Silva Fennica 1992, Vol. 26 N:o 3: 169–176

A decision theoretic approach applied to goal programming of forest management

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TIIVISTELMÄ: PÄÄTÖSTEOREETTINEN LÄHESTYMISTAPA TAVOITEOHJELMOINTITEHTÄVÄN MUOTOILUUN METSÄTALOUDEN SUUNNITTELUSSA

Kangas, J. & Pukkala, T. 1992. A decision theoretic approach applied to goal programming of forest management. Tiivistelmä: Päätösteoreettinen lähestymistapa tavoiteohjelmointitehtävän muotoiluun metsätalouden suunnittelussa. Silva Fennica 26(3): 169–176.

An alternative approach to formulating a forestry goal programming problem is presented. First, single objective optima levels are solved. The Analytic Hierarchy Process is applied in the estimation of a priori weights of deviations from the goal target levels. The ratios of the weights can be interpreted as relative importances of the goals, respectively. The sum of the weighted deviations from all single optima levels associated with the management goals is minimized. Instead of absolute deviations, relative ones are used. A case study problem of forest management planning with several objectives, measured in different units, is analyzed.

Tutkimuksessa esitetään päätösteoreettinen tapa muotoilla tavoiteohjelmoinnilla ratkaistava metsätalouden suunnittelun monitavoitteinen optimointitehtävä. Kullekin yksittäiselle tavoitteelle haetaan se tavoitearvo, joka tuottaa suurimman mahdollisen osahyödyn. Näitä osahyötyoptimeja käytetään vastaavien tavoiteohjelmoinnin rajoiteyhtälöiden tavoitearvoina. Analyyttisellä hierarkiaprosessilla estimoidaan tavoitearvoista poikkeamien suhteelliset painoarvot. Painoarvot ovat päätöksentekijän asettamat tavoitteiden suhteelliset tärkeydet. Tärkeyksillä painotettujen suhteellisten poikkeamien summaa minimoidaan. Lähestymistapa havainnollistetaan muotoilemalla metsätalouden suunnittelun esimerkkitehtävä ja ratkaisemalla se.

Keywords: forest management, forest planning, goal programming, decision analysis, optimization methods, models.
FDC 624

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Accepted November 9, 1992

1 Introduction

Goal programming (GP) is an extension of linear programming, with different problem formulation. It was first described by Charnes and Cooper (1961). A forestry application of GP was first presented by Field (1973). After that, several applications of GP to forest management planning have been presented (e.g. Rustagi 1976, Schuler et al. 1977, Field et al. 1980, Hotvedt et al. 1982, Walker 1985, Mendoza et al. 1987, Lappi 1992). Mendoza (1987) provided an overview of GP formulations and extensions with special reference to forest planning. By using goal programming, some crucial problems of standard linear programming (LP) can be avoided. However, the method of numerical solution is the same as in standard linear programming.

In standard linear programming, a single overriding management goal is assumed, which is represented by the objective function. For interpretational purposes, it is recommendable to use a single-valued objective function, the variables of which are commensurable and compatible. Other objectives are taken into account through constraints. The description of management goals by standard LP constraints may be unsatisfactory for several reasons (e.g. Allen and Gould 1986, Cocklin 1989, Hobbs and Hepenstal 1989, Mendoza and Sprouse 1989). The selection of the goal to be handled as the objective function is arbitrary. The objective function is optimized within the feasible region defined by the rigid LP constraints; thus, the constraints may, in fact, receive a greater weight than the objective. Rigid constraints may also cause infeasibility problems.

In forestry practice, there are usually more than one objective. The objectives are often measured in different measurement units, and the goals are incommensurable. According to both theoretical and empirical studies, preferences of forestry decision-makers vary considerably from one decision-maker to another (Hyberg and Holthausen 1989, Kreutzwiser and Wright 1990, Kangas 1992a).

In a goal programming approach, all the objectives are handled in the same manner: they are expressed by goal constraints. A goal constraint differs from an LP constraint by goal variables. A goal variable measures the amount by which the contribution of all activities to the goal in question falls short or exceeds the goal level (i.e. the right hand side of the constraint). The objective function of a GP problem is to minimize the sum of the weighted deviations from all target levels associated with the management goals. When goal variables are included in a constraint, the problem of infeasibility linked to the constraint is avoided.

In this study, a decision theoretic approach is presented for formulating a GP problem. The aim is to formulate a forestry GP problem that reflects the preferences of the decision-maker more accurately than GP problems formulated by methods presented previously. The approach is illustrated by a case study taken from forest management planning.

GP problem formulation

The general formulation of a GP problem is

$$\min z = \sum_{i=1}^{m} (w_i^- D_i^- + w_i^+ D_i^+) \tag{1}$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j + D_i^- - D_i^+ = G_i \qquad i = 1,\ldots,m \tag{2}$$

$$\sum_{j=1}^{n} a_{ij} x_j \le / \ge B_i \qquad i = m+1,\ldots,M \tag{3}$$

$$x_j, D_i^-, D_i^+, \ge 0 \tag{4}$$

where (1) is the goal programming objective function; (2) is a goal constraint; (3) is an LP constraint; m is the number of goals represented by goal constraints; M is the total number of constraints; D_i^- is the underachievement deviation variable; D_i^+ is the overachievement deviation variable; w_i^- is the weight given to each unit of underachievement deviation; w_i^+ is the weight given to each unit of overachievement deviation; X_j is the jth activity variable; a_{ij} is the contribution to goal i made by each unit of activity j; n is the number of activity variables; G_i is the target level for goal i

Standard LP constraint may be used if any deviation from a goal target level is unacceptable. Also, constraint with only one deviation variable, either Di^+ or D_i^-, can be applied. As scaling factors, weights wi^+ and w_i^- reduce deviations expressed in different measurement units to a common unit of measurement, and they reflect the relative importances of the goals. Thus, the objective function of GP represents the preference function of the decision-maker. The ratios of the weights are the trade-offs. Because of the scaling role, the interpretation of weights as relative importances of goals, respectively, is not unambigious, when absolute deviations are used (Keeney and Raiffa 1976).

Some problems with GP

Crucial problems when utilizing GP include: (i) specifying the target levels of the goals, (ii) determining the weights used in the objective function, and (iii) making goals measured with different units commensurable. Any mathematical programming problem has to be formulated so that it reflects as well as possible the preferences of the decision maker. Prior information on the preferences of the decision maker is needed to formulate the problem appropriately. Because there are usually no single overriding management goal in multiple-use forestry, but a set of more or less conflicting objectives having certain trade-offs, cardinal weighting is recommendable instead of ordinal weighting (e.g. Dyer et al. 1979, Field et al. 1980).

Intuitive establishment of desired goal achievement levels may lead to the generation of dominated or inferior solutions (Dyer et al. 1979). Setting the target level of one of the goals to an arbitrary very large value, as presented by Kao and Brodie (1979), overstates the actual deviation of the final solution from the single objective optimal level (Walker 1985). A third approach to determine the weights, as categorized by Walker (1985), is to set the goal levels to single objective optima (e.g. Field et al. 1980). Using this approach, the final solutions show the actual deviations of goals from their single objective optima. Knowledge of single objective optima is also of value when determining the weights of management goals.

Specifying a set of a priori relative weights for the goals is often found to be difficult. Several methods for estimating the weights through an interactive process have been presented (e.g. Nijkamp and Spronk 1980, Zeleny 1982, Korhonen and Laakso 1986a). Unfortunately, for most decision-makers it is difficult to supply the right information concerning his preferences, required to determine the search direction and the step-size in this direction. The better the first problem formulation, the faster the optimal solution can be found. In the very first problem formulation, a priori weights are always used.

In decision theoretic research, especially in the mathematical psychology approach, much attention has been paid to estimating the preference function of a decision-maker. Some help in estimating the a priori weights can be attained from utility theoretic methods of multicriteria decision making. Korhonen (1987) used a decision analysis method called the Analytic Hierarchy Process (AHP) for specifying a search direction in a visual interactive optimization method. He found the AHP a convenient way to structure requisite preference information. The AHP can also be utilized in the estimation of a priori weights for the objective function of goal programming, as will be shown in this paper.

2 A decision theoretic GP formulation

A decision theoretic approach to GP differs from normal GP procedures only with respect to problem formulation – or, rather, with respect to the determination of the goal target levels and weights of deviations from these desired achievement levels. Thus, in this paper, we mainly consider the problem formulation. We assume that the greatest possible sub-utility with respect to each objective included in the objective function is obtained at its maximum or minimum level. For the steps of GP, readers are referred to e.g. Steuer (1986), Buongiorno and Gilless (1987) and Taha (1987).

First, the greatest or smallest attainable target level of each goal constraint is solved. This target level ($G_{u\text{-max}\ i}$) is considered to represent the greatest possible sub-utility, referring to the objective in question. The goal objective is to minimize the weighted sum of relative deviations from the maximum or minimum attainable target levels, referred to as aspiration levels. Weights are determined using the Analytic Hierarchy Process.

Thus, the formulation of the GP model is

$$\min z = \sum_{i=1}^{m} (w_i D_i / G_{u-\max i}) \qquad (5)$$

subject to

$$\sum_{j=1}^{n} a_{ij} X_j + D_i^- = G_{u-\max i} \qquad i = 1,\ldots,m_k \qquad (6)$$

$$\sum_{j=1}^{n} a_{ij} X_j - D_i^+ = G_{u-\max i} \qquad i = m_{k+1},\ldots,m \qquad (7)$$

$$\sum_{j=1}^{n} a_{ij} x_j \le / \ge B_i \qquad i = m+1,\ldots,M$$

$$x_j, D_i^+, D_i^-, \ge 0$$

where $D_i = D_i^-$ when it is a question of an objective producing the greatest sub-utility at its maximum level; $D_i = D_i^+$ when it is a question of an objective producing the greatest sub-utility at its minimum level; m_k is the number of objectives producing the greatest sub-utility at their maximum levels; $m - m_k$ is the number of objectives producing the greatest sub-utility at their minimum levels; $G_{u\text{-max } i}$, $i = 1,\ldots, m_k$ is the maximum attainable value of a goal variable; $G_{u\text{-max } i}$, $i = m_{k+1},\ldots, m$ is the minimum attainable value of a goal variable

Weight determination

The Analytic Hierarchy Process (AHP) is a mathematical method for analysing complex decision problems with multiple criteria. It was originally developed by Saaty (1977,1980). Basically, the AHP is a general theory of measurement, having both mathematical and psychological features. In this study, the eigenvalue method, as applied in the AHP, is utilized in determining weights of decision criteria, which are included in goal objective function.

For estimating a priori weights of proportional deviations from the aspiration levels, pairwise comparisons between decision criteria are carried out. When making the comparisons, the question is: which of the two factors has a greater weight in decision-making, and how much greater?

Pairwise comparisons are converted into numerical form and reciprocal matrices of pairwise comparisons are constructed (Equation 8). In the standard version of the AHP, verbal comparisons are converted into an integer scale from 1 to 9. The choice of the scale is based on observations on human information processing capacity, made by psychologists (e.g. Miller 1956), and on several practical tests (e.g. Harker and Vargas 1987).

$$\mathbf{A} = (a_{ij}) = \begin{bmatrix} 1 & w_1/w_2 & \ldots & w_1/w_n \\ w_2/w_1 & 1 & \ldots & w_2/w_n \\ . & . & \ldots & . \\ . & . & \ldots & . \\ w_n/w_1 & w_n/w_2 & \ldots & 1 \end{bmatrix} \qquad (8)$$

where w_i/w_j is the ratio of the weights of decision elements i and j, respectively

Using the pairwise comparisons as input, the relative weights of elements are computed by using the eigenvalue method. The resulting weights, priorities, represent the decision-maker's perception of the relative importance, or preference, of the criteria.

Differences in measurement units do not present any difficulty when the decision theoretic approach is used in formulating a GP problem. This is because the method is based on direct comparisons of the importance of decision elements, without using any physical units, and proportional deviations are used.

Based on properties of reciprocal matrices, a consistency ratio (CR) can be calculated. CR measures the coherence of the pairwise comparisons. In human decision-making, some inconsistencies can be expected. As a rule of thumb, a CR value of 10 percent or less is considered acceptable (Saaty 1980, Saaty and Kearns 1985). Otherwise, all or some of the comparisons should be repeated to resolve the inconsistencies of the pairwise comparisons. For more details on the AHP theory and the estimation of relative priorities, readers are referred to Saaty (1977, 1980) and Saaty and Kearns (1985), and for forestry applications to Mendoza and Sprouse (1989), and Kangas (1992a, 1992b).

There has been a lively scientific discussion concerning the choice of the scale used in pairwise comparisons (e.g. Dyer 1990, Harker and Vargas 1990). The scale 1 to 9, as presented by Saaty (1977, 1980), is quite rough when there exists only a slight difference in the importance or preference of the elements to be compared. The smallest ratio differing from equal weights, which it is possible to express using that scale, is 2:1. Intuitively, the ratio 2:1 describes a rather strong preference of one element over another. Furthermore, each person has his own scale when comparing elements verbally. For example, a comparison 'objective A is strongly more important than objective B' does not represent the same importance ratio (in numerical form) for everybody.

The choice of a proper scale is difficult. In this study, the problems of verbal comparisons and converting them into numericals were avoided by using graphical interface, where barlengths expressed the relative importance of goals (see also Forman and Saaty 1986). Weights were estimated interactively using a micro computer, adjusting the lengths of bars by arrows of the keyboard. When bars are used, comparisons are made using a continuous scale. Especially when only 3 or fewer elements are to be compared, graphics are recommendable.

3 An example

The forementioned way to formulate goal programming problems was tested in the forest of a non-industrial private landowner. The forest covered 31.4 hectares in Eastern Finland. Scots pine was the dominant tree species, but Norway spruce and birch were quite common, too. The sites were of medium fertility.

The area has been divided into 26 compartments. The age class distribution of the forests was distinctly two-peaked: 16 hectares were covered by well-established plantations, with stand age less than 40 years. Another 15 hectares belonged to the age class 60–80 years.

The owner of the case study area is not financially dependent on timber sales. He aims at continuous and high income. However, his most important objective is to maintain or improve the productivity of the forest.

The method employed to prepare management plans may be divided into two steps (Pukkala 1988a, 1988b). First, one or more treatment schedules are simulated for each compartment over a 20-year planning period. Second, an optimal combination of treatment schedules is selected using mathematical programming.

The 20-year planning period consists of two 10-year management periods. Each treatment schedule is described by state and change variables. State variables include stand volume, volume increment, stumpage value, value increment, and volumes of different tree species and timber assortments after 10 and 20 years. Change variables describe the removal, income, and costs during the first and second 10-year period. All state and change variables can be used as goal or constraining variables in the optimization.

Altogether 68 treatment schedules were simulated for the 26 compartments of the case study area. Three different management plans, Plan 1, Plan 2, and Plan 3, were composed from the treatment schedules. There were four simultaneous goals in each plan: total volume after 20 years, annual volume increment after 20 years, net income during the first 10-year period, and net income during the second 10-year period (Table 1). All goals were one-sided, criteria relating only to the attaining of the specified target.

Table 1. Goals and their range of variation.

Goal	Minimum	Maximum	Unit
Total volume after 20 years	2542	8499	m^3
Annual volume increment after 20 years	139	251	m^3
Net income during the first 10-year period	0	288	1000 FIM
Net income during the second 10-year period	4.5	834	1000 FIM

Plans 1 and 2 were formulated according to Equations (5) to (6), using the highest possible amount of a goal as the aspiration level (Table 1). In Plan 3, it was assumed that a total volume of 4000 m^3 at the end of the 20-year planning period would be enough to guarantee high production in the future.

In Plan 1, the importances of different goals were considered as equal. In Plans 2 and 3, importances were worked out using pairwise comparisons and the AHP method (Table 2). The significance of 'Final volume after 20 years' was slightly different in Plans 2 and 3. In Plan 2, this criterion referred to the overall importance of the final volume. In Plan 3, it described the importance of having a sufficient standing volume after 20 years.

Problem formulations for Plans 1 and 2 yielded a high final volume and very little income during the second 10-year period (Table 3). In

Plan 3, final volume equalled the specified target level while the incomes were increasing, corresponding to the cutting possibilities in the area. Plan 3 was clearly the best one, and close to a plan that the forest owner is following.

Table 2. Importance of goals in different plans. The sum of the importance is one for each plan.

Goal	Plan 1	Plan 2	Plan 3
Total volume after 20 years	0.2500	0.3515	0.3974
Annual volume increment after 20 years	0.2500	0.2916	0.2548
Net income during the first 10-year period	0.2500	0.1838	0.1759
Net income during the second 10-year period	0.2500	0.1732	0.1719

Table 3. Information on alternative plans.

Variable	Plan 1	Plan 2	Plan 3	Unit
Total volume				
Present	4202	4202	4202	m³
After 10 years	4430	4566	4430	m³
After 20 years	5860	6523	4000	m³
Volume increment				
Present	212	212	212	m³
After 10 years	202	206	202	m³
After 20 years	235	245	203	m³
Net income				
1st 10 years	266	252	266	1000 FIM
2nd 10 years	90	9	355	1000 FIM
Removal				
1st 10 years	1753	1636	753	m³
2nd 10 years	619	60	2301	m³

4 Discussion and conclusion

When the decision theoretic approach is used, the basic nature of each objective has to be clarified before formulating the GP problem: are the goals to be maximized, minimized, or is there a target level which is not the greatest or the smallest possible amount. For the objectives, which are maximized or minimized, aspiration levels can easily be determined as maximum or minimum attainable amounts, respectively. In most cases, the sub-utilities, with respect to the objectives, are greatest at the maximum attainable levels. Determination of aspiration levels is difficult for objectives which are neither minimized nor maximized. Furthermore, in such a case, the weights of over- and underachievement deviations may be different.

The case study example revealed that the GP-formulation, as presented in this paper, is not always usable without additions or modifications. The main reason for this is that the solution is extremely sensitive to the weights given to the goal variables. This is particularly true if the scaled goal variables are nearly linearly related to each other with the slope of –1. Taking this into account, some of the goals may need a target level different from the single-objective minimum or maximum. Another way to overcome some of these problems is to add ordinary LP-constraints to the GP-model to guarantee a specified limit of a goal variable.

Weights determined by the AHP procedure can be interpreted as relative importances of management goals. However, dealing with target levels expressed in quantitative units may cause difficulties concerning the cardinal interpretation of the weights, and concerning tradeoff analyses. The lower the aspiration level of an objective, the greater the weight of a deviation of one measurement unit of that objective.

Because of the difficulty of dealing with qualitative objectives, goal programming, as well as all the other mathematical programming methods, has only limited applications in forest planning. However, qualitative criteria can, sometimes, be quantified using artificial measures (e.g. Korhonen 1987, Pukkala 1988b, Kangas 1992a). Besides mathematical programming, evaluation methods appropriate in dealing with qualitative criteria are also often needed in forest planning. In addition, utility is not usually linearly related to quantitative measures, such as yields of timber and berries, populations of wildlife, or even money units, as is expected when goal programming is used.

We cannot expect the goal programming approach, nor any other mathematical programming method, to attain the real optimum solution to a problem in forest management planning – especially at the first attempt. However, goal programming has proved to be an efficient method in generating satisfactory solutions (e.g. Mendoza and Sprouse 1989), as well as in learning about the decision situation. Information on single objective optima and results of different weighting schemata provide a basis for studying the decision problem.

Goal programming is frequently used in iterative and interactive optimization approaches (e.g. Nijkamp and Spronk 1980, Masud and Hwang 1981, Korhonen and Laakso 1986b, Korhonen and Wallenius 1990). The decision theoretic problem formulation, as presented in this paper, can be used in the first step of an iterative optimization. By using this formulation, we can expect a solution nearer to the real optimum solution than can be attained using conventional GP formulations, and the iterative optimization process can be shortened. In addition, using this formulation, we can produce solutions approaching near the real optimum solution, and these can be further evaluated by the AHP, for example.

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