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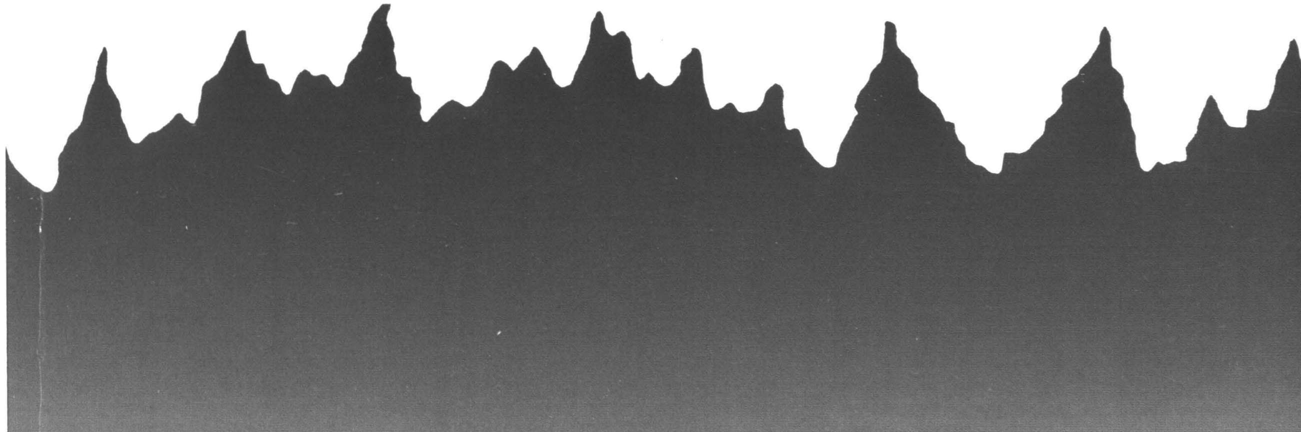

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Effect of spatial pattern of trees on the growth of a Norway spruce stand. A simulation model

Timo Pukkala & Taneli Kolström

TIIVISTELMÄ: TILAJÄRJESTYKSEN VAIKUTUS KUUSIKON KASVUUN. SIMULOINTIMALLI

Pukkala, T. & Kolström, T. 1991. Effect of spatial pattern of trees on the growth of a Norway spruce stand. Tiivistelmä: tilajärjestyksen vaikutus kuusikon kasvuun. Simulointimalli. Silva Fennica 25(3): 117–131.

The simulation model consists of a method to generate theoretical Norway spruce (*Picea abies* (L.) Karst.) stands, and a spatial growth model to predict the growth of these stands. The stand generation procedure first predicts the tree diameters from a few stands characteristics and from tree locations. Tree age and height are predicted using spatial models. Spatial growth models were made for both diameter growth and basal area growth. Past growth was used as a predictor in one pair of models and omitted in another pair. The stand generation method and the growth models were utilized in studying the effect of tree arrangement and thinning method on the growth of Norway spruce stand.

Simulointimalli koostuu menetelmästä, joka tuottaa teoreettisia kuusikoita (*Picea abies* (L.) Karst.), ja spatiaalisesta kasvumallista, jolla näiden mallimetsiköiden kasvu ennustetaan. Metsikköä generoitaessa ennustetaan ensin puun läpimitta metsikkötunnusten ja puiden koordinaattien avulla. Puun ikä ja pituus lasketaan spatiaalisilla malleilla. Tutkimuksessa esitetään spatiaalisia kasvumalleja läpimitan ja pohjapinta-alan kasvulle. Yhdessä malliparissa menneen kauden kasvu on yksi selittäjä, toisessa mennyttä kasvua ei ole käytetty. Mallimetsiköiden generointimenetelmää ja spatiaalisia kasvumalleja käyttäen selvitettiin puiden tilajärjestyksen, ajourien ja harvennustavan vaikutusta kuusikon kasvuun.

Keywords: spatial distribution, simulation models, growth, biological competition, *Picea abies*.
FDC 56

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Symbols

Stand characteristics

A	Hopkins' grouping index
$D_{<5}$	Unweighted mean diameter of trees within a 5-m circle around the subject tree, subject tree included, cm
D_g	Mean diameter, weighted by basal area, cm
$D_{<5(g)}$	Mean diameter (weighted by basal area) of trees within 5 m from the subject tree, subject tree included, cm
D_{max}	Maximum diameter, cm
D_{min}	Minimum diameter, cm
G	Stand basal area, m^2/ha
$G_{<5}$	Basal area of neighbors nearer than 5 m, subject tree excluded, m^2/ha
$G_{<5>d}$	Basal area of neighbors nearer than 5 m and greater than the subject tree, m^2/ha
H_g	Mean height, weighted by basal area, cm
H_{dom}	Dominant height, m
N	Number of trees per hectare
$N_{<5}$	Number of trees per hectare within a 5-m circle around the subject tree (subject tree included)
R	Grouping index of Clark and Evans
$S_{<5}$	Mean distance of neighbors nearer than 5 m, m; if there are no neighbors $S_{<5} = 6$
s_d^2	Variance of diameter within the circle, cm^2
T	Age at breast height, weighted by basal area, years
$\Sigma d_j/s_j$	Sum of ratios of diameter and distance of neighbors nearer than 5 m, subject tree excluded, cm/m
$\Sigma d_j/s_j(q3)$	Sum of ratios of diameter and distance of neighbors nearer than 5 m, the neighbors are selected with relascope using basal area factor 3 m^2/ha, cm/m
$\Sigma d_j/s_j(q2)$	Sum of ratios of diameter and distance of neighbors nearer than 5 m, the neighbors are selected with relascope using basal area factor 2 m^2/ha, cm/m

Tree characteristics

d	Diameter at breast height, cm
g	Basal area, cm^2
h	Height, m
i_d	Future 5-year diameter growth including bark, cm
i_{d-5}	Past overbark 5-year diameter growth, cm
i_g	Future 5-year basal area growth including bark, cm^2
i_{g-5}	Past overbark 5-year basal area growth, cm^2
t	Age at breast height, a

Others

R^2	Degree of determination
$s_e\%$	Relative standard error of estimate, $100\sqrt{(\exp(s_f^2/2)-1)}$
s_f	Standard deviation of the dependent variable around the function

1 Introduction

The results of empirical studies are most conveniently expressed as mathematical models which allow a free and detailed exploration of the relationships found in the investigation. When studying the effect of spatial pattern of trees on stand productivity the target model is a distance dependent single tree growth model. It enables the prediction of stand growth with different tree arrangements.

If one wishes to analyze theoretical stands and situations, there is also a need for a method to produce the theoretical stands. Even though these stands are theoretical, they should be sensible and conceivable. This means that the relationships between stand and tree characteristics in the model stands should be similar to that which natural birth, growth and death processes can produce. Otherwise the results of simulations cannot be generalized to practical situations.

The methodology to theoretically examine the effect of spatial pattern on stand productivity has recently been developed for Scots pine (Pukkala 1989a, 1989b). In the first step of this method, the tree dimensions are generated from tree locations and some stand characteristics. The growths of the resulting trees are predicted in the second step using a spatial growth model.

This study aimed at developing the corresponding models for Norway spruce (*Picea abies* (L.) Karst.) stands. The models prepared were used to analyze the effect of spatial pattern and thinning method on the stand growth.

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2 Methods

2.1 Generation of model stands

Definition of a model stand

For yield investigation purposes the stand is defined accurately enough if the following characteristics are known for each tree: location, species, age, diameter at breast height and height. The age is usually required as the breast height age since growth models commonly use this predictor.

When studying the effect of spatial distribution of trees on stand productivity it is hoped that everything but the spatial pattern is similar in the stands which are to be compared, i.e. stands have exactly the same age, diameter distribution, dominant height and stand volume. However, this is usually unrealistic since spatial pattern affects the development of stand characteristics and their relations. Only immediately after systematic thinnings (e.g. row harvesting) can the diameter distribution and stand characteristics be similar in different tree arrangements.

When it is not logical to have all the stand parameters except tree locations similar in all model stands, the stands should at least have the same growing site, age and density. Otherwise it is not easy to distinguish the effect of spatial distribution from the effects of other factors.

In older stands, stand density is usually expressed as basal area. Site fertility can be defined as the relationship between age and dominant height. This relationship is not greatly affected by stand density, spatial pattern of trees and usual thinnings.

Therefore, if a simulation compares stands with the same forest site type, age and density, it can be assumed that the following stand characteristics are known when the generation of a model stand starts: tree species, stand age, dominant height and basal area or number of trees per hectare.

Steps in the generation of model stands

The adopted method of producing Norway spruce model stands is divided into the following steps:

(1) Decide the stand age (T), dominant height (H_{dom}) and stand basal area (G). Obtain a sensible number of trees per hectare (N) either as a model prediction or from other sources.
(2) Generate the coordinates of trees as a realization of a suitable spatial process.
(3) Calculate the tree diameters (d) from T, H_{dom}, G, N and tree locations.
(4) Calculate the breast height age (t) of each tree from d, T, H_{dom}, G, N and tree locations.
(5) Calculate tree heights (h) from d, t, T, H_{dom}, G, N and tree locations.

The prediction models for tree characteristics (d, t, and h) make a recursive set of equations where model predictions appear as predictors in the subsequent models. This does not improve the predictions from what can be obtained with the help of T, H_{dom}, G, N and tree locations only, but it assures that diameter, height and age are logically related to each other. In the absence of a proper growth model, this height model facilitates the simulation of height development better than a model which omits the tree age.

Models used to generate tree dimensions explain only part of the variation in tree dimensions. Therefore, to get the tree dimensions to vary in model stands as much as the variation in real stands, residual variation must be added as a stochastic component to the model prediction. In this study, stochastic component was added to the diameter and age models. The height model was used in simulations for incrementing the tree height, and a normally distributed stochastic term could have created negative growths.

Even though the model stands generated by the system may have the same input values for T, H_{dom}, G and N, their realized values are seldom exactly the same. This is because of the stochasticity of the method.

Prediction of tree diameter

Two different approaches were used to predict tree diameter as a function of T, H_{dom}, G, N and tree locations (Pukkala 1989a). In the first method (referred to as Method 1), diameter was predicted directly from stand characteristics and the number and distance of close neighbors. A stochastic term corresponding to the residual variation was added to the prediction.

The second method (Method 2) first predicted the local diameter distribution and then randomly took one diameter from it. Local diameter distribution was calculated for each tree from a circular area, the subject tree located in the center. The beta distribution was used as the theoretical local distribution. It can be calculated from the minimum, mean, maximum and variance of diameter. These variables were predicted from stand characteristics (T, H_{dom}, G and N) and the number of trees in the circle.

2.2 Spatial growth model

Common predictors of empirical diameter growth models are: site fertility, present diameter, age of the tree, relative size of the tree, stand density or competition, sometimes also past growth and removal (e.g. Eriksson 1976, Pukkala 1989b, Tham 1989). Relative size can be expressed as the ratio of tree diameter and the mean diameter of the stand. Stand density or competition may be described as the stand basal area. Site fertility index may be a separate predictor or its effect can be expressed through stand age and other characteristics.

In a spatial model part of the predictors, such as relative size and stand density, are computed separately for each tree from the locations and dimensions of close neighbors. Local stand density is usually expressed with a competition index which takes into account the size, proximity and number of competing neighbors.

Trees which are taken into account when calculating competition indices can be selected with different criteria. The simplest way is to take all trees nearer than a certain limit into the index. Another possibility is to use a relascope so that bigger trees are included from longer distances than small individuals. It is also possible to take into account the size difference between the subject tree and the neighbor e.g. by computing the index separately for bigger neighbors and smaller neighbors (Tham 1989). All these approaches were also utilized in the present study when searching for predictors of distance dependent growth models and other spatial tree models.

All the ways to calculate competition index need a limiting distance beyond which trees are no longer considered as competitors. Because competition indices are usually computed in such a way that the contribution of a tree to the index sharply decreases with distance, the indices or the growth models are not particularly sensitive to changes in the limiting distance. The distance affects more the measurement and the use of the study material. The limit defines the width of the border zone which is needed when measuring empirical sample plot material or selecting observations: the shorter the limit, the nearer the plot edge the observation can be situated and the bigger the number of trees that can be used for constructing the models.

In this study the effect of limiting distance was examined by calculating different growth models where the limiting distance was 4 m, 5 m or 6 m. The residual variation usually decreased with increasing limiting distance but the difference between 5 m and 6 m was negligible. Therefore, 5 m was selected as the competition distance of this study. This limiting distance was also used in models for tree dimensions.

3 Material

3.1 Measurements

The models were computed from 16 plots measured in pure Norway spruce stands in North Karelia. Another set of 7 plots were used for testing the models. The plot size varied according to the number of stems per hectare; the number of trees further than five meters from the nearest plot edge was 40–80 in each plot. The stand age, density and spatial distribution of trees varied considerably (Table 1). The site fertility was either good (*Oxalis-Myrtillus* type) or medium (*Myrtillus* type). Part of the plots were situated in naturally regenerated stands

Table 1. Stand characteristics of the study plots. Parameters R and A are the grouping indices of Clark and Evans, and Hopkins, respectively.

Plot	N	G	T	H_{dom}	D_g	H_g	R	A
Study plots								
1	1592	41.0	42.7	22.2	23.5	18.0	1.19	0.80
2	767	35.0	37.4	21.5	25.6	19.6	1.26	0.67
3	778	22.0	34.9	20.2	21.6	18.2	1.30	0.69
4	733	27.1	36.3	20.5	23.1	18.8	1.29	0.60
5	830	8.6	16.5	9.9	18.9	9.1	1.11	1.33
6	883	19.3	29.3	17.8	22.8	15.4	1.12	1.26
7	647	22.7	70.6	24.6	26.5	21.0	1.18	0.87
8	208	20.5	70.5	24.7	39.4	24.1	1.25	0.70
9	587	22.7	41.2	20.6	23.4	18.8	1.35	0.46
10	448	22.5	38.9	21.8	27.8	20.5	1.21	0.68
11	208	18.4	77.8	24.9	37.2	24.3	1.26	0.75
12	1053	30.0	40.1	20.3	21.6	18.0	1.17	1.48
13	506	27.5	38.1	21.2	28.0	19.7	1.17	0.88
14	660	22.7	42.0	19.7	22.0	18.5	1.32	0.52
15	840	12.5	15.4	11.5	15.3	10.3	1.08	2.56
16	258	13.1	66.1	21.5	35.8	20.8	1.22	0.60
Test plots								
1	517	30.1	54.0	25.8	36.9	23.2	0.93	1.23
2	1130	30.5	77.4	25.5	34.5	22.0	0.86	1.41
3	1317	30.7	59.6	19.6	22.8	16.8	1.00	0.81
4	760	15.2	35.5	17.5	21.8	15.8	0.91	1.01
5	1300	27.8	59.5	24.3	27.7	20.2	1.16	0.63
6	1967	28.5	44.8	19.4	22.7	15.6	0.93	0.78
7	1767	16.1	64.3	16.3	15.4	12.2	0.83	2.39

and another part in plantations. The stands had been managed normally. The time since previous thinning was always at least five years.

The diameter and coordinates were measured for each tree and stump. Trees further than five meters from the plot edge were measured also by height, age and the annual radial growths of the past 10 years. However, if the tree top was difficult to see, the height was not measured but predicted with the help of diameter using a model which was based on the measured heights and diameters. Age and growth were measured from a core taken at breast height. If the core was unclear, the growth measurements were omitted and the age was predicted from diameter. The bark thickness was measured from 20 trees per plot.

The total number of trees further than five meters from the nearest plot edge was 944. The breast height age was measured for 861 trees, height for 822 trees and radial growth for 860 trees.

3.2 Computations

In the first step of the computations of a plot the measurements were used for models that predicted the tree height, age, bark thickness or radial growth as a function of diameter. The model was always a linear model ($y = a + bx$) except the height model which was the Näslund's (1936) height curve ($y = 1.3 + x^2/(a + bx)^2$). These models were utilized if a measurement was missing.

The measured or estimated tree dimensions were used for computing the following stand characteristics: age, dominant height, basal area and number of stems per hectare. Additional predictors for models that predicted tree dimensions (diameter, age and height) were computed in the second step. In the diameter model, when Method 1 was used (see above), the predictors described the number and proximity of neighbors (see Pukkala 1989a). With Method 2 the number of trees and the minimum, mean, maximum and variance of diameter were computed for each tree from a circular area around the tree.

Table 2. Range and mean of some characteristics within 860 trees which were used for constructing spatial growth models.

Variable	Minimum	Maximum	Mean	Unit
i_d	0.0500	7.470	1.515	cm/5 a
i_g	0.8000	243.8	42.11	cm²/5 a
d	0.3400	55.42	18.75	cm
t	3.0000	93.00	34.44	a
h	1.3700	29.31	15.79	m
$G_{<5}$	0.0000	54.46	20.05	m²/ha
$\sum d_j/s_j$	0.0000	137.6	33.85	cm/m

Several predictors were computed for the age and height models from tree locations and diameters. They described the amount of competition that the tree faces and were similar to the competition indices of spatial growth models. Only those trees for which the age or height was measured were used for constructing the model. The rest of the trees were used for computing predictors.

The future five-year overbark growth was selected as the predicted variable of the spatial growth model. In order to predict future growth, tree dimensions five years earlier were derived. Double bark thickness and 5-year diameter growth (without bark) were subtracted from the present diameter. Overbark diameter and overbark diameter growth were computed assuming that the proportion of bark had been the same for the past five years. The future five-year diameter growth 10 years ago (past five-year growth 5 years ago) was computed similarly.

The tree dimensions five years ago were used for calculating stand characteristics, different competition indices and other potential predictors of the spatial growth model (Table 2).

4 Results

4.1 The method for generating model stands

Models

In the present study material the number of stems per hectare was dependent on stand age, dominant height and basal area as follows:

$$
\begin{array}{llllr}
 & \text{Coefficient} & & \text{Predictor} & \text{t-value} \\
\ln(N) = & 8.409 & & & 36.0 \\
 & -0.01674 & \times & T & 5.3 \\
 & +1.368 & \times & \ln(T) & 7.4 \\
 & -3.885 & \times & \ln(H_{dom}) & 20.8 \\
 & +1.729 & \times & \ln(G) & 38.6 \quad (1)
\end{array}
$$

$$R^2 = 0.780 \quad s_f = 0.245 \quad s_e\% = 17.5$$

The number of stems per hectare of the model stands to be generated by the system should be close to the prediction of Equation (1). Otherwise the predicted tree dimensions may be unreliable or biased.

With Method 1 the tree diameter is predicted directly from stand characteristics and the number and vicinity of neighbor trees:

$$
\begin{array}{llllr}
 & \text{Coefficient} & & \text{Predictor} & \text{t-value} \\
d^{0.5} = & 7.995 & & & 7.4 \\
 & +0.2369 & \times & S < 5 & 5.2 \\
 & -0.3911 & \times & \ln(N_{<5}) & 5.7 \\
 & -0.06986 & \times & G & 4.9 \\
 & +3.342 & \times & \ln(G) & 9.0 \\
 & -1.376 & \times & \ln(H_{dom}) & 4.9 \\
 & -0.9925 & \times & \ln(N) & 7.9 \quad (2)
\end{array}
$$

$$R^2 = 0.460 \quad s_f = 0.730$$

where $S_{<5}$ = mean distance of neighbors nearer than 5 m (m) if there are no neighbors $S_{<5=6}$
$N_{<5}$ = number of trees per hectare within 5 m (subject tree included)

The model explains 46 % of the variation in tree diameter. In simulations, the residual variation should be generated by adding to the predicted $d^{0.5}$ a normally distributed stochastic term which has a zero mean and a standard deviation equal to 0.730.

When using Method 2 for generating tree diameters the following set of models are needed to calculate the local diameter distribution:

$$
\begin{array}{llllr}
 & \text{Coefficient} & & \text{Predictor} & \text{t-value} \\
\ln(D_{<5}) = & 4.162 & & & 19.4 \\
 & -0.1443 & \times & \ln(T) & 6.5 \\
 & -0.02475 & \times & G & 8.2 \\
 & +1.230 & \times & \ln(G) & 19.2 \\
 & -0.4496 & \times & \ln(N) & 17.2 \\
 & -0.1479 & \times & \ln(N_{<5}) & 9.7 \quad (3)
\end{array}
$$

$$R^2 = 0.773 \quad s_f = 0.162 \quad s_e\% = 11.5$$

$$
\begin{array}{llllr}
 & \text{Coefficient} & & \text{Predictor} & \text{t-value} \\
\ln(D_{min}) = & 20.62 & & & 21.3 \\
 & -0.1272 & \times & T & 18.9 \\
 & +6.564 & \times & \ln(T) & 19.7 \\
 & +1.035 & \times & H_{dom} & 20.6 \\
 & -23.06 & \times & \ln(H_{dom}) & 23.3 \\
 & -0.153 & \times & G & 18.4 \\
 & +4.639 & \times & \ln(G) & 19.0 \\
 & -0.3577 & \times & \ln(N) & 7.0 \\
 & -0.1815 & \times & \ln(N_{<5}) & 6.9 \\
 & +1.292 & \times & \ln(D_{<5}) & 24.3 \quad (4)
\end{array}
$$

$$R^2 = 0.822 \quad s_f = 0.261 \quad s_e\% = 18.6$$

$$
\begin{array}{llllr}
 & \text{Coefficient} & & \text{Predictor} & \text{t-value} \\
\ln(D_{max}) = & -22.41 & & & 24.3 \\
 & +0.128 & \times & T & 23.5 \\
 & -6.038 & \times & \ln(T) & 23.6 \\
 & -1.074 & \times & H_{dom} & 26.1 \\
 & +22.55 & \times & \ln(H_{dom}) & 27.2 \\
 & +0.2122 & \times & G & 23.6 \\
 & -5.24 & \times & \ln(G) & 23.6 \\
 & -0.001406 & \times & N & 14.8 \\
 & +1.145 & \times & \ln(N) & 16.3 \\
 & +0.1065 & \times & \ln(N_{<5}) & 7.9 \\
 & +0.04652 & \times & D_{<5} & 34.1 \quad (5)
\end{array}
$$

$$R^2 = 0.828 \quad s_f = 0.134 \quad s_e\% = 9.5$$

$$
\begin{array}{llllr}
 & \text{Coefficient} & & \text{Predictor} & \text{t-value} \\
\ln(s_d^2) = & 3.491 & & & 26.3 \\
 & -0.00008286 & \times & N & 6.3 \\
 & +0.0003596 & \times & N_{<5} & 15.4 \\
 & -0.6721 & \times & \ln(N_{<5}) & 31.7 \\
 & +0.9987 & \times & \ln(D_{max}-D_{min}) & 574.9 \quad (6)
\end{array}
$$

$$R^2 = 0.999 \quad s_f = 0.094 \quad s_e\% = 6.7$$

where $D_{<5}$ = unweighted mean diameter of trees within a 5-m circle around the subject tree, subject tree included (cm)
D_{min} = minimum diameter within the circle (cm)

D_{max} = maximum diameter within the circle (cm)

s_d^2 = variance of diameter within the circle (cm^2)

The beta distribution is computed from $D_{<5}$, D_{min}, D_{max} and s_d^2 (Loetsch et al. 1973, p. 53). This distribution is sampled to obtain a diameter for the subject tree. Equations (3)–(6) again constitute a recursive chain of models. This means that the predictions for D_{min}, D_{max} and s_d^2 are not as good as the presented R^2, s_f and $s_e\%$ suggest. The use of estimated $D_{<5}$ for predicting D_{min}, D_{max} and s_d^2 assures that these parameters are logically related to each other. Another way to take into account correlations between D_n, D_{min}, D_{max} and s_d^2 is to compute covariances of residuals of models for D_n, D_{min}, D_{max} and s_d^2, these models being based on known predictors only. When using the models, four stochastic components are sampled from the joint distribution of the residuals.

The spatial models for predicting tree age and height complete the method for generating model stands:

	Coefficient		Predictor	t-value	
$\ln(t) =$	-0.2733			4.6	
	$+0.1208$	×	$d^{0.5}$	10.2	
	$+0.006497$	×	$G_{<5>d}$	10.2	
	$+0.2859$	×	$d/D_{<5(g)}$	9.8	
	$+0.8883$	×	$\ln(T)$	43.7	(7)

$R^2 = 0.847 \quad s_f = 0.181 \quad s_e\% = 12.9$

	Coefficient		Predictor	t-value	
$\ln(h-1.3) =$	3.667			205.7	
	-19.72	×	$1/(d+5)$	51.3	
	-7.732	×	$1/(t+5)$	14.3	
	$+0.002599$	×	$G_{<5}$	5.4	(8)

$R^2 = 0.901 \quad s_f = 0.150 \quad s_e\% = 10.6$

where $G_{<5>d}$ = basal area of neighbors nearer than 5 m and greater than the subject tree (m^2/ha)

$D_{<5(g)}$ = mean diameter (weighted by basal area) of trees within 5 m from the subject tree, subject tree included (cm)

$G_{<5}$ = basal area of neighbors nearer than 5 m, subject tree excluded (m^2/ha)

The models for tree diameter indicate that an increase in the number of trees within a subarea of a stand shifts the diameter distribution towards smaller diameters. A tree which faces much competition is older and taller than a tree with the same diameter and less competition.

Validity of the models

The validity of the stand generation method was tested by computing the diameter, breast height age and height of each tree in the 7 test plots with Equations (2)...(8) and comparing the stand characteristics computed from these predictions to those computed from the measurements. The residual variation of the models was simulated with normally distributed random numbers.

Both the basal areas and the mean heights computed from the predictions frequently differ from the measured values (Fig. 1). This is partly due to the stochasticity of the stand generation method and partly due to inadequacies of the models. The differences between measured and predicted stand dimensions are greatest in test plot 6 where both methods produced too small trees, especially Method 2. Test plot 6 has more stems per hectare than any of the study plots used to compute the models. It seems that, outside the range of variation in the study material, the models for diameter and diameter distribution over estimate the effect of number of stems.

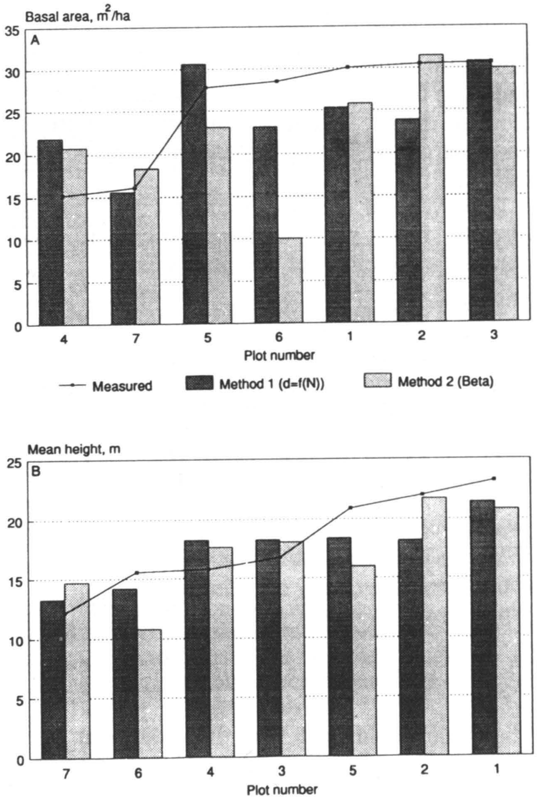

Fig. 1. Measured and predicted stand basal area (A) and mean height (B) of the seven test plots. Predicted stand characteristics are based on tree dimensions calculated from stand basal area, number of stems per hectare, age, dominant height and tree locations.

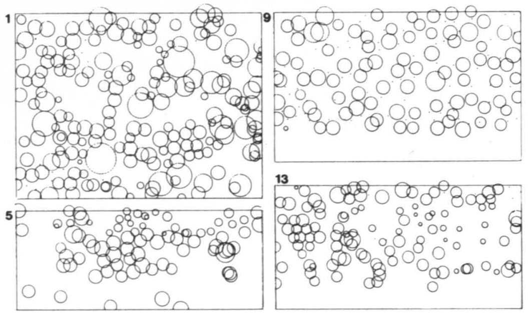

Fig. 2. Crown maps of study plots 1, 5, 9 and 13. Crown width (dc, m) is calculated by equation dc = 0.5 + 0.1 × d, where d is breast height diameter (cm). The small dots represent stumps.

Plots 1, 5, 9 and 13 were used for a more thorough analysis of the diameter models (Fig. 2). Both methods to generate tree diameters produced distributions more or less similar to the measured ones (Fig. 3). In plot 9 both methods produced too many small diameters (9–15 cm), and in plot 5 the distribution generated by Method 2 was too narrow.

The methods were tested at tree level by computing correlation coefficients between diameter and variables describing the number and proximity of neighbors; the coefficients should be similar in real and generated stands (Table 3). The results indicate that in this respect Method 1 (direct prediction of diameter) is better than Method 2 (estimation of local diameter distribution). Because Method 2 is was not bet-

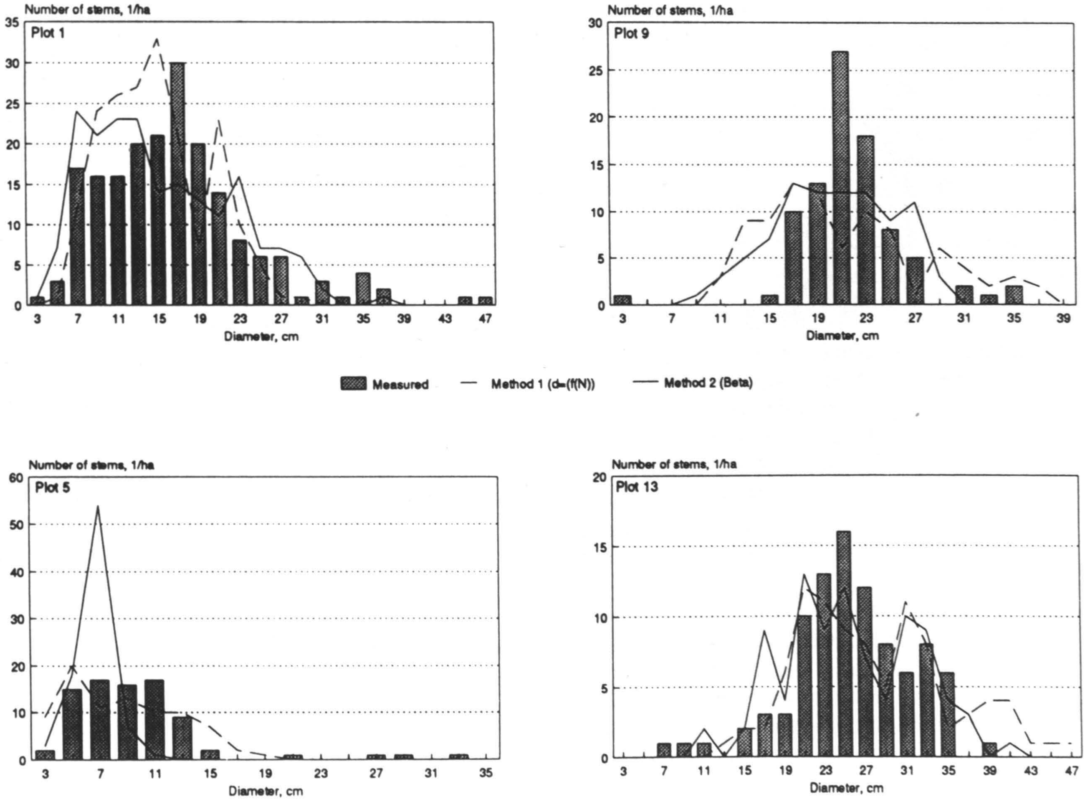

Fig. 3. Measured and predicted diameter distribution of study plots 1, 5, 9 and 13. Diameters were predicted with Equation (2) (Method 1) or Equations (3)–(6) (Method 2).

Table 3. Correlation coefficient between tree diameter and some variables describing the number and proximity of neighbor trees. $S_{<5}$ is the mean distance of neighbors nearer than 5 m. $\sum 1/s_j$ is the sum of inverted distances of these neighbors. In Method 1 the diameter is predicted from the number and locations of neighbors and in Method 2 is the sample from the predicted local diameter distribution.

Plot	Measured	Method 1	Method 2	Measured	Method 1	Method 2
	Correlation with $S_{<5}$			Correlation with $\sum 1/s_j$		
1	0.273	0.065	0.083	–0.180	–0.110	0.001
5	0.242	–0.148	–0.092	–0.170	–0.316	0.016
9	0.016	0.127	–0.122	–0.193	–0.024	–0.218
13	–0.074	0.342	0.174	–0.045	–0.429	–0.212
Mean	0.114	0.097	0.011	–0.147	–0.220	–0.103
1 + 5 + 9 + 13	0.333	0.404	–0.105	–0.506	–0.591	–0.162

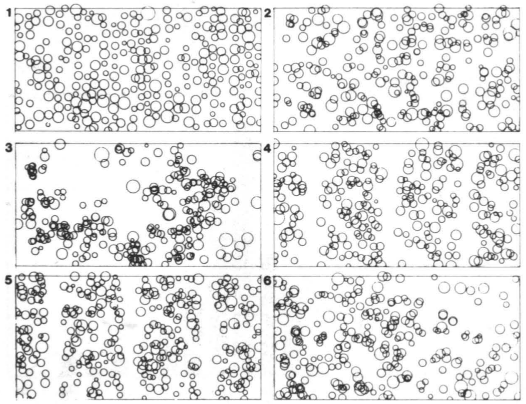

Fig. 4. Examples of stands (crown maps) generated by Method 1. In stand 5 the extraction roads are new. In the other stands the spatial distribution has remained unchanged for several years. Stands 1, 2, 3, 4 and 6 were used in the simulation experiment of Section 5.1. The area of each plot is 40 m × 80 m.

ter than Method 1 at the stand level, Method 1 should be preferred in producing Norway spruce model stands. Method 1 has the additional advantage of being much simpler than Method 2.

The use of the above models for generating tree dimensions presupposes that the present spatial arrangement has affected the tree growth for a long time; the latest thinning has taken place at least five years ago. The procedure can be combined with simulated thinnings to create such differences in the spatial distribution which have a recent origin.

Examples of model stands generated by Method 1 are given in Fig. 4. In stand 5 of Fig. 4 the extraction roads are new (created after the generation of diameters), and in stand 4 they have an older origin.

4.2 Spatial growth models

Spatial growth models were computed for a five-year basal area growth (i_g) and diameter growth (i_d). One pair of models was made by using past growth as a predictor and another pair without past growth:

	Coefficient		Predictor	t-value	
$\ln(i_g) =$	4.102			25.5	
	+0.9323	×	$d^{0.5}$	16.4	
	–0.0009224	×	g	7.5	
	+1.231	×	$\ln(1/(t+5))$	20.3	
	–0.01452	×	$\sum d_j/s_j$	15.4	
	+1.037	×	$d/D_{<5(g)}$	9.5	(9)

$R^2 = 0.738 \quad s_f = 0.490 \quad s_e\% = 35.7$

	Coefficient		Predictor	t-value	
$\ln(i_g) =$	0.5679			7.1	
	–0.005786	×	ig-5	9.5	
	+1.196	×	$\ln(i_{d\text{-}5})$	40.3	
	+0.7016	×	$d^{0.5}$	23.1	
	–0.0004445	×	g	6.1	
	–0.006379	×	$\sum d_j/s_j$	11.2	
	+0.2683	×	$d/D < 5(g)$	3.9	(10)

$R^2 = 0.905 \quad s_f = 0.296 \quad s_e\% = 21.2$

	Coefficient		Predictor	t-value	
$\ln(i_d) =$	4.336			19.8	
	+0.04982	×	h	7.8	
	+1.372	×	$\ln(1/(t+10))$	19.6	
	–0.01506	×	$\sum d_j/s_j(q3)$	16.5	
	+0.9481	×	$d/D_{<5(g)}$	10.6	(11)

$R^2 = 0.595 \quad s_f = 0.472 \quad s_e\% = 34.3$

	Coefficient		Predictor	t-value	
$\ln(i_d) =$	–0.1335			2.1	
	–0.1650	×	$i_{d\text{-}5}$	8.0	
	+1.176	×	$\ln(i_{d\text{-}5})$	29.5	
	+6.357	×	$1/(t+10)$	5.0	
	–0.006213	×	$\sum d_j/s_j(q2)$	11.1	
	+0.3047	×	$d/D_{<5(g)}$	5.9	(12)

$R^2 = 0.850 \quad s_f = 0.287 \quad s_e\% = 20.5$

where i_g = future 5-year basal area growth including bark (cm^2)
g = basal area of tree (cm^2)
$\sum d_j/s_j$ = sum of ratios of diameter (cm) and distance (m) of neighbors nearer than 5 m, subject tree excluded (cm/m)
$D_{<5(g)}$ = mean diameter (weighted by basal area) of trees within 5 m of the subject tree, subject tree included (cm)
$i_{g\text{-}5}$ = past overbark 5-year basal area growth (cm^2)
$i_{d\text{-}5}$ = past overbark 5-year diameter growth (cm)
i_d = future 5-year diameter growth (cm)
h = tree height (m)
$\sum d_j/s_j(q3)$ = sum of ratios of diameter (cm) and distance (m) of neighbors nearer than 5 m, the neighbors are selected with a relascope using basal area factor 3 m^2/ha (cm/m)
$\sum d_j/s_j(q2)$ = sum of ratios of diameter (cm) and distance (m) of neighbors nearer than 5 m, the neighbors are selected with a relascope using basal area factor 2 m^2/ha (cm/m)

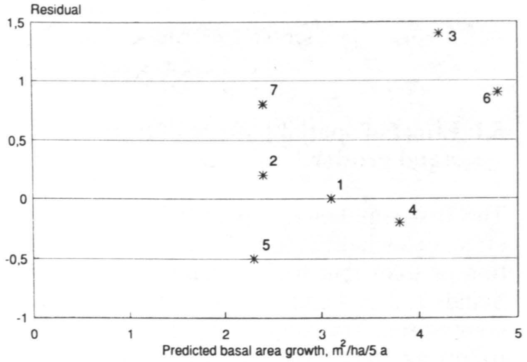

Fig. 5. Error in the predicted basal area growth ($m^2ha^{-1}a^{-1}$) of the test plots 1–7 as a function of prediction. Predicted growth is calculated by Equation (9).

Using past growth as an additional predictor improved the models considerably. However, spatial predictors were still needed. This indicates that there are temporal changes in the competitive status of a tree. These changes may belong to a normal stand development or they may have been created through thinnings.

When the aim is to predict the volume growth of a stand as accurately as possible, it is preferable to predict basal area growth instead of diameter growth. In practical situations the past growth is seldom available for growth prediction. Equation (9) can therefore be considered as the most important growth model of the study.

Equation (9) was tested by predicting the future 5-year growth (five years ago) of each tree in the 7 test plots. The basal area growth of the plot was computed both from the predicted and measured diameter growths (Fig. 5). The predictions were near the measured growths in plots 1, 2, 4 and 5, but in plots 3, 6 and 7 underestimation was notable. These plots have a rather high number of stems per hectare and the trees are quite small (Table 1). This indicates that Equation (9) overestimates the effect of competition in dense stands consisting of small trees. In plot 7 an additional reason might be the high degree of aggregation of trees (Table 1) the effect of which may not be fully reflected in the growth model.

5 Simulation experiments

5.1 Effect of spatial pattern of trees on stand growth

The first simulation experiment examined the effect of such differences in the spatial distribution of trees that had existed for a long time. Stands 1, 2, 3, 4 and 6 of Fig. 4 were used in the comparison. The trees were generated from the following stand characteristics: basal area (G) 25 m²/ha, breast height age (T) 40 years, dominant height (H_{dom}) 20 m and number of stems (N) 848 trees/ha (prediction of Equation 1). The tree coordinates were generated as follows.

(1) Systematic grid but with some mortality and random deviations from the grid points (a tree plantation).
(2) Poisson distribution.
(3) Trees are in clusters which are Poisson distributed. There are 15 trees in each cluster (average) and the radius of a cluster is 7 m.
(4) Five meters wide extraction roads 25 m apart in a non-stationary Poisson process with 1 m hard core. Number of stems per hectare decreases by 20 % from the middle of two roads to the road edge. The minimum distance (hard core) between any two trees is 1 m.
(6) Non-stationary Poisson distribution where the number of stems per hectare changes in a trend-like manner. The relative number density is 1 in the lower left hand corner and decreases linearly down to 0.25 in the upper right hand corner.

The grouping indices of Clark and Evans (1954) and Hopkins (1954) were used to describe the degree of clustering of the stands. The index of Clark and Evans (R) is

$$R = v^2\sqrt{\mu} \tag{13}$$

where v is the mean distance from a tree to the nearest neighbor and μ is the stand density (trees/m²). The index of Hopkins (A) is computed by the formula

$$A = \Sigma a_i^2/\Sigma b_j^2 \tag{14}$$

where a_i is a random 'point to nearest tree' distance and b_j is a random 'tree to nearest neighbor' distance. The number of both distances is the same. For Poisson distribution both indices have an expected value of one. For a regular patter R is greater than one and A less than one, and for aggregated distributions R is less than one and A greater than one.

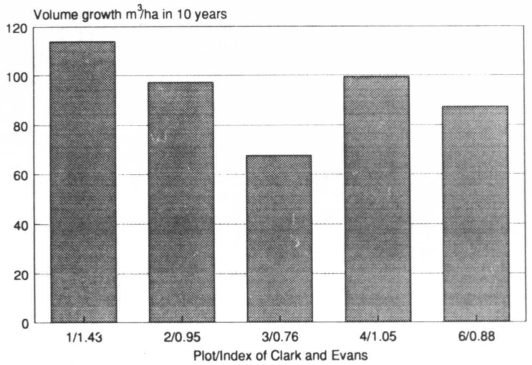

Fig. 6. Predicted growth of stands 1, 2, 3, 4 and 6 of Fig. 4.

For the model stands used in the simulation, indices R and A of these stands were as follows:

Stand	R	A
1	1.43	0.36
2	0.95	0.94
3	0.76	3.41
4	1.05	1.25
6	0.88	1.48

According to the indices, stand 1 is the most regular and stand 3 clearly the most clustered.

The growth of each stand was simulated for 10 years using Equation (9). The height corresponding to the incremented diameter was calculated using Equation (8) and the stem volume using the volume function of Laasasenaho (1982). It was supposed that the plots had similar plots on all sides.

The predicted growth of the model stands clearly decreased with clustering (Fig. 6). The growth of stand 3 was 40 % lower than in the most regular, stand 1. Five meter wide extraction roads decreased the growth by about 15 % when many trees had been cropped from the ride sides (stand 4). The correlation coefficient of growth prediction is 0.909 with grouping index R and –0.964 with index A.

5.2 Effect of thinning method on stand growth

The second simulation studied the effect of thinning on stand productivity. Before the thinning treatment the stand basal area was 31.8 m²/ha, age 38.9 years, dominant height 20.6 m and number of trees per hectare 2063. The spatial distribution was similar to plot 3 in the previous simulation experiment (clustered). This stand was thinned to a remaining basal area of 22 m²/ha according to the following principles (Fig. 7):

(1) Removed trees were selected at random.
(2) 4.5 meter wide extraction roads were cleared at 30 meter distances. The rest of the removal (4 m²/ha) was taken at random.
(3) Trees were removed on the basis of diameter only, starting from the biggest tree (thinning from above).
(4) Trees were removed on the basis of diameter only, starting from the smallest tree (thinning from below).
(5) The smaller tree of each pair closer than 1 m was removed, after which rule (4) was applied.

After thinning the grouping indices R and A indicated a slightly regular distribution for thinning method 5 and clustered patterns for the other methods:

Stand	R	A
1	0.85	2.15
2	0.77	2.14
3	0.83	1.64
4	0.89	1.62
5	1.08	0.78

The best growth was obtained for rule 5, with a thinning regime which removed small trees and aimed, at the same time, for a regular spatial distribution (Fig. 8). The growth prediction was poorest for the method that removed the biggest trees without considering the spatial distribution (rule 3). Rule 5 closely resembles a practical execution of a thinning treatment. In this simulation experiment the correlation coefficient between the predicted growth and grouping index R was 0.877, and –0.741 between growth and index A.

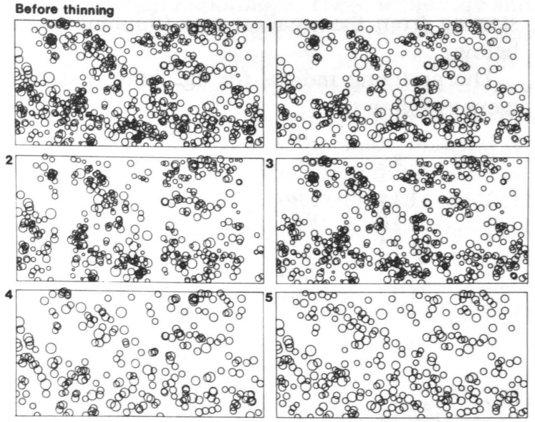

Fig. 7. Crown map of a model stand before thinning and after 5 different thinnings. The area of the plot is 40 m × 80 m.

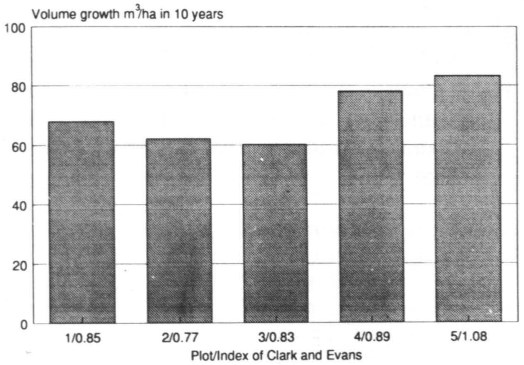

Fig. 8. Predicted stand growth after 5 different thinnings. The crown maps after thinnings are presented in Fig. 7.

5.3 Effect of the width of extraction road on stand growth

The last simulation example studied the effect of the width of extraction road on the growth of a planted Norway spruce stand. The stand basal area was 37.5 m²/ha prior to thinnings, the breast height age 34 years, dominant height 20.1 m and number of stems 2583 trees/ha. The width of the harvest road was 10 m (stand 1), 8 m (stand 2), 6 m (stand 3), 4 m (stand 4) or 2 m (stand 5) and the distance between the roads 30 m (Fig. 9). In all the cases stand basal area was decreased to 25 m²/ha. After cutting the extraction roads, the remainder of the removal was taken systematically according to diameter starting from the smallest tree (thinning from below).

The grouping indices for the thinned stands were as follows.

Stand	R	A
1	1.14	1.96
2	1.10	1.51
3	1.12	1.21
4	1.15	0.84
5	1.17	0.61

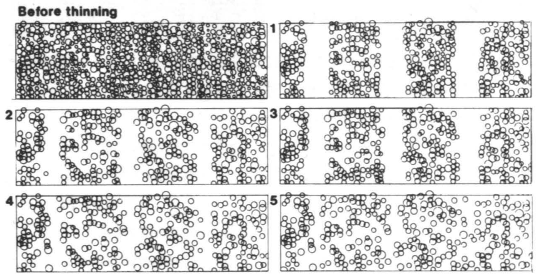

Fig. 9. Crown map of a model stand before thinning and after 5 different thinnings with varying width of extraction road. The area of the plot is 30 m × 100 m.

As expected, the growth is better the narrower the extraction road (Fig. 10). With 6-m rides the growth is 6.4 % lower than with 2-m rides, and 10-m wide rides decrease the yield by 15 %. In this simulation the growth prediction correlates better with grouping index A (–0.998) than with index R (0.614).

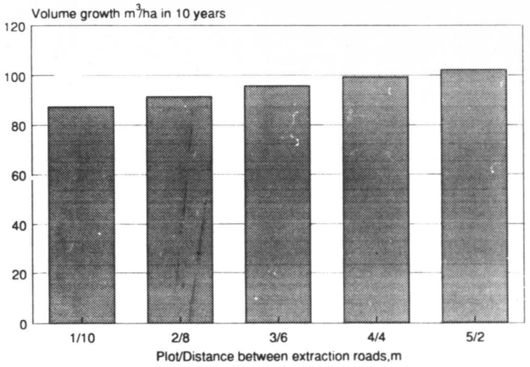

Fig. 10. Predicted stand growth after 5 different thinnings. The crown maps after thinnings are presented in Fig. 9.

6 Discussion

The method for producing model stands, when combined with simulated thinnings, provides a flexible tool for generating material for simulation experiments. The main drawback is that the study material was not variable enough for extreme cases: strictly regular or very irregular stands, or exceptionally young, old, sparse and dense stands. The practical limits for stand characteristics are 300–1500 stems per hectare, 20–80 years for age, 10–35 m²/ha for basal area and 10–28 m for dominant height, and the relationships between these characteristics should correspond to normal Norway spruce stands. Grouping index R should be between 1.40 and 0.80 and grouping index A between 0.35 and 3. Otherwise the predicted tree dimensions may not be reliable. The same limits apply to the spatial growth models. Also within these limits the stand generation method can produce stand basal areas substantially different from the aimed basal area. The produced diameters may therefore need scaling to adjust the stand basal area to a specified value.

The obvious way to improve the simulation model is to collect more data, especially from exceptional stands with respect to spatial pattern, site, stand density and age. Uneven-aged stands in the study material would allow the simulation of the yield and development of a selection forest.

The residual variation of the basal area growth model was 0.490 when past growth was not used as a predictor. This compares well with Tham's (1989) much more complicated model where the residual variation was 0.412. The degree of determination of the diameter growth model was 0.595 when past growth was not used as a predictor. In Mielikäinen's (1978) spatial growth model for spruce the degree of determination was 0.435–0.442.

The residual variation of all the presented models were of about the same magnitude as in the corresponding models of Scots pine (Pukkala 1989a, 1989b). Past growth seems to improve the growth model more with Norway spruce than with Scots pine. Another difference is that spatial predictors do not seem to be equally important for Norway spruce. The advantage of spatial predictors is that, although they do not decrease the residual variation notably, they allow the study of spatial problems.

The small significance of within-stand variation in stand density was also reflected in the diameter models: the correlation between tree diameter and number of stems around the tree was very weak. This, as well as the small contribution of spatial predictors in the growth models, is partly a consequence of the study material: the within-stand variation of the sample plots was small compared to the between-stand variation.

The removal variable (stumps) did not improve the growth models (cf. Mielikäinen 1978). Also the directional distribution of competitors was of small importance. These results are similar to those obtained for Scots pine stands (Pukkala 1989a, 1989b). The reason for the small effect of removal and directional distribution might be that these factors correlate with other variables, and also that the study material did not contain enough variation in these respects.

The models of this study were estimated with an ordinary least squares technique. The shortcoming of the analysis was that observations were not independent of each other because several trees were measured in each plot. An improved model type for this kind of data i a mixed linear model, which divides the stochastic variation in diameter growth into between-stand and within-stand components (Lappi 1986). In this case, the fixed parameters can be estimated using the generalized least squares technique. This type of model can be calibrated by estimating the random stand effects for a particular stand.

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