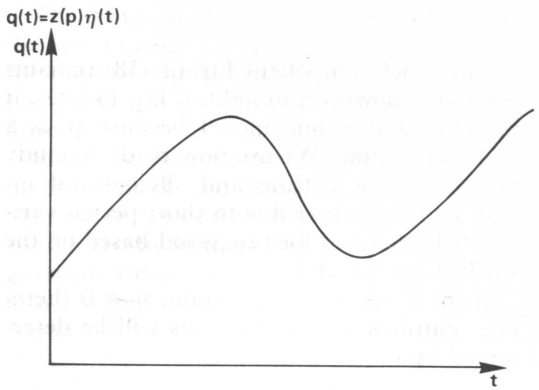

Figure 3. Short-term changes in the demand for roundwood.

$$(3-36)\quad \dot{V} = G(\overset{*}{V}(t),\overset{*}{S}(t)) - q^* = 0$$

$$(3-37)\quad \dot{S} = -sS + I^* = 0$$

is attained. Notice that the quantity of cuttings in this equilibrium is greater than the initial static equilibrium. The increase in cuttings is as a result of silvicultural investments that were undertaken when forestry was in a state of disequilibrium. However, since growing stock growth function is concave, the rate of silvicultural investments declines as the price increases. The expansion in the growing stock depends on the magnitude of condition Eq. (3–34).

Based on Eq. (3–23), if we assume a simple demand cycle Figure 3; then the quantity of cuttings will be different according to whether

$$(3-38)\quad G_V = \delta - \eta \gtreqless 0.$$

Finally, a decrease in demand for rawwood implies $\eta < 0$ which will generate the same pattern of disequilibrium behaviour as we have just elucidated but in the opposite direction.

Short-term changes in the demand for rawwood that are part of the changes in aggregate demand for commodities in the national economy lead to the decision by forest owners of either to cut or postpone cuttings, invest in silviculture or postpone such investments. Decisions made in the short-term have got only long-term impact on the growing stock. Silvicultural expenditures that are as a result of decisions made in the short-term cannot be a basis for stumpage appraisal as the cost-recovery appraisal method implies in certain cycles of forest owners that practise intensive silviculture. Opportunity cost of silviculture and opportunity cost of waiting determine whether or not a forest owner will cut timber or invest in silviculture. As soon as capital is invested in the forest growth, all silvicultural expenditures momentarily become sunk costs.

32. Information – based stumpage appraisal model

In section 2, the quantity of timber to be supplied by a forest owner was uniquely determined as assumed to be equal to marginal growth on his growing stock. The limit theorem established a correspondence between the core solution set and the competitive equilibrium market prices. This was achieved under the restrictive assumptions underlying partial equilibrium analysis. We shall now move away from partial equilibrium framework and consider the demand for stumpage as part of the aggregate demand in a closed economy with flexible prices.

The equilibrium stumpage supply for a given forest owner corresponds to the supply curve $q^* - q^*$ in Figure 4.

Due to protean nature of forest growing stock, exigencies of the moment that emerge as a result of cyclical shifts in demand can be met by a forest owner either by cutting more than or less than the marginal increment of the growing stock. Based on already derived functions

$$\pi_1'(p) > 0$$

$$p = p(0)\, e^{n(t)}$$

the forest owner's supply curve $q^* - q^*$ changes to $s - s$, Figure 4.

The supply function corresponding to the curve $s - s$, in Figure 4 is

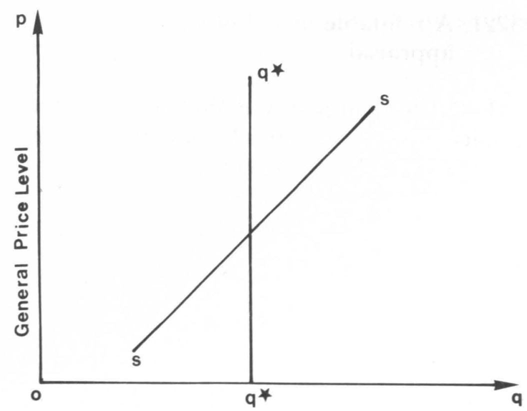

Figure 4. Stumpage supply curve.

$$(3-39)\quad q_t = \alpha(p_t - p_t^e) + q_t^*$$

where

q_t = cuttings in period t
p_t = stumpage price in period t
p_t^e = forest owner's perception of the general price level in period t
q_t^* = equilibrium stumpage supply in period t
α = parameter.

From the general price level P, a forest owner is able to determine the opportunity cost of silviculture and opportunity cost of waiting. A forest owner will cut

$$(3-40)\quad q_t \gtreqless q_t^* \text{ whenever } (3-41)\quad p_t \gtreqless p_t^e,$$

so that

$$(3-42)\quad q_t \gtreqless q_t^* \equiv G(\overset{*}{V}(t),\overset{*}{S}(t)).$$

According to Eq. (3–42) we are only able to unequivocally state whether a forest owner is overcutting or undercutting if we know the marginal increment of the growing stock and the level of silviculture.

321. Model

Following the approach of Lucas (1972a), (1972b), and (1973), stumpage price p_t, relative to the general price level p_t, is given by

$$(3-43)\quad p_t = P_t + \eta(t)$$

where $\eta(t)$, is the relative demand shift. The best estimate of the general price level from a forest owner's viewpoint is

$$(3-44)\quad E_{t-1} \equiv E(P_t/\Omega_{t-1}).$$

This identity states that the rational expectation of P_t, is formed at period t−1 by taking the mathematical expectation E, conditional on the latest information set Ω_{t-1} including knowledge of causal dynamics in P_t or failing that some estimation of it. Hence,

$$(3-45)\quad p_t^e = \tilde{P} + (\sigma_p^2/\sigma_p^2 + \sigma_\eta^2)(p_t - \tilde{P}_t)$$

where $\tilde{P}$, is the unconditionally expected level of prices, σ_p^2 and σ_η^2 is the variance of p and η respectively.

Substituting Eq. (3–45) into Eq. (3–39) results in

$$(3-46)\quad q_t = \alpha(1 - \beta)(p_t - P_t) + q^*$$

where

$$\beta = \frac{\sigma_p^2}{\sigma_p^2 + \sigma_p^2}\ .$$

By transferring p_t, in Eq. (3–46) to the left-hand side and subtracting the lagged general price level from both sides we obtain the model

$$(3-47)\quad p_t - P_{t-1} = P_t - \tilde{P}_{t-1} + \alpha(1 - \beta)(q_t - q_t^*).$$

322. Interpretation of the model based on the nature of stumpage market

Stumpage price in the roundwood market is an outcome of two decision makers determined in the following way

$$(3-48) \quad p = \gamma_1 H + \gamma_2 X + \xi$$

where

H = forest owner's policy instrument
X = owner of the wood processing firm policy instrument
ξ = stochastic term
γ_1, γ_2 = parameters.

If we postulate the strategy of a forest owner and wood processing firm entrepreneur to be respectively

$$(3-49) \quad H_t = b_t p_{t-1} + g_t$$

$$(3-50) \quad X_t = c_t p_{t-1} + m_t$$

where g_t and m_t are stochastic terms, then the controlled model behaves as

$$(3-51) \quad p_t = a_0 + (a_1 + \gamma_1 b_t + \gamma_2 c_t)p_{t-1} + (\xi + \gamma_1 g_t + \gamma_2 m_t).$$

Non-causality arises in the stumpage appraisal issue because of the joint dependence between H_t and X_t in that both determine p_t. Consequently, for either a forest owner or wood processing firm entrepreneur to achieve optimality the simultaneity between H and X must be accounted for as a whole at any given time t. Eq. (3–51) constitutes the Nash equilibrium of the roundwood trade and is part of the contract curve.

If on the other hand, the two participants fail to react to each other, or to any endogenous or endogenized variable in the controlled model, causality arises. This will imply

$$(3-52) \quad \frac{\partial H_t}{\partial X_t} = 0$$

$$(3-53) \quad \frac{\partial X_t}{\partial H_t} = 0$$

This behaviour violates the requirements of our information-based model. Moreover, such behaviour can only occur if the decision makers do not attempt to learn from their past mistakes so that policy revisions in terms of

$$(3-54) \quad p_t - E_{t-1}p_t = \xi$$

are non existent. Therefore, Cournot strategy implied by Eqs. (3–52) and (3–53) is unstable and suboptimal as such, unacceptable.

3221. A refutable model of stumpage appraisal

Following Sargent and Wallace (1975) approach, we may specify an independent strategy for a forest owner as

$$(3-55) \quad p_t = \varrho H_t + \alpha(q_t - E_{t-1}q_t) + \eta_t$$

where

$$\alpha = 0 < \alpha < 1.$$

According to Eq. (3–55), if qt = $E_{t-1}q_t$, then $p_t = P_t$. Otherwise, a relative demand increase will imply $p_t > P_t$ and vice versa for a relative demand decrease.

A wood processing firm may have the strategy

$$(3-56) \quad q_t = \gamma X_{t-1} + u_t$$

where

q_t = quantity of stumpage demanded in period t
u_t = a stochastic term.

We may rewrite Eq. (3–56) as

$$(3-57) \quad E_{t-1}q_t = \gamma X_{t-1} + u_t$$

and combine Eq. (3–57) with Eq. (3–55) to obtain a two-equation

$$(3-58a) \quad q_t = \gamma X_{t-1} + u_t$$

$$(3-58b) \quad p_t = \varrho H_t + \alpha u_t + \eta_t$$

The first equation says that only unexpected demand influences stumpage price, while the second equation says that the expected stumpage price is equal to the predictable component of the process determining demand.

Solving the first part of Eq. (3–58) in terms of u_t and substituting in the second part results in

$$(3-60) \quad p_t = \varrho H_t + \alpha q_t - \alpha\gamma X_{t-1} + \eta_t.$$

This is the stumpage appraisal rule that a forest owner will apply based on his expectations and will continue to do so until such a time that his expectations are proved incorrect. When this happens, he reformulates his expectations, enacts a new rule and he proceeds as before.

Notice how the assumption of rational expectations constraints the parameters of X_{t-1} in Eq. (3–60) to be $-\alpha\gamma$. A negation of this assumption will imply that the parameters of X_{t-1} in Eqs. (3–60) and (3–56) should be independent of each other and assume any magnitude. However, if a forest owner formulates his expectations according to the rational expectations hypothesis, the independence vanishes. Expectations must be formed in a way which is restricted to be in accordance with the process which demand for stumpage actually follows.

To determine whether the assumption of rational expectations is significant, a likelihood ratio-test statistic is computed and compared with the chi-square distribution with the appropriate degrees of freedom. The same computation has to be done for an equivalent specification of unrestricted model in order to find out if the restricted model performs better than the unrestricted model or not to be able to accept or reject the restricted model (see, e.g., Attfield et al. 1981a and 1981b).

4. Concluding remarks

We may appraise stumpage in many different ways as long as we have no theory about it and be forgiven. However, if there happens to be a theory behind our appraisal methods, our policies cease to be free from reproachment. Institutional factors, and appraisal methods that are based on such factors, cannot be explained adequately within the framework of economic theory. The best thing we can do is to treat an appraisal policy based on economic theory as a benchmark of judging the efficiency of any kind of non-economic stumpage appraisal policy.

The explanatory power of price-determined stumpage appraisal methods diminishes as we move away from frontier economy situations to environments in which forest resource becomes a scarce commodity whose availability is ensured by introduction of silviculture in the growing stock. The philosophy of derived demand is not explicit on the issue of the existance of producers of capital goods. In the case of forestry, it fails to explain the crucial issue of when an efficient forest owner is absorbing inefficiency accruing out of a mismanaged wood processing firm. Price as a unit of measurement is a derived concept; therefore, in determining prices we must somehow envisage derived demand for whatever commodity under consideration. It seems to be more realistic to picture a forest owner as a quantity adjuster whose actions are based on the behavioural assumption of profit maximization just like any other entrepreneur in the national economy.

Stumpage price has got undoubtful important influence on the level of intensive silviculture a forest owner will undertake. There is nothing outrageous about taking a viewpoint of absolute rent when pricing timber that is a capital asset. However, it is the demand for timber and the ability of a forest owner to discern his opportunity cost of silviculture and opportunity cost of waiting from the expected general price level that will influence the market value of stumpage and not the magnitude of silvicultural expenditures incurred.

A forest owner's behaviour with respect to when to cut timber, and whether to cut more than or less than the marginal growth of the growing stock is influenced by the prevailing aggregate demand in the national economy. Relative demand shifts as a significant feature of the national economy account for the observed differences in stumpage prices among spatially distributed forest owners. This view to stumpage appraisal has an economic meaning as opposed to the cutting policy determined by Faustmann formula based on either physical or financial data. This policy cannot account for disequilibrium behaviour hence, the evolution of a growing stock from an arbitrary forest working circle.

In traditional theory, the assumption of rationality underlies the prediction of economic behaviour forthcoming from agents operating in a timeless equilibrium. In time dependent economic activities, the assumption of rationality should be construed *ex ante*. It is therefore worthwhile to interpret the stochastic terms in the derivation of the trading rule as the difference between the actual estimate and an optimal estimate of stumpage price, rather than as the estimate and the actual value of a future price. Equilibrium prices based on *ex ante* rationality will always be Pareto optimal regardless of the number of participants in the trade, *ceteris paribus*.

The gist of this study has been to present an analytical approach to determination of forest land in a given nation that is socially optimal and stumpage price level that will maintain a state of balance between land under forestry versus land under other economic activities. Zivnuska (1975, pp. 16–23) outlined the two problems but fell short of demonstrating how optimal solutions corresponding to the two problems may be obtained.

The stock-supply model provides the solution to the problem of determining a socially optimal land that should be under forest management. In the long-run equilibrium, the stock-supply model determines the optimum growing stock V*. Cuttings from land occupied by V* are able to meet the national rawwood requirements because the optimal growing stock has been determined on the basis of market forces operating in the aggregate economy.

The stock-supply model provides a clear approach to the issue of how much land should be under forest management as compared to some methods that have been put forward so far. For example, econometric studies begin by performing a gap analysis, then translate the national quantity of forest products requirements into its roundwood equivalent and finally translate the roundwood quantities into land requirements. This is a painstaking approach that takes cutting and investment behaviour from the veiwpoint of a forest owner for granted. On the other hand, it is very easy to prove that the forest working circle delineated by a residual stumpage model puts more land under forest management than is socially optimal (see, e.g., Ledyard and Moses 1976, pp. 141–157).

Finally, it is very unfortunate that at the national level outside forestry cycles, in formulating forest policy, more emphasis is put on externalities than on the impact of fiscal and monetary policies. The aim of such forest policy is to justify the existence of forests rather than timber production. For example, a monetary policy that does not take into consideration the impact of real interest rates on forest conservation reinforces von Thunen (1875) fear that the discount rate could lead to the destruction of forests. To promote forest conservation, a good monetary policy has to ensure that the ratio

$$\theta = \frac{\delta}{\varkappa}$$

where δ, is the real discount rate and $\varkappa$, is the mean growth of the forests; does not exceed unit.

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