Silva Fennica 1986, vol. 20 n:o 3: 245–250

A Comparison of replacement strategies in continuous forest inventory

Peng Shikui

TIIVISTELMÄ: KOEALOJEN KORJAUSMENETELMIEN VERTAILU JATKUVASSA METSIEN INVENTOINNISSA

Shikui, P. 1986. A comparison of replacement strategies in continuous forest inventory. Tiivistelmä: Koealojen korjausmenetelmien vertailu jatkuvassa metsien inventoinnissa. Silva Fennica 20(3): 245–250.

Three replacement strategies in continuous forest inventory of the Enso-Gutzeit Company have been presented and discussed. The first strategy adopts data from only the last two inventory occasions; the second strategy employs data from all four occasions, in which there are two groups of permanent plots measured on the first three occasions and independently on the last two occasions; the third strategy also utilizes data from all four occasions, but includes only one group of permanent plots measured on all four occasions. Results indicate that the last strategy is best for the efficiency. The difference between the first two strategies is small.

Kolme korjausmenetelmää Enso-Gutzeit-yhtiön jatkuvassa metsien inventoinnissa on esitetty ja arvioitu. Ensimmäinen ryhmä koealoja mitattiin kolmessa ensimmäisessä inventoinnissa ja jälkimmäinen ryhmä edellisestä riippumatta kahdessa viimeisessä. Ensimmäinen menetelmä käyttää vain kahden viimeisen inventoinnin aineistoja; toinen menetelmä hyödyntää kaikkien neljän inventoinnin aineistoja; kolmas menetelmä hyödyntää myös kaikkien neljän inventoinnin aineistoja, mutta vain yhden koealaryhmän osalta. Tulokset osoittavat, että viimeinen menetelmä on paras tehokkuudeltaan. Kahden ensimmäisen menetelmän ero on pieni.

Keywords: CFI, estimation, SPR
ODC 524.63
Author's address: University of Helsinki, Department of Forest Inventory. Unioninkatu 40 B, SF-00170 Helsinki, Finland
Approved on 10. 12. 1986

1. Introduction

Theoretically, principles and basic strategies of the sampling with partial replacement (SPR) on two or more than two occasions in continuous forest inventory (CFI) have been developed and formulated by Ware and Cunia (1962), and Cunia and Chevrou (1969). In practice, however, as with increase of inventory times, a variety of replacement strategies might appear in CFI, and therefore how to appraise and treat the possibly encounted strategies should be taken into account. The main purpose of this paper is to appraise and compare some strategies of SPR met in CFI in the Enso-Gutzeit Company in Finland.

With the total forest land area of 3450 sq. km., of which, the forest area is 3200 sq. km., the Enso-Gutzeit Company has made four continuous forest inventories. The first inventory was made in 1958–1959. More than 20 000 sample plots (line-plot) were measured, of which about one tenth are permanent plots. In the second inventory in 1965 only permanent plots were remeasured. For the third inventory made in 1975, besides the remeasurement of permanent plots, about 10 000 new plots were established, of which, 1246 plots were marked. During the most recent inventory, in 1983, only the permanent plots which were established in 1975 were remeasured. The sampling design, the surveying method, final results and analysis concerning these continuous forest inventories in the Enso-Gutzeit Company, especially the first three inventories, have been described and discussed in detail by Nyyssönen (1967, 1972, 1981).

According to the sample information available from the four inventories in the company, three replacement strategies of SPR on four occasions will be discussed.

First, we should explain the following points:

(a) The estimation formulae in the present paper, except those in strategy 1, have been newly derived for the cases met in the company under the condition of a given population variance, according to the way developed by Ware and Cunia (1962) and Cunia and Chevrou (1965, 1969).

(b) The population variance is not available for a large forest area, so sample variances and covariances (or correlation coefficients) will be used in the estimation formulae instead of population variances and covariances.

(c) The main symbols employed in this paper are explained as follows:

$\bar{x}^{(i)}_{j_1 j_2 \ldots}$ = mean volume of a sample on the ith occasion, $j_1 j_2 \ldots$ refer to the occasions on which the sample is measured and remeasured

S^2_j = sample variance of the volume on the jth occasion

S_{jk} = sample covariance of the volume between the jth and the kth occasions

r_{jk} = sample correlation coefficient of the volume between the jth and the kth occasions

$n_{j_1 j_2 \ldots}$ = number of sample plots for the sample which is measured and remeasured on occasions j_1 and j_2 . . . .

Table 1 presents the sample information from four inventories in the Enso-Gutzeit

Table 1. Sample information from four inventories in Enso-Gutzeit

Occasion	Sample or sample plot number	Mean volume of sample	Occasion 1 (Variance and covariance)	2	3	4
1	n_1	$\bar{x}^{(1)}_1$	S^2_1	S_{12}	S_{13}	0
2	n_{123}	$\bar{x}^{(2)}_{123}$		S^2_2	S_{23}	0
3	n_3	$\bar{x}^{(3)}_3$			S^2_3	S_{34}
	n_{123}	$\bar{x}^{(3)}_{123}$				
	n_{34}	$\bar{x}^{(3)}_{34}$				
4	n_{34}	$\bar{x}^{(4)}_{34}$				S^2_4

2. Replacement strategies and formulae

On the basis of the sample information available from the Enso-Gutzeit Company, two replacement strategies will be used to estimate the current volume on the last occasion and the change between the last two occasions. In addition, for comparison and analysis an alternative replacement strategy will be assumed.

2.1. Strategy 1 — using only the sample information available on the last two occasions

The estimator of current volume and its variance

The estimator $\bar{x}$ of current volume is

$$\bar{x} = \bar{x}^{(4)}_{34} - b_1 (\bar{x}^{(3)} - \bar{x}^{(3)}_{34}) \qquad (81)$$

where

$\bar{x}^{(3)}$ = mean volume of all sample plots on the third occasion including both permanent plots and temporary plots

$b_1 = r_{34} S_4/S_3$.

Its variance estimator $S^2_{\bar{x}}$ is

$$S^2_{\bar{x}} = S^2_4 \left(\frac{1-r^2_{34}}{n_{34}} + \frac{r^2_{34}}{T_3}\right) \qquad (2)$$

where $T_3 = n_3 + n_{34} + n_{123}$.

The estimator of change and its variance

The estimator $\bar{\Delta}$ of change is

$$\bar{\Delta} = \bar{x}^{(4)}_{34} - b_2 \bar{x}^{(3)}_{34} - (1-b_2)\bar{x}^{(3)} \qquad (3)$$

where $b_2 = b_1 = r_{34}S_4/S_3$.

Its variance estimator $S^2_{\bar{\Delta}}$ is

$$S^2_{\bar{\Delta}} = \frac{S^2_4(1-(1-n_{34}/T_3)r^2_{34})}{n_{34}} + \frac{S^2_3 - 2r_{34}S_3S_4}{T_3} \qquad (4)$$

The above four formulae were presented and used by Nyyssönen (1967).

2.2. Strategy 2 — using all sample information available on four occasions

The estimator of current volume and its variance

The estimator $\bar{x}$ of current volume is

$$\bar{x} = a_1 (\bar{x}^{(1)}_1 - \bar{x}^{(1)}_{123}) + b_1 (\bar{x}^{(3)} - \bar{x}^{(3)}_{34}) + c_1 (\bar{x}^{(3)} - \bar{x}^{(3)}_{123}) + \bar{x}^{(4)}_{34} \qquad (5)$$

Its variance estimator $S^2_{\bar{x}}$ is

$$S^2_{\bar{x}} = \frac{S^2_4}{n_{34}} \left(1 - \frac{b_1 r_{34} S_3 (n_3 + n_{123})}{S_4 T_3} + \frac{c_1 r_{34} S_3 n_{34}}{S_4 T_3}\right) \qquad (6)$$

In formulae (5) and (6) a_1, b_1 and c_1 are unknown coefficients which are available, on the basis of the principle of minimizing variance (Cunia and Chevrou 1969), by solving the following equations:

$$\sum L' = K'$$

where

$$L = [a_1 \; b_1 \; c_1]$$

$$K = [0 \;\; S_{34}(1/n_{34} - 1/T_3) \;\; -S_{34}/T_3]$$

$$\sum = \begin{bmatrix} S^2_1 (1/n_1 + 1/n_{123}) & -S_{13}/T_3 & S_{13}(1/n_{123} - 1/T_3) \\ -S_{13}/T_3 & S\,(1/n_{34} - 1/T_3) & -S^2_3/T_3 \\ S_{13}(1/n_3 - 1/T_3) & -S^2_3/T_3 & S^2_3(1/n_{123} - 1/T_3) \end{bmatrix}$$

The estimator of change and its variance

The estimator of change $\bar{\Delta}$ is

$$\bar{\Delta} = a_2(\bar{x}^{(1)}_1 - \bar{x}^{(1)}_{123}) + b_2(\bar{x}^{(3)} - \bar{x}^{(3)}_{34}) + c_2(\bar{x}^{(3)} - \bar{x}^{(3)}_{123}) + \bar{x}^{(4)}_{34} - \bar{x}^{(3)} \qquad (7)$$

Its variance estimator $S^2_{\bar{\Delta}}$ is

$$S^2_{\bar{\Delta}} = S^2_4/n_{34}(1-b_2r_{34}S_3(n_3+n_{123}(/S_4T_3 + c_2r_{34}S_3n_{34}/S_4/T_3) + S^2_3(1+a_2r_{13}S_1/S_3)/T_3 - 2S_3S_4r_{34}/T_3 \quad (8)$$

Where a_2, b_2 and c_2 can be obtained by solving the equations

$$\sum L' = K'$$

where $L_2 = [a_2\ b_2\ c_2]$ and $K = [-S_{13}/T_3\ S_{34}(1/n_{34}-1/T_3\ -S_{34}/T_{34}]$

Note that there is no sample information on the second occasion in formulae (5)– – –(8) due to that there are no temporary plots on the second occasion, although a sample of permanent plots exits on this occasion.

It should be pointed out that in formulae (5) and (7) $\bar{x}^{(3)}_3$ can be used instead of $\bar{x}^{(3)}$, in turn, the associated formulae (6) and (8) on variances would be formally changed. The reason for using $\bar{x}^{(3)}$ is to maintain consistency with formulae (1) and (3) in the form. In fact, formulae (5) and (7) can be rewritten in several ways for convenience.

2.3. Strategy 3 — an assumed strategy

As a contrast, let us now consider another strategy of sample replacement. Sample n_{34} is replaced by sample n_{1234} which is drawn from sample n_{123} and correlated to sample n_{123}. It is just this point which differs from n_{123} of strategy 2. Other samples of the strategy are the same as those in replacement strategy 2.

The estimator of current volume and its variance

The estimator of current volume is

$$\bar{x} = a_1(\bar{x}^{(1)}_1-\bar{x}^{(1)}_{123}) + b_1(\bar{x}^{(1)}_{1234}-\bar{x}^{(1)}_{123}) + c_1(\bar{x}^{(3)}_3-\bar{x}^{(3)}_{123}) + d_1(\bar{x}^{(3)}_{1234}-\bar{x}^{(3)}_{123}) + \bar{x}^{(4)}_{1234} \quad (9)$$

Its variance estimator is

$$S^2_{\bar{x}} = S^2_4/n_{1234} - a_1S_{14}/n_{123} + b_1S_{14}(1/n_{1234}-1/n_{123}) - c_1S_{34}/n_{123} + d_1S_{34}(1/n_{1234}-1/n_{123} \quad (10)$$

Here, a_1, b_1, c_1 and d_1, are unkown coefficients which are available from solving the following equations:

$$\sum L' = K'$$

where

$$\sum = \begin{bmatrix} S^2_1(1/n_1+1/n_{123}) & 0 & S_{13}/n_{123} & 0 \\ 0 & S^2_1(1/n_{1234}-1/n_{123}) & 0 & S_{13}(1/n_{1234}-1/n_{123}) \\ S_{13}/n_{123} & 0 & S^2_3(1/n_3+1/n_{123}) & 0 \\ 0 & S_{13}(1/n_{1234}-1/n_{123}) & 0 & S^2_3(1/n_{1234}-1/n_{123}) \end{bmatrix}$$

$$L = [a_1\ b_1\ c_1\ d_1]$$

$$K = [S_{14}/n_{123}\ -S_{14}(1/n_{1234}-1/n_{123})\ S_{34}/n_{123}\ -S_{34}(1/n_{1234}-1/n_{123})]$$

Note that in the above formulae n_{1234} is a part of n_{123} and $T_3 = n_3 + n_{123}$ instead of $T_3 = n_3 + n_{123} + n_{34}$ in strategy 2.

The estimator of change and its variance

The estimator of change is

$$\bar{\Delta} = a_2(\bar{x}^{(1)}_1-\bar{x}^{(1)}_{123}) + b_2(\bar{x}^{(1)}_{1234}-\bar{x}^{(1)}_{123}) + c_2\bar{x}^{(3)}_3-\bar{x}^{(3)}_{123}) + d_2(\bar{x}^{(3)}_{1234}-\bar{x}^{(3)}_{123}) + \bar{x}^{(4)}_{1234}-\bar{x}^{(3)}_3 \quad (11)$$

Its variance estimator is

$$S^2_{\bar{\Delta}} = S^2_4/n_{1234} - a_2S_{14}/n_{123} + b_2S_{14}(1//n_{1234}-1/n_{123}) - c_2S_{34}/n_{123} + d_2S_{34}(1/n_{1234}-1/n_{123}) + S^2_3/n_3(1-c_2) \quad (12)$$

where a_2, b_2, c_2 and d_2 can be obtained by solving the following equations:

$$\sum L' = K'$$

and $L = [a_2\ b_2\ c_2\ d_2]$

$$K = [S_{14}/n_{123}\ -S_{14}(1/n_{1234}-1/n_{123})\ S_{34}/n_{34}+S^2_3/n_3\ -S_{34}(1/n_{1234}-1/n_{123})]$$

3. Results and discussion

Table 2 demonstrates results about estimates and variances in the three replacement strategies. The mean value $\bar{I}$ of total growth, which here includes ingrowth, growth of survivals and removals, and its variance $S^2_{\bar{I}}$ for the first two strategies were calculated by formulae (3), (4) and (7) (8) respectively. $S^2_{\bar{I}}$ for strategy 3 was estimated by formula (12).

From Table 2 it is observable that the estimation efficiency of strategy 2 is better than those in strategy 1 since its variance of either current volume or its change is less than that in strategy 1. This result can be expected because strategy 2 has more sample information. The differences of the variances, however, are not significant. The reason for this is that there is no sample which can directly connect information of sample n_1 on the first occasion with the estimator on the fourth occasion. Sample n_1 can affect the estimator of change between the third and fourth occasions through sample n_{123} to some extent but it can not affect the estimator of current volume on the fourth occasion. In fact, the most effective sample is sample n_{34} for estimating either current volume or change. This is why $S^2_{\bar{\Delta}}$ of two strategies are nearly eqal in table 2. Consequently, in the present case of the sample replacement strategy in the Enso-Gutzeit Company, strategy 1 is a feasible strategy since its loss of efficiency is small and its estimation formulae are simple.

For strategy 3 there is a permanent plot sample n_{1234} which has a set of measured values for each occasion, from the first occasion to the fourth occasion. Then sample n_1, together with sample n_3, can directly affect the estimator of the fourth occasion through sample n_{1234}. As a result, its efficiency is not only better than strategy 1 but also better than strategy 2. The results in Table 2 are consistent with the above analyses.

It should be explained that since strategy 3 is only used for comparison, and is not put into the practice, some necessary information for calculating $S^2_{\bar{I}}$ such as the correlation coefficient r_{14} of the volume between the first and the last occasions had to be approximately estimated (r_{14}=.13 for change and .32 for total growth).

Generally speaking, if other conditions are the same, the efficiency of the replacement strategy using a sample of permanent plots which are remeasured for each occasion is not less than that of the replacement strategy using two or more samples of permanent plots which are connected with each other and independent on each other from the first occasion to the last occasion (Peng 1982). In addition, another advantage of keeping a permanent plot sample with a series of observation values in succession is to be able to provide much useful information about growth and mortality for forest management, construction of yield tables and prediction of growth. A drawback for this strategy is the possibly biased estimation caused by conspicuous permanent plots if without any replacement for a long period.

In fact, it is difficult to maintain a sample of permanent plots and not to change any part of it because of changes in borders and the loss of permanent plots. So long as there is a permanent plot sample big enough in the period concerned, it does not matter too much whether additional materials from other periods will be involved

Table 2. Estimates and variances of three strategies

Replacement strategy	Current volume (m^3/ha)		Change (m^3/ha)		Total growth (m^3/ha)	
	$\bar{x}$	$S^2_{\bar{x}}$	$\bar{\Delta}$	$S^2_{\bar{\Delta}}$	$\bar{I}$	$S^2_{\bar{I}}$
1	67.83	3.0585	4.3866	2.8090	24.7780	.4670
2	68.01	3.0529	4.4345	2.8085	24.8061	.4636
3	—	2.9654	—	2.6320	—	.4399

4. Conclusions

Using all sample information available in multitemporal forest inventories contributes to the increase in efficiency for estimating either current volume or change, but if the permanent plots were divided into two independent parts by periods like the case met in the Enso-Gutzeit Company, the replacement strategy which utilizes only the sample information on the last two occasions is feasible since its formulae are simple and loss of the efficiency is small. From a point of the estimation efficiency, the replacement strategy including a sample of permanent plots remeasured during every inventory is better than that including two or more independent samples of permanent plots by periods if temporary sample plots in every period are invovled.

5. Acknowledge

The author expresses his sincere thanks to the helps given by Professor A. Nyyssönen, Professor S. Poso, the Department of Forest Mensuration and Management of the University of Helsinki and the Enso-Gutzeit Company, and to Mr. Antti Penttinen of Jyväskylä University who checked the statistical methods.

References

Cunia, T. 1965. Continuous forest inventory with partial replacement of samples and multiple regression. For. Sci. 11(2): 480–502.

— & Chevrou, R. B. 1969. Sampling with partial replacement on three or more occasions. For. Sci. 15(2): 204–224.

Nyyssönen, A. 1967. Remeasured sample plots in forest inventory. Medd. Norske Skogforsøksvesen 84: 180–220.

— 1972. Forest inventory with remeasured sample plots. Proc. International Scientific Conference, Zvolen, pp. 299–312.

— 1981. Remeasurement of sample plots in management plan inventories: some experiences. Proc XVII IUFRO Wold congress in Japan.

Peng Shikui. 1982. A study of continuous forest inventory on more than two occasions. (In Chinese, the abstract in English), Journal of Nanjing Technological College of Forest Products 1982(1): 1–19.

Ware, K. D. & Cunia, T. 1962. Continuous forest inventory with partial replacement of samples. For. Sci. Monog. 3: 1–40.

Kirjoitusten julkaiseminen Silva Fennica-sarjassa

Julkaistavaksi aiotusta käsikirjoituksesta lähetetään Suomen Metsätieteellisen Seuran sihteerille *kolme* täydellistä, viimeisteltyä kopiota, joihin sisältyy myös kopiot kaikista kuvista ja taulukoista. Originaaliainestoa ei tässä vaiheessa pidä lähettää.

Seuran sihteeri lähettää vastaanotetut käsikirjoitukset valitsemilleen ennakkotarkastajille. Suomen Metsätieteellisen Seuran hallitus päättää kirjoituksen julkaisemisesta ennakkotarkastajien lausuntojen ja muiden ilmenneiden seikkojen perusteella.

Kirjoituksen tekijä on vastuussa kirjoituksen tieteellisestä asiasisällöstä ja kieliasusta. Tekijä ei saa julkaista muualla kyseistä kirjoitusta ilman seuran suostumusta eikä seuran sarjoihin pidä tarjota aiemmin julkaistuja kirjoituksia.

Seuran hallitus päättää kirjoituksen julkaisemisesta neljän kuukauden kuluessa siitä, kun täydelliset käsikirjoituskopiot ovat seuralle tulleet.

Tekijän on otettava huomioon käsikirjoitusta korjatessaan ennakkotarkastajien ja seuran hallituksen korjausesitykset tai ennen toimittajalle jättämistä ilmoitettava eriävä mielipiteensä seuran sihteerille tai hallitukselle, joka tarvittaessa ottaa asian uuteen käsittelyyn. Käsikirjoitukseen ei saa enää hyväksymisen jälkeen tehdä olennaisia muutoksia ilman seuran sihteerin lupaa. Suuret muutokset edellyttävät uutta hyväksymiskäsittelyä.

Tekijän tulee antaa korjattu, alkuperäinen käsikirjoitus ja kuvaoriginaalit seuran toimittajalle tai seuran toimistoon kahden kuukauden kuluessa seuran hallituksen hyväksymispäätöksestä. Käsikirjoituksen saatteesta tulee selvästi ilmetä, että käsikirjoitus on lopullinen, kirjapainoon tarkoitettu kappale.

Silva Fennica 20(1):ssä (1986, s. 75–81) on julkaistu laajemmat ohjeet käsikirjoitusten hyväksymismenettelystä ja laatimisesta. Julkaisemisen edellytyksenä on, että näitä ohjeita on noudatettu. Ohjeita on saatavissa seuran toimistosta.

Manuscripts for Silva Fennica should be addressed to the Society of Forestry in Finland in three copies. Author's full name and address must be stated. The typography should be in accordance with the latest volumes in the series.

KANNATTAJAJÄSENET – SUPPORTING MEMBERS

CENTRALSKOGSNÄMNDEN SKOGSKULTUR
SUOMEN METSÄTEOLLISUUDEN KESKUSLIITTO
OSUUSKUNTA METSÄLIITTO
KESKUSOSUUSLIIKE HANKKIJA
OY WILH. SCHAUMAN AB
KEMIRA OY
G. A. SERLACHIUS OY
KYMI-STRÖMBERG OY
KESKUSMETSÄLAUTAKUNTA TAPIO
KOIVUKESKUS
A. AHLSTRÖM OSAKEYHTIÖ
TEOLLISUUDEN PUUYHDISTYS
OY TAMPELLA AB
KAJAANI OY
KEMI OY
MAATALOUSTUOTTAJAIN KESKUSLIITTO
VAKUUTUSOSAKEYHTIÖ POHJOLA
VEITSILUOTO OSAKEYHTIÖ
OSUUSPANKKIEN KESKUSPANKKI OY
SUOMEN SAHANOMISTAJAYHDISTYS
OY HACKMAN AB
YHTYNEET PAPERITEHTAAT OSAKEYHTIÖ
RAUMA REPOLA OY
OY NOKIA AB NOKIAN PAIKALLISHALLINTO
JAAKKO PÖYRY OY
KANSALLIS-OSAKE-PANKKI
SOTKA OY
THOMESTO OY
SAASTAMOINEN OY
OY KESKUSLABORATORIO
METSÄNJALOSTUSSÄÄTIÖ
SUOMEN METSÄNHOITAJALIITTO
SUOMEN 4H-LIITTO
SUOMEN PUULEVYTEOLLISUUSLIITTO R.Y.
OY W. ROSENLEW AB
METSÄMIESTEN SÄÄTIÖ
SÄÄSTÖPANKKIEN KESKUS-OSAKE-PANKKI
ENSO-GUTZEIT OY

Hämeenlinna 1986, Karisto Oy:n kirjapaino