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Predicting diameter growth in even-aged Scots pine stands with a spatial and non-spatial model

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TIIVISTELMÄ: LÄPIMITAN KASVUN ENNUSTAMINEN TASAIKÄISESSÄ MÄNNIKÖSSÄ SPATIAALISELLA JA EI-SPATIAALISELLA MALLILLA

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The single tree growth models presented in this study were based on about 4000 trees measured in 50 even-aged Scots pine sample plots with varying density, spatial pattern of trees and stand age. Predictors that used information about tree locations decreased the relative standard error of estimate by 10 percentage points (15 %), if past growth was not used as a predictor, and about 15 percentage points (30 %) when past growth was one of the predictors. When ranked according to the degree of determination, the best growth models were obtained for the basal area increment, the next best for relative growth, and the poorest for diameter increment. The past growth decreased the relative standard error of estimate by 15...20 percentage points, but did not make the spatial predictors unnecessary. The degree of determination of the spatial basal area growth model was almost 80 % if the past growth was unknown and almost 90 % if the past growth was known. Variables that described the amount of removed competition did not improve the growth models.

Tutkimuksessa esitetyt yhden puun kasvumallit perustuvat noin 4000 puuhun, jotka on mitattu 50 tasaikäisestä männiköstä. Männiköiden ikä, tiheys ja tilajärjestys vaihtelivat laajoissa rajoissa. Muuttujat, jotka ottivat huomioon puiden sijainnin, pienensivät kasvuennusteen suhteellista keskivirhettä 10 %-yksiköllä (15 %), kun mennyt kasvu ei ollut selittäjänä. Pienennys oli noin 15 %-yksikköä (30 %), jos mennyt kasvu oli yhtenä selittäjänä. Paras kasvumalli saatiin pohjapinta-alan kasvulle, toiseksi paras suhteelliselle kasvulle ja heikoin läpimitan kasvulle, kun malleja arvosteltiin selitysasteen perusteella. Menneen kauden kasvu alensi ennusteen suhteellista keskivirhettä huomattavasti, mutta ei tehnyt tarpeettomaksi spatiaalisia selittäjiä. Pohjapinta-alan kasvumallin selitysaste oli lähes 80 %, jos mennyt kasvu ei ollut selittäjänä, ja lähes 90 %, jos myös mennyt kasvu oli mukana mallissa. Kasvumalleja ei voitu parantaa poistuman määrää ja sijaintia kuvaavien muuttujien avulla.

Keywords: growth models, spatial distribution, growth prediction, tree models. ODC 561

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Symbols

Stand characteristics

A_j	Horizontal angle from the subject tree to the stem of tree j calculated at breast height, radian
$D(g)$	Mean diameter (weighted by basal area), cm
$D_{<a}(g)$	Mean diameter of trees nearer than a meters from the subject tree (subject tree included), cm
G	Basal area, m^2/ha
G_a	Basal area of neighbors whose distance is less than a meters, m^2
H_{dom}	Dominant height (mean height of 100 thickest trees/ha), m
$H(g)$	Mean height, m
H_{ref}	Reference height (dominant height in an average site class), m
N	Number of trees per hectare
SI	Site index (H_{dom}/H_{ref})
$T(g)$	Mean age at breast height, a
V	Volume, m^3/ha

Tree characteristics

d	Diameter at breast height, cm
d_j	Diameter of a neighbor nearer than 5 m, cm
g	Basal area, cm^2
h	Height, m
i_d	Diameter growth during the coming 5-year period, cm
i_{dp}	Diameter growth during the past 5-year period, cm
i_g	Basal area growth during the coming 5-year period, cm^2
i_{gp}	Basal area growth during the past 5-year period, cm^2
p_d	Relative 5-year diameter growth, %
p_g	Relative 5-year basal area growth, %
s_j	Distance of a neighbor nearer than 5 m, m
t	Age at breast height, a

Others

n	Number of observations
R^2	Degree of determination, %
s_e%	Relative standard error of estimate $(100\sqrt{(\exp(s_f^2/2) - 1)}$
s_f	Standard deviation of the dependent variable about the function

1. Introduction

In an even-aged Scots pine stand different trees of the same size grow at very different rates. The variation in diameter growth can be much greater within one diameter class than between different diameters, even if the diameter varies considerably (Mielikäinen 1978, Pukkala and Kolström 1987). This means that it is impossible to predict the growth of an individual tree accurately with a model that does not account for the reasons of the growth variation within diameter class.

One important reason for the growth variation is the spatial variation in stand density or competition (e.g. Lin 1974, Eriksson 1976). Variation in the site fertility may be another important factor (Rudra 1980). Differences in the number of competitors removed in thinnings from the vicinity of a tree can also create growth differences as well as the time elapsed since thinning (Bucht 1981). Finally, the variation in genotype is also reflected in diameter growth.

The variation in competition can be taken into account by using spatial growth models instead of non-spatial ones. In the spatial models the number, size and distance of competitors are used for growth prediction. The site variation could be accounted for by a site index, based on age and dominant height, which is calculated for small subareas of the stand rather than for the whole stand. Removed competition can be incorporated directly in the growth model in the same way as the amount of existing competition (Eriksson 1976). Genotype together with other factors creating growth variation is taken into account by using the past growth during a certain period or during the lifetime as a predictor of the growth model.

This study examined the ability of different methods to predict the diameter growth of Scots pine. The main emphasis was on the comparison of spatial and non-spatial growth models: how much the growth prediction could be improved by using the spatial growth models instead of the non-spatial ones. The study also presented a set of spatial and non-spatial growth models applicable over a wide range of different Scots pine stands.

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2. Predictors of tree growth

In previous studies the following variables have been commonly used as predictors of a single-tree diameter growth model (Lundell 1973, Eriksson 1976, 1977, Rudra 1980, Mielikäinen 1985, Ojansuu 1987): diameter or height of the tree, age, stand density, size of the tree in relation to other trees, and the fertility of the site. In a non-spatial growth model the stand density is described e.g. by the stand basal area (Nyyssönen and Mielikäinen 1978, Ojansuu 1987), and the relative size, which describes the competitive status of the tree, as the ratio between the tree size and the average size of all trees (Saramäki 1977, Mielikäinen 1980).

In a spatial growth model the stand density is usually expressed as a point density in the vicinity of the tree, either as the basal area of the competitors (Mielikäinen 1980) or in terms of different competition indices (Lundell 1973, Eriksson 1976). The aim of constructing competition indices is to describe how growth depends on different components of competition, i.e. the number, size, distance and azimuth of neighbors (Pukkala 1989a). These effects can be incorporated in only one or several separate measures of competition.

The site is implicitly incorporated in the model if e.g. the height and the age are both used as predictors (Nyyssönen and Mielikäinen 1978). Explicitly the site effect is accounted for by forest site types or different site indices. The indices are a better alternative because they are usually continuous and more easily measurable than the forest site type.

In this study, diameter growth was predicted with the absolute and relative size of the subject tree, the age of the tree and the stand, the density of the stand and a site index. In the non-spatial models the stand density and the relative size of the subject tree were calculated at the stand level and in the spatial models for a subarea around the subject tree. The competition indices were the same as or near to the most promising ones found in literature (Hegyi 1974, Lin 1974, Eriksson 1976, Saramäki 1977, Mielikäinen 1978, Pukkala and Kolström 1987, Pukkala 1988, 1989a). The joint effect of distance and size was described either by the ratio between the diameter and distance of the competitor (Hegyi 1974, Mielikäinen 1978) or by a horizontal angle from the subject tree to the stem of the competitor (Lin 1974, Pukkala and Kolström 1987). The directional distribution of competitors was described by the distance of the competition center as proposed by Pukkala (1989a).

The site fertility (SI) was described by the following ratio:

$$SI = H_{dom}/H_{ref} \quad (1)$$

i.e. the dominant height (average height of 100 largest trees per hectare) divided by a reference height. The reference height was calculated by

$$H_{ref} = 28/(1 + 2412\ (T(g) + 12)^{-2.033}) \quad (2)$$

where T(g) is the mean breast-height age of the stand (mean age weighted by basal area). Equation (2) expresses the development of the dominant height of a naturally normal Scots pine stand in the *Vaccinium* site (forest site type VT, Koivisto 1959) with the assumption that the difference between the total age (from seed germination) and breast-height age is 12 years. Equation (1) or SI is thus approximately the ratio between the actual dominant height and that in a *Vaccinium* site with the same breast-height age. SI is not constant for a particular stand but changes as the stand develops.

The amount of removed competition was described in the same way as the amount of existing competition, e.g. by calculating the basal area of removed trees in the surroundings of the subject tree, or with different competition indices that depended on the size, distance and number of harvested neighbors (Eriksson 1977).

The time since thinning was taken into account by multiplying the measure of removed competition by a factor which varied between -1 and 1. If the thinning was executed just before the measurement, the multiplicator was -1 because the removed trees had been competitors during the whole 5-year growth period of the study. If the age of the stumps was 2.5 years the factor was 0, since the harvested neighbors had been existing competitors for 50 % of the time and removed competitors for another 50 %. Between stump ages 5 and 10 years the multiplicator was 1, after which it began to decline by 10 percentage points per year approximately describing the decline of the thinning response with time (Jonsson 1974, Isomäki 1986).

3. Material and methods

31. Study material and measurements

The study material consisted of the same 50 sample plots as used in Pukkala (1989b). The area of the plots varied between 800 and 3000 m^2. The plots represented the whole range of variation in age, density and spatial pattern of naturally regenerated Scots pine stands that have passed the seedling phase (Appendix 1). When selecting the material, special attention was paid to getting much variation in the amount and type of the spatial distribution of trees. The plots were nearly even-aged and the site was uniform within one plot.

Each tree was measured by coordinates and diameter. About 20 trees of different sizes were measured on each plot for height, breast-height age, bark thickness and stump diameter to calculate how these characteristics depend on the breast-height diameter, or how the breast-height diameter depends on the stump diameter. In ten out of 50 plots the height was measured from all trees.

A radial growth core was bored from all trees further than five meters from the plot edge. In 7 of the plots, a core was taken from the border zone trees as well. The radial growth without bark during the last (7 plots) or last two (43 plots) 5 year periods were measured from the core.

The diameters and coordinates of stumps were measured on 40 plots if there had been a thinning within 15 years (asterisk in Appendix 1). The year of the thinning was also recorded. In most of the thinned stands the stumps were about five years old.

32. Reconstruction of the stands five years earlier

The predicted variable of the growth models of this study was the future diameter increment or basal area increment with bark during a 5-year period. Because the plots were measured only once, their state 5 years earlier had to be estimated from the present tree dimensions and the growth measurements.

The sample trees and the growth measurements of each plot were used for calculating regression models for bark thickness, tree height, breast-height diameter (of stumps), breast-height age and radial growth (without bark) during the past 5 years. These plotwise regression models were usually of the form y=a+bx where y is the predicted variable, x is the predictor, and a and b are parameters. Näslund's (1936) height curve $y = 1.3 + x^2/(a + bx)^2$ was used as a height model. The predictor of the models was the breast-height diameter except in the breast-height diameter model where it was the stump diameter.

The diameter model was utilized for calculating the breast- height diameter for the removed trees (stumps). The bark, age and height models were used for estimating these characteristics for the non-sample trees. The growth model, which was constructed from trees of which the core was bored, was used for estimating the past growth of the trees on the border zone.

To obtain the tree dimensions 5 years earlier the doubled bark thickness and radial growth without bark were subtracted from the present diameter. The overbark diameter 5 years earlier was calculated by assuming that the proportion of bark of the total diameter had not changed during the 5-year period. The height 5 years earlier was estimated by subtracting from the present height the difference of the height estimates corresponding to the present diameter and that 5 years earlier. Finally, the 5-year diameter increment with bark was calculated as a difference of the present overbark diameter and that 5 years earlier. The past growth 5 years ago (in years 10 – 6 from the present) was calculated in the same way. Only trees the diameter increment of which was measured from the core were used as observations. This amounted to 4001 observations for the spatial growth models and 4597 observations for the non-spatial ones. The rest of the trees (mainly those on the buffer zone) were used for calculating predictors for the growth models. The number of such observations, of which the past growth was known, was 3708 for the non-spatial models and 3521 for the spatial ones.

After reconstructing the stands 5 years earlier, the stand characteristics (Appendix 1) and other predictors of the growth models were computed (Appendix 2).

4. Results and discussion

41. Competition zone

When constructing spatial growth models it must be decided which of the neighboring trees should be considered as competitors. This question is most easily solved by defining a limiting distance beyond which the trees are considered as noncompetitors. For the sake of the effective use of the study material, the distance should be reasonably short, because the material usually consists of rather small sample plots.

In previous studies it has been found that it suffices to take trees nearer than 3...7 meters into account when describing the competition of a tree (Eriksson 1976, Bucht 1981, Pukkala and Kolström 1987, Pukkala 1988). The dependence of the effect of a competitor on its distance was studied in the present study material by calculating a spatial growth model where separate regression coefficients were computed for basal areas in different one-meter-wide zones around the subject tree (see Eriksson 1976):

$$\ln(i_g) = -4.832 - 0.2606d + 2.833d\sqrt{d} + 26.34\ 1/(t+5) + 1.181\ d/D_{<8}(g) - 0.4622\ \ln(d/D_{<8}(g)) - 10.41G_1 - 5.191G_2 - 3.856G_3 - 2.682G_4 - 2.089G_5 - 1.801G_6 - 0.9772G_7 - 0.4831G_8 \quad (3)$$

$R^2 = 77.3\ \%$ $s_f = 0.493227$ $s_e\% = 52.57$

where i_g = 5-year basal area growth (cm²),
d = diameter (cm),
t = breast-height age (a),
$D_{<8}(g)$ = mean diameter weighted by basal area in a circular area (radius 8 m) around the subject tree; the subject tree is included in $D_{<8}(g)$ (cm),
G_1 = basal area of neighbors whose distance is less than 1 m (m²),
G_2 = basal area of neighbors whose distance is at least 1 m but less than 2 m (m²),
$G_3 \ldots G_8$ are defined correspondingly.

The regression coefficients indicate that the effect of a certain amount of competition (certain basal area) decreases exponentially with increasing distance and is rather small beyond 6 m (Fig. 1).

It should be noted that the results of Eqn (3) do not describe the effect of stand density at different distances but that of an individual tree of certain size. Had the basal areas of competitors been expressed as per land area units, the decrease in the absolute value of regression coefficient had been slower and more linear than in Fig. 1.

The calculations show that the expected effect of a competitor is negligible if it is further away than 6 m, and increases rapidly from distances 4...5 m towards the tree.

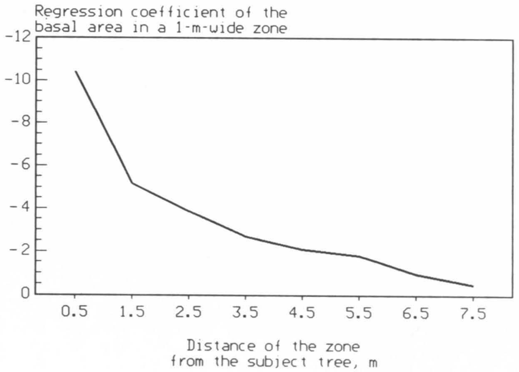


Fig. 1. Effect of the total basal area of trees in different 1-m-wide zones around a tree on the 5-year basal area increment. The y-axis shows the regression coefficient (Eqn 3) of basal area at different distances from the object tree.

For this reason, the studied limiting distances or competition zones were taken as either 5 m or 6 m. Out of these distances 5 m gave constantly a little better growth models than 6 m with the used predictors, the difference in the degree of determination being typically around one percentage point in favor of the 5-m competition zone.

Based on these results, 5 m was selected as the competition zone of this study, i.e. neighbors nearer than 5 m were defined as competitors.

42. Comparison between different models

To compare the possibilities of different ways of predicting the diameter growth of Scots pine the models were calculated for different predicted variables using either non-spatial or spatial predictors, in addition to a constant collection of tree characteristics. In the non-spatial models the competition was described at the stand level only, in spatial models by the competition within 5 m from the subject tree. The 6 best predictors were selected in each model in a stepwise regression analysis. Increasing the number of predictors to 7 or 8 did not change the order of the models when ranked according to the degree of determination. Neither did the additional predictors increase the degree of determination by more than 1...2 %.

Table 1. Non-spatial growth models where the past growth is not used as a predictor.

	Predicted variable							
Predictor	$\ln(i_d)$		$\ln(i_g)$		$\ln(p_d)$		$\ln(p_g)$	
	Coeff.	t	Coeff.	t	Coeff.	t	Coeff.	t
Constant	0.6833	5.8	-7.847	13.4	1.043	1.9	-2.763	3.7
d	..		-0.8604	10.5	-0.6971	8.9	..	
ln d	..		-3.995	9.4	-5.058	12.3	..	
$\sqrt{d}$	0.9551	28.7	9.016	12.4	7.4547	10.7	..	
g	-0.001951	21.5	0.003974	6.1	0.003212	5.0	..	
t	..		..		..		-0.03335	14.5
ln t	-0.7642	25.9	..		-0.7469	30.3	1.979	9.8
1/(t+5)	..		..		..		63.69	13.1
1/(t+10)	..		41.33	31.0	..		..	
G	..		..		..		-0.04136	17.5
T(g)	..		..		..		0.01184	8.2
ln G	-0.5692	18.6	-0.5349	16.5	-0.5312	17.6	..	
ln T(g)	..		..		..		-0.4528	6.2
d/D(g)	0.5373	6.2	..		..		..	
ln d/D(g)	-0.3899	7.0	..		..		..	
n	4597		4597		4597		4597	
R^2	0.435		0.723		0.591		0.591	
s_f	0.541		0.562		0.538		0.571	
$s_e\%$	58.4		61.0		58.0		62.1	
Equation	(4)		(5)		(6)		(7)	

$s_f^2/2$ should be added to the prediction due to the logarithmic transformation.

The use of spatial predictors increased the degree of determination most when diameter increment ($\ln(i_d)$) was the predicted variable, from 43.5 % to 56.3 % (29 %) if the past growth was unknown (Tables 1 and 2), and from 62 % to 78 % (26 %) if the past growth was known. For the basal area growth the increase in the degree of determination due to the spatial predictors was 8...11 %. The degree of determination was highest for the basal area growth, because of the strong correlation between the diameter and basal area growth.

Spatial predictors decreased the standard error of estimate by 10 percentage points (15 %), if the past growth was not used as a predictor. The decrease was greater, about 15 percentage points (30 %), when past growth was one of the predictors (Fig. 2, Table 3).

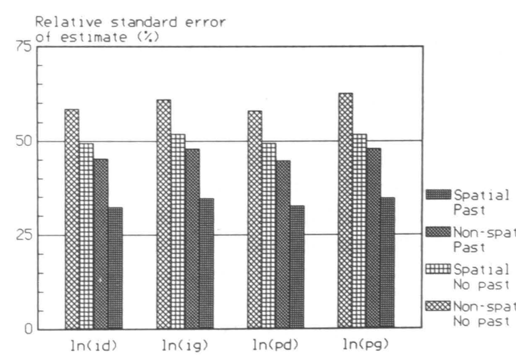


Fig. 2. Relative standard error of estimate of the growth models with different predicted variables when the past 5-year growth is either known (Past) or unknown (No past). Each model includes six predictors which were selected in a stepwise regression analysis.

Table 2. Spatial growth models where past growth is not used as a predictor.

Predictor	Predicted variable							
	$\ln(i_d)$		$\ln(i_g)$		$\ln(p_d)$		$\ln(p_g)$	
	Coeff.	t	Coeff.	t	Coeff.	t	Coeff.	t
Constant	1.297	19.1	- 3.093	22.5	4.221	26.7	7.626	112.0
d	..		- 0.1815	27.0	- 0.1997	11.8	..	
ln d	..		..		- 2.468	13.3	- 0.8289	22.6
$\sqrt{d}$	0.3913	13.3	2.052	33.6	2.517	10.8	..	
g	- .001062	13.6	..		..		..	
ln t	- 0.6970	26.6	..		- 0.7652	32.6	- 0.6375	23.3
1/(t + 10)	..		38.85	29.2	..		..	
$\Sigma_5 A_j(d/d_j)$	..		..		..		- 0.8183	28.8
$\Sigma_5 d_j/s_j$	- 0.009238	24.8	- 0.009561	25.1	- 0.009710	27.5	..	
$\Sigma^5 A_j \cdot (d/d_j)^{1.6}$	..		..		..		0.1727	25.1
$d/D_{<5}(g)$	0.9762	20.6	1.043	20.9	1.020	21.7	0.7418	15.5
G	..		- 0.01173	5.6	..		..	
In G	- 0.1712	5.8	..		..		- 0.2653	8.8
n	4001		4001		4001		4001	
R^2	0.563		0.782		0.706		0.716	
s_f	0.467		0.487		0.466		0.487	
s_e%	49.4		51.8		49.3		51.7	
Equation	(8)		(9)		(10)		(11)	

Σ_5 means that trees nearer than 5 m are included in the sum and Σ^5 that five nearest trees within 5 m distance are included in the sum.

$s_f^2/2$ should be added to the prediction due to the logarithmic transformation.

The regression equations in Tables 1...3 show clearly that the following four variables are the most important predictors of the future growth: absolute size (diameter), relative size, age and stand density (or competition index). The site index was only seldom selected in the models, presumably because its effect could be predicted through the age and tree dimensions.

43. Validity of the models

For studying the accuracy of the predictions of the growth models, four plots were separated from the study material (plots 3, 8, 14 and 37) and the models to be tested were recalculated without them. In this way the four plots served as independent test material (Fig. 3). All of the test plots are

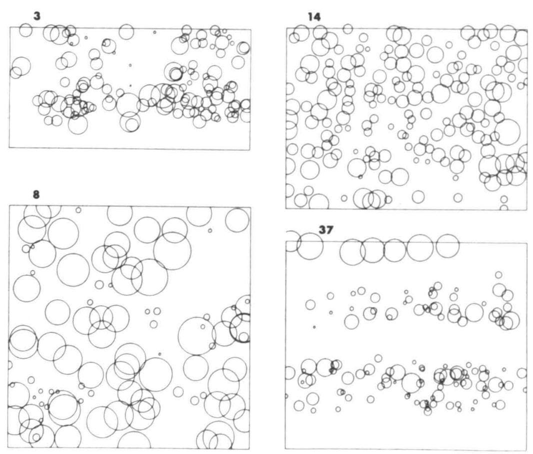


Fig. 3. Crown maps of the four study plots used for testing the growth models.

Table 3. Models for 5-year basal area growth with the past growth as a predictor.

Predictor	Non-spatial model		Spatial model	
	Coeff.	t	Coeff.	t
Constant	0.1676	2.2	- 2.377	7.3
i_{dp}	0.5297	31.3	..	
ln i_{dp}	..		0.8742	72.6
i_{gp}	0.007027	8.1	..	
ln d	1.164	40.2	..	
ln g	..		0.4596	54.3
ln t	- 0.2780	9.1	..	
G	- 0.02520	12.4	..	
H(g)	0.02698	10.0	..	
$\Sigma_5 d_j/s_j$	..		- 0.006178	22.8
$d/D_{<5}(g)$	..		0.3261	9.1
SI	..		- 2.936	9.4
ln SI	..		- 2.724	8.0
n	3708		3521	
R^2	0.799		0.886	
s_f	0.455		0.335	
s_e%	47.9		34.5	
Equation	(12)		(13)	

Σ_5 means that trees nearer than 5 m are included in the sum.

$s_f^2/2$ should be added to the prediction due to the logarithmic transformation.

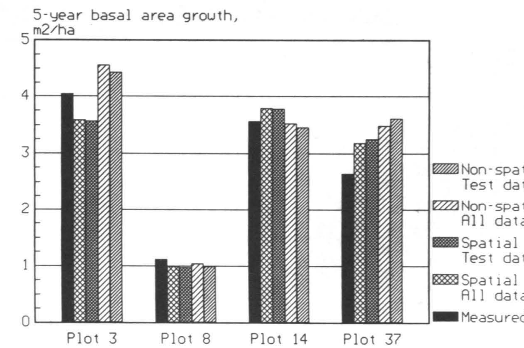


Fig. 4. Measured and predicted basal area growth of the sample plots presented in Fig. 3. 'Spatial' refers to a spatial growth model and 'Non-spat.' to a non-spatial model. 'All data' means that the model is computed from all data, and 'Test data' that plots 3, 8, 14 and 37 were not used as study material.

different from each other in some respects: plots 3 and 37 are very irregular and plots 8 and 14 fairly regular; plot 8 is the oldest plot and the rest among the youngest (Appendix 1). The basal area growth of each plot was computed from the measurements, from the estimates of the non-spatial and spatial models (Eqns 5 and 9) and from the estimates of the corresponding models calculated without the test material.

The predicted basal area growths are about the correct magnitude in all plots and for both the spatial and non-spatial models (Fig. 4). The prediction seems to be as accurate for the independent material as within the study material indicating that the models are applicable also outside the study material. There is no marked difference in the average accuracy of the non-spatial and spatial model when the total growth of all trees is concerned. It is noteworthy, however, that the non-spatial models overestimate the growth of an aggregated stand in both cases (plots 3 and 37).

The errors in the predicted 5-year diameter growth indicate that the non-spatial test model (calculated without plots 3, 8, 14 and 37) tends to underestimate the growth of trees which have little competition, and overestimate the growth if the tree faces much competition (Fig. 5). The standard deviation of the error in the predicted diameter growth was 12 % greater with the non-spatial test model when compared to the spatial model.

44. Growth prediction with a spatial and non-spatial model

The predictions of a spatial and non-spatial growth model are presumably most different in stands whose spatial distribution is somehow untypical. To demonstrate this and to compare the models, the growth of four different model stands were simulated with a non-spatial (Eqn 5) and spatial model (Eqn 9). The stands were generated as follows. The spatial distribution was first generated for 1500 trees/ha as a realization of a suitable spatial process (see e.g. Pukkala 1988). The diameters of trees were estimated by the method of Pukkala (1989b,

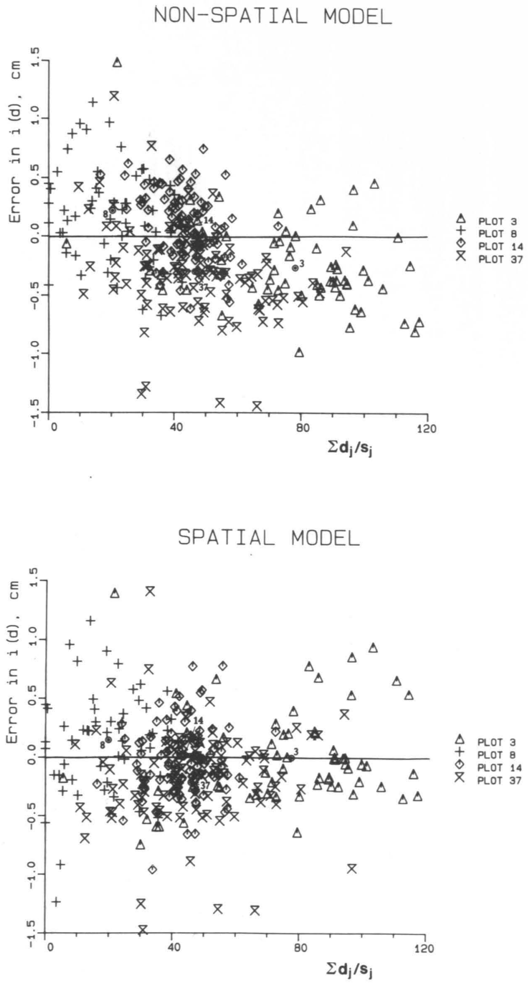


Fig. 5. Errors in the predicted 5-year diameter growth (measured growth - predicted growth) as a function of competition (Σdj/sj). The predictions were calculated with basal area growth models which were constructed without trees presented in the figure. The mean errors in different plots are shown as small dots.

Eqns 11...14) to obtain a total stand basal area of 15 m²/ha (Fig. 6). The breast height age of each tree was calculated by formula t = a + 0.8d (t is the breast-height age in years and d diameter in cm; 0.8 was a typical regression coefficient in the even-aged plots of this study). Parameter a was chosen so that the age and size of the trees approximately corresponded to a *Vaccinium* site.

The height of each tree was calculated by the following spatial height model which was calculated from the sample trees of the present study material:

	Coefficient	t-value	
$\ln(h-1.3)=$	4.624	52.0	
	$-\ 21.45 \cdot 1/(d+5)$	28.7	
	$+\ 0.015345 \cdot \Sigma(d/d_j)(1/s_j)$	10.7	
	$-\ 0.8804 \cdot d/D_{<5}(g)$	16.1	
	$+\ 0.5088 \cdot \ln(d/D_{<5}(g))$	11.3	
	$+\ 0.01663 \cdot D(g)$	6.6	
	$-\ 15.08 \cdot 1/(T(g)+10)$	10.0	
	$-\ 0.004330 \cdot T(g)$	7.1	(14)

$R^2 = 87.4\,\%$ $\quad s_f = 0.188 \quad s_e\% = 19.07$

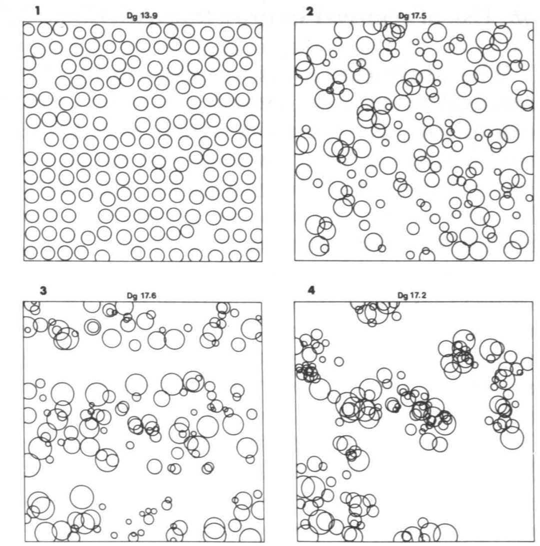
where h = height (m),
d = diameter (cm),
d_j = diameter of a neighbor nearer than 5 m (cm),
s_j = distance of a neighbor nearer than 5 m (m),
$D_{<5}(g)$ = mean diameter of trees nearer than 5 m from the subject tree (subject tree included) weighted by basal area (cm),
D(g) = mean diameter of the stand weighted by basal area (cm),
T(g) = mean breast-height age of the stand weighted by basal area (a).

The 5-year diameter growth of each tree was calculated by Equations (5) and (9). The tree height corresponding to the new diameter was estimated by Equation (14) and the stem volume by Equation 61.3 of Laasasenaho (1982). Each time when the growth or height was calculated, a buffer zone was generated around the simulated 40 m x 40 m plot by assuming that the plot has a similar plot on all sides.

As expected, the estimated development of the stand volume according to the spatial and non-spatial model is most different in very regular and very irregular stands (Fig. 7). The two intermediate plots (stands 2 and 3 in Figs. 6 and 7) seem to correspond to typical plots in the present study material, because the non-spatial model gives almost the same estimate as the spatial model.

Although it cannot be said that the estimated stand development along the spatial

Fig. 6. Crown maps of the model stands used for comparing the growth estimates of a spatial and non-spatial model. The area of each plot is 40 m x 40 m.



model is correct, it can be assumed to be rather near to it. The results of Fig. 7 thus roughly indicate the magnitude of the expected error which the use of the non-spatial models could cause in different cases. The non-spatial models underestimate the growth of a regular stand and strongly overestimate it in a very grouped stand (see Figs. 3, 4, and 5). The errors in the differentiation of tree size and in the amounts of wood assortments might be still greater than in the volume growth.

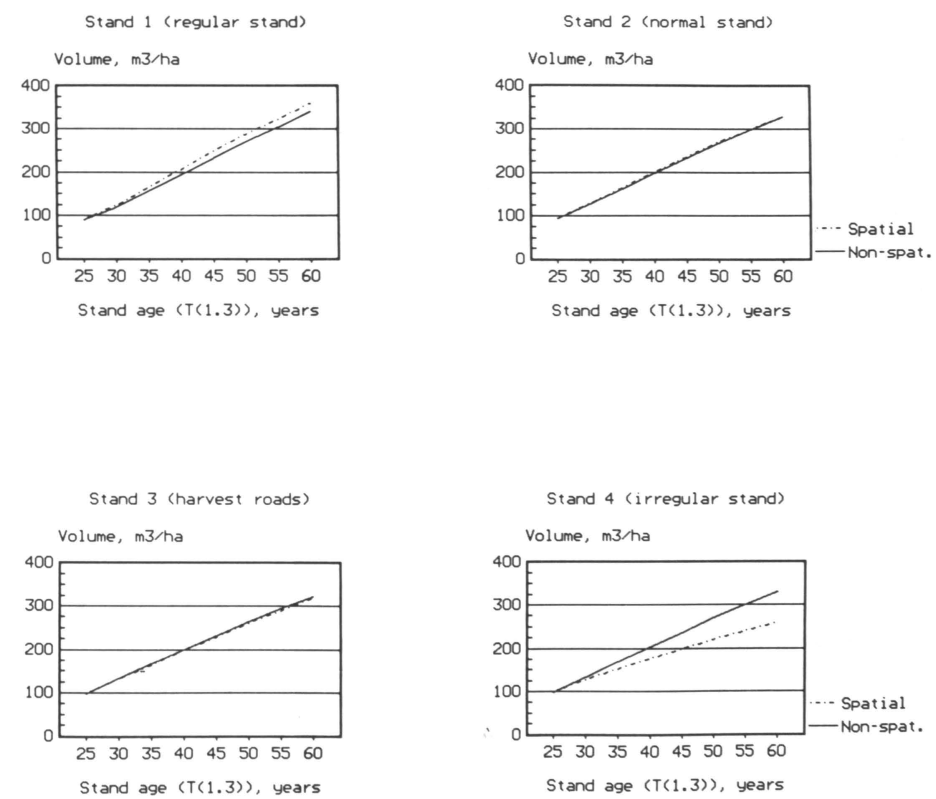


Fig. 7. Predicted development of stand volume along a spatial and non-spatial growth model in stands presented in Fig. 6.

45. Use of removal in growth prediction

The ability of measures describing the removal to improve growth prediction was examined with the plots which were thinned within 15 years prior to the measurement, (marked with asterisk in Appendix 1). The removal from the vicinity of the subject tree (from nearer than 5 m) did not at all increase the degree of determination or decrease the relative standard error of estimate (Fig. 8). Variables that described the amount of removal were not always selected among the 6 predictors in a stepwise regression analysis.

The small effect of removal is a somewhat surprising result, since previous studies indicate that a certain amount of removed competition increases growth as much as the same amount of existing competition decreases it (Eriksson 1976, Mielikäinen 1978). Growth correlates positively with the number and size of stumps also in the present study material, the correlation coefficient being usually around 0.2 between the diameter increment and the different measures of removed competition.

One reason for the small importance of removal may be that the thinnings executed in the study material have been rather light and always from below, which means that mainly such trees were removed which had not been very strong competitors of the remaining trees. Another explanation could be that the effect of harvested trees could be explained through other variables. Thinnings alter the relation between the tree size, tree age and the amount of existing competition in a predictable manner, so that it is possible to estimate the thinning effect from these variables without knowing the amount of removal.

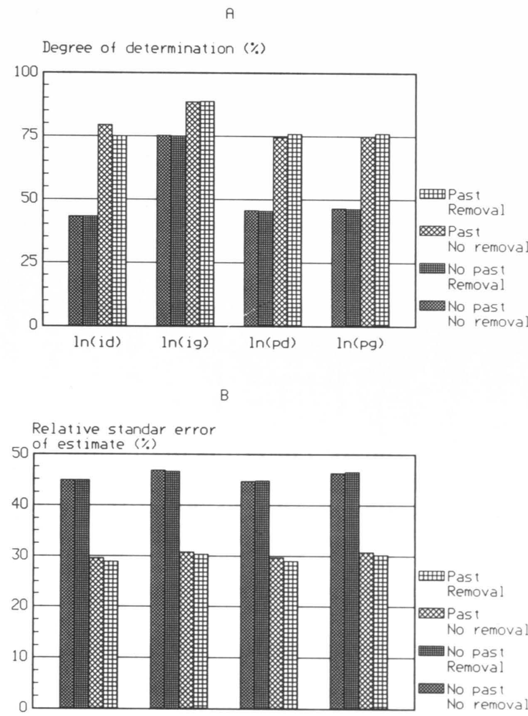


Fig. 8. Degree of determination (A) and relative standard error of estimate (B) of a 6-predictor growth model if either past 5-year growth or removal or both of them are known, or if neither of them is known.

5. Conclusions

According to the present study the use of the spatial growth models can increase the accuracy of growth prediction considerably, especially in stands of an exceptional spatial distribution of trees. Even when the improvement in accuracy is not very pronounced for the volume growth at the stand level, the estimated growth of individual trees may be essentially different from the estimate of a non-spatial model, making it easier to predict reliably the change in the size distribution of trees.

The past 5-year growth improved the prediction more than the spatial information, but it is only seldom recorded in practical stand measurements. The information about removal did not increase the accuracy of growth prediction, which was assumed to have been partly a consequence of the fact that a thinning treatment is

reflected in the relationships between the tree dimensions and the remaining competition.

The spatial growth models accounted for almost 80 % of the variation in the logarithm of the basal area growth, and almost 90 % if the past growth was used as a predictor. The degree of determination was thus very good. It was not possible to increase it notably by changing the predictors or increasing their number without drastically decreasing the level of significance.

Part of the residual variation has arisen from errors in the field measurements and in the estimation of the breast-height age, tree height and bark growth. In addition, the growth of a tree was measured from one increment core per tree, which gives only a sample of the increase in tree size. The growth rate may be different at different heights and sides of the stem (Isomäki 1986), especially in irregular stands, which were fairly common in the present study.

Because the residual variation includes errors due to the measurements, the degree of determination may underestimate the reliability of the predictions obtained by the models. If the models are used for simulating the stand development, it may not be correct to add the total residual variation as a stochastic component to the estimate but only part of it to obtain the closest resemblance between the real and simulated stand development.

Fig. 9. Annual growth indices of Scots pine stands in North Karelia. Indices are the mean values of 10 index series calculated for 40...150-year old Scots pine stands. Each series was based on 10...15 trees.

The directional distribution of competitors (on one side or on all sides) seems to be of a rather small importance, because the distance of the competition center from the object tree was not selected in any of the spatial growth models (cf. Pukkala 1989a). The reason may again be that the directional distribution of competitors correlates closely with other variables and is therefore not needed as a separate predictor. In a typical case a tree having fewer competitors than on the average grows beside a harvest road or some other gap. The spatial models can therefore give fairly reliable growth estimates also for trees near harvest roads, although the directional distribution of competitors was not explicitly included into the models.

The spatial growth models presented in this study can be used in simulation studies to examine e.g. the effect of a thinning method or spatial pattern of trees on the stand productivity. Because the study material covers a reasonably wide range of different stands, the simulations can be extended over prolonged time periods and over different stand structures. This is a clear improvement from the previous models (e.g. Mielikäinen 1978, Pukkala 1988, 1989a), which usually have described the growth variation within one or a few stands only, giving unreliable results in other stands. The present models are most reliable in North Karelia on the most typical growing sites of Scots pine.

The prediction (and some of the predictors) of the models should be corrected to a level which corresponds to normal or average climatic conditions, especially if the models are used in practical growth predictions. Since the purpose of the present computations was to compare different models within the same material, this was not necessary in this study.

The mean annual growth index - calculated from a rather small material - of the years 1983...1987 was 99.9 which indicates that there is no need to correct the growth prediction (Fig. 9). If the past growth is used as a predictor, it should be corrected to a level of 1.043, because the mean index of the years 1979...1982, during which the past growth of this study had formed, was 104.3.

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Appendix 1. The main characteristics of the study plots at the beginning of the growth period (5 years ago). Stumps were measured in plots marked with asterisk.

Plot	N trees/ha	G m²/ha	V m³/ha	D(g) cm	H(g) m	T(g) a	H_{dom} m	SI m/m
1*	1109	20	153	19	15	64	18	0.89
2*	756	18	131	20	15	58	17	0.84
3	1829	19	127	15	13	39	16	1.03
4	2019	20	131	13	13	41	16	1.00
5*	1040	18	157	18	18	46	21	1.19
6*	686	18	161	20	18	50	20	1.08
7*	489	19	173	24	20	75	21	0.96
8	375	23	256	36	24	128	25	1.00
9	523	29	299	34	22	118	24	0.97
10	792	24	231	21	20	50	21	1.14
11*	584	17	159	21	19	49	20	1.13
12	509	25	271	32	23	114	25	1.02
13	578	27	282	27	22	50	24	1.30
14	1509	17	119	14	14	38	16	1.06
15	978	18	158	26	18	99	22	0.90
16	2217	19	116	13	12	38	15	0.99
17*	844	15	94	17	12	48	14	0.77
18	408	23	226	31	21	69	22	1.05
19	1520	13	81	13	12	33	16	1.14
20	2011	13	71	11	11	25	14	1.27
21*	532	24	248	27	22	57	23	1.19
22*	190	11	119	29	24	72	25	1.16
23	1977	5	18	7	6	8	7	1.54
24*	471	13	110	21	18	43	19	1.12
25*	871	12	80	16	14	43	16	0.96
26*	553	13	117	21	18	51	20	1.10
27	880	27	273	31	22	92	25	1.05
28	631	15	130	26	18	110	20	0.81
29	1182	9	47	13	10	24	12	1.13
30*	960	20	169	17	18	44	19	1.12
31*	840	20	159	19	16	50	17	0.96
32*	1017	14	98	16	14	33	16	1.19
33	356	25	259	31	23	137	23	0.91
34*	1280	12	77	13	13	27	16	1.35
35*	906	18	142	17	16	49	18	1.02
36*	584	12	91	19	15	60	18	0.88
37	1587	7	34	10	10	23	14	1.38
38	1829	16	90	13	10	17	6	0.74
39	3813	5	17	5	5	7	6	1.58
40	2533	18	122	14	13	43	18	1.09
41	1051	17	127	16	16	55	18	0.93
42	2194	20	144	15	14	54	18	0.94
43	612	11	81	18	15	58	18	0.89
44	1269	18	140	19	16	58	19	0.98
45	702	18	161	21	18	61	20	1.00
46	727	19	167	20	18	60	20	1.01
47	571	18	172	23	20	63	21	1.04
48	2957	14	62	9	8	21	9	0.98
49	1191	15	105	17	14	50	17	0.92
50	3827	18	87	9	9	20	10	1.11

Appendix 2. Range, mean and standard deviation of some of the predicted variables and predictors of the study at the beginning of the 5-year growth period. See Tables 1 – 3 for the explanation of variable symbols.

Variable	Minimum	Maximum	Mean	Standard deviation	Unit
d	0.6	50.1	13.3	8.1	cm
h	1.6	33.0	12.6	5.7	m
t	3.7	172.2	42.2	26.6	a
i_d	0.05	5.3	1.1	0.7	cm
i_g	0.3	151.1	25.0	20.0	cm^2
p_d	0.3	330.1	12.5	16.8	%
p_g	0.6	1750.4	29.3	59.9	%
$G_{<5}$	0.1	73.8	20.2	9.6	m^2/ha
$\Sigma_5 d_j/s_j$	0.0	75.1	16.0	10.0	cm/m
$d/D_{<5}(g)$	0.04	1.7	0.8	0.3	-