

Modeling Forest Growth with Management Data: A Matrix Approach for the Italian Alps

Paola Virgilietti and Joseph Buongiorno

Virgilietti, P. & Buongiorno, J. 1997. Modeling forest growth with management data: A matrix approach for the Italian Alps. *Silva Fennica* 31(1): 27–42.

This paper reports on the possibility and difficulties in building growth models from past Forest Administration records on cut and growth in the Italian Alps. As a case study, a matrix model was calibrated for uneven-aged forests in the Valsugana valley of the Trentino province. The model gave reliable predictions over 30 years, and plausible long-term forest dynamics, including steady-states that are similar to virgin forests. The results support the view that the current forests are deeply altered as to composition, relative to what would obtain from natural growth. They also support the concept of long cyclic changes in natural stands, gradually approaching a climax state. Shortcomings of the data are that they do not come from an experimental design, they are not always accurate, and they must be supplemented with other information, especially concerning mortality. Still, these cheap and available data can lead to workable models adapted to local conditions, with many management applications.

Keywords growth models, matrix models, forest dynamics, Italian Alps

Authors' address *Virgilietti* Università degli Studi di Padova, Dipartimento Territorio e Sistemi Agroforestali, Padua, Italy. *Buongiorno* University of Wisconsin-Madison, Department of Forestry, 120 Russell Laboratories, 1630 Linden Drive, Madison WI 53706-1598, USA **Fax** (Buongiorno) +1 608 262 9922 **E-mail** jbuongio@facstaff.wisc.edu

Accepted 7 January 1997

1 Introduction

Uneven-aged forests constitute about 25 % of the Italian forests (excluding coppices), covering more than 554 400 ha, and irregular forests cover another 391 900 ha, altogether 41 % of the high forests (ISAFMA-MAF 1988).

Although forms of uneven-aged management were used in some areas of the Alps for centuries, foresters began to be really interested in them after the Second World War, following the spread of the nature oriented silviculture. The view is that there is a better chance to guarantee the sustainability of cultivated forests, with less expenditures in energy and money, if they are

managed to get structures and functions similar to those of natural forests (Bernetti 1977, Susmel 1980, Ciancio and Nocentini 1994).

In temperate oceanic and suboceanic climates, where beech, fir and spruce find optimal living conditions, the multispecific, uneven-aged formations represent the final stage (climax) of a long evolution. In the subcontinental climates, for the forests of spruce, fir and larch, the final structures are uneven or irregular, while in continental climates, even-aged forests dominate (Susmel 1980). Therefore, in the Alps, uneven-aged forests are, in most cases, the natural formation toward which, according to the principle of nature oriented silviculture, the management of forests should tend.

One of the first regions where these ideas were applied in Italy was the Trentino Province, where from the fifties, the *méthode du contrôle*¹ was used widely. The method requires precise and frequent inventories, and accurate data of harvests in the interval between two inventories (ISEA 1986).

The results of the application of this method for about 40 years in Trentino are altogether positive, with a growth in wood biomass, an increase in forest biodiversity, a decrease in the need for artificial regeneration and a reduction of pathologic attacks. The problems are the high management costs, due particularly to the necessity of having frequent complete inventories (Wolynski 1993).

In this context it seems worthwhile to research methods of predicting future stand states and the effects of management, with minimum data and expenses. Growth models could serve for that purpose; and matrix models in particular, because of their relative simplicity and the possibility to build them with inventory and harvest data that are already available.

The purpose of this paper is to report on the possibilities and difficulties in building such a matrix growth model with the data collected in the past by the Forest Administration for management purposes. In the specific forests of Valsugana, as in all the Italian Alps, there are no permanent uneven-aged plots that can serve for this.

The first part reviews briefly model taxonomy, with particular attention to matrix models, and it describes the specific model structure chosen here, followed by a comparison with the cultural model applied currently. The second part presents the calibration of the matrix model with data from forests of the Valsugana valley in Trentino province, and describes the difficulties caused by some data, and their solution. Part three reports the results of model validation for 30 years forecasts, and the long term stand dynamics predicted by the model. The steady state implied by the matrix model is also computed, and compared to the tree distribution in virgin forests, and that assumed by the current cultural model.

2 Choosing a Model Structure

As used here, the term "growth model" refers to a set of mathematical equations, to predict the growth of a tree, stand, or forest. Growth models require empirical data to be realistic. After calibration for a specific area, they can be of great help in forest management (Vanclay 1995).

There are three general types of growth models, with a number of intermediate forms. "Whole stand models" group all the trees in a single class, with little detail on its composition. They may be equations that predict future stand basal area as a function of current stand basal area. Or, consist of equations to forecast the parameters of a tree distribution function. They are useful with simple stand structures, with few species, and they are most suitable for even-aged stands.

Instead, in "stage class" models, trees are divided in several classes of species and size, and the future state is predicted for each class. There are explicit equations for the growth of trees between size class and for regeneration. They are more flexible than whole stand models and fit even, uneven, mono, or multi specific formations. They have enough detail for many applications, remaining relatively simple and computationally efficient.

Last, "single tree models" describe a forest by a list of trees, each one defined by several parameters (species, diameter, height, location and so on). These models are very detailed, but they

¹ Gurnauds *méthode du contrôle*, as applied in the Trentino requires that the harvest be decided stand by stand, according to forest conditions which must be closely monitored (Gurnaude 1886, Bernetti 1977).

are costly to develop and present problems of tractability, given the numerous equations and their complexity.

There is an obvious continuum among model types: increasing the number of classes in a stand class model to one class per tree gets a single tree model, and vice versa. A specific stage class model, with a matrix structure, was selected here, because it strikes a balance between the detail required to identify meaningful management alternatives and the tractability needed to develop and apply the model (Vanclay 1995, Getz and Haight 1989).

2.1 Matrix Models

A general form of a linear matrix model is:

$$\mathbf{y}_{t+1} = \mathbf{G} (\mathbf{y}_t - \mathbf{h}_t) + \mathbf{c} \quad (1)$$

where \mathbf{y}_t is a vector of the number of trees per unit area in a stand at time t , \mathbf{h}_t is the number of trees harvested at time t , \mathbf{G} is a matrix of coefficients and \mathbf{c} is a vector of constant parameters. These coefficient are derived from two sets of data: the fraction of trees that move from one diameter class to another and an equation that relates ingrowth to the stand state (Buongiorno and Gilles 1987, p. 101).

The first matrix models were used by Leslie and Lewis to investigate the effect of age structure on the growth of animal populations (Lewis 1942, Leslie 1945 and 1948). Usher (1966 and 1969) adapted Leslie's model to forest stands. In these first models, recruitment was directly proportional to the number of trees after harvest. To find the maximum sustainable yield, a growing stock constraint was needed to bound the problem. Buongiorno and Michie (1980) avoided this by expressing ingrowth as a linear function of the number of trees and total basal area, relying on Ek's (1974) observations. This linear matrix model structure can readily accommodate stands of many species (e.g. Lu and Buongiorno 1993).

Like ingrowth, transition probabilities may also depend on stand state, and a common criticism of linear matrix models is that they assume a stationary upgrowth matrix (Harrison and Michie 1985). To avoid this, Solomon et al. (1986), and

Mengel and Roise (1990) made the matrix \mathbf{G} a function of stand state. Buongiorno et al. (1995) verified, for uneven-aged forests in the French Jura, the existence of an inverse relationship between transition probabilities and basal area. With these relations, the model is no longer linear, although it is linear for a given basal area. Non linearities complicate considerably the study of management alternatives.

Growth models have many uses. A direct application, and a check of the realism of the model, is to predict stand dynamics and find the steady state in natural conditions, that is the climax, if it exists. For linear matrix models, the steady state can be computed directly from the equations of the model, and it is independent of initial stand conditions (Buongiorno and Michie 1980).

Moreover, one can study the consequences of different cutting regimes, and thus find those that best meet the management objectives. A fundamental goal in forest management is to produce a constant periodic harvest without depleting the growing stock (sustained yield conditions). But other objectives are generally present. Though early applications dealt mostly with economic or productivity objectives, more recently the models have also been applied to optimize ecological objectives such as diversity (Gove et al. 1994, Buongiorno et al. 1994, 1995). This trend reflects an increasing environmental concern in forest management, which is bound to continue in the future.

2.2 Matrix Model Structure

The growth model adopted for the Valsugana forests is analog to that of Buongiorno et al. (1995) for the similar forest in the French Jura. But, the final model is linear. As explained below, though non linearities were tested, their effects were found to be too small to matter in practical applications. The model structure is as in Equation (1), where the stand state at time t is represented by the vector:

$$\mathbf{y}_t = [y_{ijt}]$$

where y_{ijt} is the number of trees per hectare that are alive of species i ($=1, 2, \dots, m$) and size j ($=1,$

Table 1. Summary of stand statistics.

		Spruce	Fir	Larch	Beech
Trees/ha	Mean	147.4	78.3	24.8	19.9
	SD	58.1	69.6	35.4	19.7
Basal area (m ² /ha)	Mean	20.4	8.5	2.6	1.0
	SD	7.1	7.5	3.2	1.0
Volume (m ³ /ha)	Mean	209.0	105.4	32.1	7.9
	SD	92.08	93.3	38.5	8.5
Growth(m ³ /ha/yr)	Mean	4.7	1.5	0.6	0.16
	SD	3.9	2.1	0.7	0.43
		N	84	77	47

Note: SD = standard deviation
N = number of observations in 30 compartments measured at various times from 1960 to 1990.

culated². Then, comparing the current distribution with the normal, the harvest for each diameter class is decided; in any case this amount should not be more than 25 % of the volume of the stand. The fundamental difference between Susmel's cultural model and the model sought here is that the former is normative. It defines the structure that the manager should work for: it tells what the forest ought to be. The latter instead is positive. It is meant to predict how a stand will grow, given its current structure, and what will result from different managements: it tells what the forest will be, not what it should be.

3 Estimation of the Matrix Growth Model

3.1 The Data

The matrix model was calibrated with data from 30 compartments located in three forests of the central Alps, in the Valsugana valley of the Trentino Province, Italy. This region has a very long

forestry tradition, and harvest and stand data have been collected regularly for management purposes, though not for research, for more than thirty years. The specific compartments were in four communes: Telve (5 compartments), Telve di Sopra (4 compartments), Castel Tesino (9 compartments), and Cinte Tesino (12 compartments), all near the town of Borgo Valsugana. The compartments were selected with the help of Forest Administration technicians and were meant to be representative of the forests in the four communes. In total, they covered 541.95 ha, or 8.5 % of the total forest area in the four communes. They averaged 18 hectares, were located at altitudes from 950 to 1700 m (average 1320 m), and most had steep slopes. They all had an uneven-aged or irregular and multistratified structure³ on more than half of their area. During the past 30 years the compartments were managed with selective cuttings.

The stands consisted mostly of spruce (*Picea abies* L.), fir (*Abies alba* Miller), larch (*Larix europea* L.) and beech (*Fagus sylvatica* L.). The data, summarized in Table 1, came from periodic inventories and records of harvested and salvaged trees. In all the compartments the first inventory was done between 1960 and 1962 and the last between 1990 and 1992. Twelve compartments had four inventories and eighteen had three, leading to 72 pairs of successive observations, or sequences. Three sequences were dropped due to changes in the compartment boundary, leaving 69 sequences.

According to the management rules, dead trees should be recorded and salvaged. However, the records for the salvaged trees clearly did not reflect all the mortality. For example, in the oldest records there was often no salvage for a decade. One reason could be that the steep slope of many compartments made salvaging operations difficult and costly. For beech, salvaged trees were very seldom recorded and inventories and harvest records were also less accurate, probably because beech had the least commercial value.

² The normal distribution is determined by first choosing q and D_{max} . Then, a unit distribution is computed, with one tree in the largest diameter class and the number of trees in each successive smaller class increasing at the geometric rate q . The normal distribution is obtained by multiplying the number of trees in each diameter class of the unit distribution by B/B_u , where B is the normal basal area (obtained from the dominant height H), and B_u is the basal area of the unit distribution.

³ In the management plans a forest is classified as uneven-aged only when trees are distributed according to an inverse J curve. Irregular and multistratified forests present some irregularities in the diameter distribution, but are still considered uneven-aged forests, since there are trees of all ages in a small area.

The parameters of the model were calibrated for a time interval (t to $t+1$) of 10 years, the usual cutting cycle in these forests. Ingrowth was defined as the number of trees that passed the 17.5 cm diameter threshold and survived in 10 years, because smaller trees were not recorded. To minimize the probability that a tree would grow more than one class in one interval, diameter classes of 10 cm were used. In fact, the oldest records recognized three broad classes only, and trees had to be distributed by 10 cm classes in proportion with earlier observations. Then, for each species and sequence ingrowth and transition probabilities were calculated, and standardized for 10 years by linear interpolation. The sequences for which either the ingrowth or the transition probabilities were negative were dropped. This occurred the most for larch and beech, the species with fewer trees. In all, 81 % of the sequences gave usable data for spruce, 74 % for fir, 39 % for larch, and 44 % for beech.

3.2 Ingrowth Equations

The ingrowth equations predict the number of trees of each species that enter the smallest size class and that are still alive after 10 years. The parameters, estimated by ordinary least squares, are in Table 2. The underlying hypothesis was that ingrowth of a species would be negatively affected by stand basal area and positively by the number of trees of that species (Buongiorno et al. 1995). The estimated coefficients had the expected signs. For all the species, the positive effect of the number of trees of the same species was significant at least at the 5 % significance level. Instead, the negative effect of basal area was not significant for fir and beech, and significant at 10 % level only for larch. One possible explanation is that beech and fir are shade tolerant species, so that their ingrowth was not influenced by stand density, at least in the range of basal areas within the data.

The coefficient of determination, R^2 , was between 0.31 and 0.44 for spruce, fir and larch, while it was negligible for beech (0.09). Therefore, for beech, little would be lost if the ingrowth were assumed constant, independent of stand density and number of trees. Experiments

Table 2. Equations of ingrowth, by species.

	Independent variable Species (tree/ha)	Basal area (m ² /ha)	Constant
<i>Spruce</i>			
Coefficient	0.15	-1.47	35.27
SE	0.047	0.395	11.273
t	3.183**	-3.727**	3.128**
R ²	0.305		
N	54		
<i>Fir</i>			
Coefficient	0.1226	-0.397	10.747
SE	0.02	0.271	6.172
t	6.033**	-1.463	1.741
R ²	0.414		
N	50		
<i>Larch</i>			
Coefficient	0.076	-0.22	6.069
SE	0.018	0.122	2.861
t	4.145**	-1.794	2.121*
R ²	0.439		
N	26		
<i>Beech</i>			
Coefficient	0.3159	-0.318	16.133
SE	0.1483	0.476	11.43
t	2.13*	-0.668	1.411
R ²	0.094		
N	28		

Note: SE = standard error.
t = Students statistic, * and ** = significant at the 5 % and 1 % level.
R² = coefficient of determination.
N = number of observations.

with different data sets showed that the regression for this species was not robust. Therefore, the final model did assume that ingrowth of beech was constant and equal to the average recorded ingrowth per decade. There are reasons for beech to differ from the other species, since it is present mostly as coppice.

3.3 Upgrowth Equations

The upgrowth hypothesis was that the probability of transition (trees moving from one size to the next) was affected by stand density and tree size. The regression results in Table 3 did find a negative effect of basal area on the upgrowth rate of spruce and fir, and a significant negative

Table 3. Equations of the probability of transition in 10 year.

	Independent variable		Constant
	Basal area (m ² /ha)	Diameter (cm)	
<i>Spruce</i>			
Coefficient	-0.0033	0.0007	0.411
SE	0.0014	0.0005	0.039
t	-2.342*	1.324	10.48**
R ²	0.018		
N	269		
<i>Fir</i>			
Coefficient	-0.0075	0.00004	0.505
SE	0.0023	0.00076	0.063
t	-3.252**	-0.05	8.01**
R ²	0.036		
N	229		
<i>Larch</i>			
Coefficient	0.0013	-0.0054	0.514
SE	0.0035	0.00117	0.093
t	0.37	-4.615*	5.489*
R ²	0.134		
N	126		
<i>Beech</i>			
Coefficient	-0.005	0.0056	0.246
SE	0.007	0.0034	0.195
t	-0.69	1.626	1.261
R ²	0.044		
N	66		

Note: SE = standard error.
 t = Students statistic, * and ** = significant at the 5 % and 1 % level.
 R² = coefficient of determination.
 N = number of observations.

relation between transition probabilities and the size of larches, suggesting a slower growth rate for bigger trees. However, the very low R² demonstrate that stand basal area and tree size explain very little of the variation in the rate of upgrowth. So the transition probabilities could be assumed constant without much loss in predictive accuracy. In the final model, the transition probabilities were set at their mean for spruce, fir and beech, and at their mean for a given diameter for larch.

3.4 Mortality

The average recorded salvage rate was 0.4 % per year for spruce and fir, and 0.2 % per year for larch. The data for beech did not allow to calculate mortality, because salvaged trees were recorded only twice. The recorded salvage rates were considerably less than the true mortality for this type of forest, estimated to be between 0.8 % and 1.2 % per year (Volin and Buongiorno 1996). Therefore, an overall mortality rate of 1 % per year was assumed here, for spruce, fir and larch. In the Alps windfalls are frequent. Even though the recorded data of mortality could not serve to establish an accurate equation, the available records showed that mortality was more frequent in the largest diameter classes. Moreover, for similar forests in the French Jura, where snow breaks and windfalls are also an important cause of mortality, there is evidence of a positive correlation between mortality and tree size (Buongiorno et al. 1995). Thus, in the final model, mortality was assumed to increase with tree diameter at the same rate as in the forests of the French Jura. The Jura study did not find any effect of basal area on mortality, and none was assumed here. For larch, absent in the Jura study, the equation for spruce was used, as it was deemed to be the closest species, ecologically.

Finally in a coppice mortality is higher, due to the intense competition in sprout clumps (Hawley and Smith 1954, p. 194). Thus, an average mortality rate of 2.8 % per year was assumed for beech, varying with size at the rate shown in the equations of Buongiorno et al. (1995). The complete final model parameters are shown in Table 4.

4 Model Validation and Application

4.1 Short-term Validation

A formal validation of the model would require prediction of the growth of an entirely different set of compartments. However, this was not possible here. All the available data were needed to develop the model. So, as in much practical modeling, there is not a sharp distinction between

model construction and testing.

Still, it is worth examining how the final model, a composite of several local regressions (for ingrowth and upgrowth) and extraneous estimates (for mortality) predicts the growth of stands in the forests where the model is meant to be applied. To that end, the parameters in Table 4 were used to predict the evolution of the compartments with a full series of observations over thirty years. Thus, the simulated compartments were a subset of the compartments used to calibrate the model, but still representative of the forests of interest. Equation (1) was applied recursively three times to predict the stand state at the last inventory (1990/92), given the state at the first inventory (1960/62), and given the intermediate harvests.

Fig. 1 shows the average stand state predicted by the model after thirty years, and the distribution of the observed stands. In general the predicted average number of trees of each species and diameter was within the 95 % confidence interval of the average observed number of trees. Only for spruce was the predicted number of trees significantly smaller than the observed, in the smallest size class. Thus, the model seems able to make acceptable thirty-year forecasts of forest growth, but it is more accurate in total than for each separate species.

4.2 Long-term Stand Dynamics

The model was also used to predict how a minimally disturbed forest would evolve, over a very long time period. The parameters in Table 4 have been computed with data from managed forests observed over thirty years only. Moreover, since these forests have always been managed in the past, they are far from natural forests in structure and composition. Thus, the most appropriate application of the model should be to analyze only slightly different management regimes over a relatively short time.

Nevertheless, it is useful to see if the implications of the model under the extreme conditions of no harvest for a very long time correspond, at least in a general way, to existing knowledge of forest succession. Checking the model behavior in such extreme conditions is a good test of

model robustness, since stands characteristics such as basal area, number of trees and species composition should remain within limits consistent with prior biological knowledge. Thus, equation (1) was applied recursively with no harvest ($h_t = 0$ for all t), for as long as 500 years.

The initial stand state for the simulation was the average of the stand states observed between 1960 and 1980 (Year = 0 in Fig. 2). The model predicted that if the harvests stopped, the number of trees in the intermediate classes would increase, over the years, and the number of largest trees would rise as well. The increased density would cause a decrease in regeneration, and therefore in the number of trees in the smallest diameter classes. So, between 50 and 100 years the forest would assume a structure similar to that of an even-aged forest, with a high basal area. Around the year 200 the stand would consist mostly of a few very large spruces and firs and many small beech. After that, a sufficient number of trees would die to create openings large enough for new regeneration to take place, and in about 300 years the forest would assume again a typical uneven-aged distribution, but more of the largest trees than in the current forest. This kind of distribution is similar to that of virgin mixed forests of fir, beech and spruce (Susmel 1980, p. 68).

Basal area would peak around year 75, when the medium and large size trees dominate, at about 45 m²/ha, and it would bottom around year 150, at some 20 m²/ha (Fig. 3). Basal area would stabilize in the long run at about 25 m²/ha. These ranges of basal areas are possible for these kind of forests, even though basal areas of 45 m²/ha are infrequent. Susmel (1951) found mixed forests of spruce and fir in another area of the Alps with basal areas ranging from 10 to 50 m²/ha. In the compartments used to calibrate the model, the lowest basal area was 8.5 m²/ha and the highest was 37 m²/ha.

According to the simulations, the number of trees would increase during the first two decades, to a maximum of 290 trees/ha, then decrease gradually until the year 200 to a minimum of 125 trees/ha, increasing then again (Fig. 3). In the long run, the number of trees would converge towards about 170 trees/ha. The number of trees predicted is lower than the numbers usually recorded for these forests, though it is possible. In

Table 4. Matrix **G** and vector **c** for a time interval of 10 years.

Species	diam.	Spruce					Fir					c		
		22.5	32.5	42.5	52.5	62.5	>67.5	22.5	32.5	42.5	52.5		62.5	>67.5
Spruce	22.5	0.68	0.03	-0.06	-0.17	-0.30	-0.50	-0.06	-0.12	-0.21	-0.32	-0.45	-0.65	
	32.5	0.35	0.57	0	0	0	0	0	0	0	0	0	0	
	42.5	0	0.35	0.56	0	0	0	0	0	0	0	0	0	
	52.5	0	0	0.35	0.54	0	0	0	0	0	0	0	0	
	62.5	0	0	0	0.35	0.53	0	0	0	0	0	0	0	
	>67.5	0	0	0	0	0.35	0.86	0	0	0	0	0	0	
Fir	22.5	-0.02	-0.03	-0.06	-0.09	-0.12	-0.18	0.74	0.09	0.07	0.04	0.00	-0.05	
	32.5	0	0	0	0	0	0	0.31	0.62	0	0	0	0	
	42.5	0	0	0	0	0	0	0	0.31	0.6	0	0	0	
	52.5	0	0	0	0	0	0	0	0	0.31	0.58	0	0	
	62.5	0	0	0	0	0	0	0	0	0	0.31	0.56	0	
	>67.5	0	0	0	0	0	0	0	0	0	0	0.31	0.85	
Larch	22.5	-0.01	-0.02	-0.03	-0.05	-0.07	-0.10	-0.01	-0.02	-0.03	-0.05	-0.07	-0.10	
	32.5	0	0	0	0	0	0	0	0	0	0	0	0	
	42.5	0	0	0	0	0	0	0	0	0	0	0	0	
	52.5	0	0	0	0	0	0	0	0	0	0	0	0	
	62.5	0	0	0	0	0	0	0	0	0	0	0	0	
	>67.5	0	0	0	0	0	0	0	0	0	0	0	0	
Beech	22.5	0	0	0	0	0	0	0	0	0	0	0	0	
	32.5	0	0	0	0	0	0	0	0	0	0	0	0	
	42.5	0	0	0	0	0	0	0	0	0	0	0	0	
	52.5	0	0	0	0	0	0	0	0	0	0	0	0	
	62.5	0	0	0	0	0	0	0	0	0	0	0	0	
	>67.5	0	0	0	0	0	0	0	0	0	0	0	0	
Species	diam.	Larch					Beech					c		
	22.5	32.5	42.5	52.5	62.5	>67.5	22.5	32.5	42.5	52.5	62.5		>67.5	
Spruce	22.5	-0.06	-0.12	-0.21	-0.32	-0.45	-0.65	-0.06	-0.12	-0.21	-0.32	-0.45	-0.65	35.27
	32.5	0	0	0	0	0	0	0	0	0	0	0	0	
	42.5	0	0	0	0	0	0	0	0	0	0	0	0	
	52.5	0	0	0	0	0	0	0	0	0	0	0	0	
	62.5	0	0	0	0	0	0	0	0	0	0	0	0	
	>67.5	0	0	0	0	0	0	0	0	0	0	0	0	
Fir	22.5	-0.02	-0.03	-0.06	-0.09	-0.12	-0.18	-0.02	-0.03	-0.06	-0.09	-0.12	-0.18	10.75
	32.5	0	0	0	0	0	0	0	0	0	0	0	0	
	42.5	0	0	0	0	0	0	0	0	0	0	0	0	
	52.5	0	0	0	0	0	0	0	0	0	0	0	0	
	62.5	0	0	0	0	0	0	0	0	0	0	0	0	
	>67.5	0	0	0	0	0	0	0	0	0	0	0	0	
Larch	22.5	0.61	0.06	0.04	0.03	0.01	-0.02	-0.01	-0.02	-0.03	-0.05	-0.07	-0.10	6.07
	32.5	0.4	0.55	0	0	0	0	0	0	0	0	0	0	
	42.5	0	0.37	0.58	0	0	0	0	0	0	0	0	0	
	52.5	0	0	0.33	0.61	0	0	0	0	0	0	0	0	
	62.5	0	0	0	0.28	0.62	0	0	0	0	0	0	0	
	>67.5	0	0	0	0	0.26	0.86	0	0	0	0	0	0	
Beech	22.5	0	0	0	0	0	0	0.57	0	0	0	0	0	12.83
	32.5	0	0	0	0	0	0	0.35	0.49	0	0	0	0	
	42.5	0	0	0	0	0	0	0	0.35	0.41	0	0	0	
	52.5	0	0	0	0	0	0	0	0	0.35	0.33	0	0	
	62.5	0	0	0	0	0	0	0	0	0	0.35	0.26	0	
	>67.5	0	0	0	0	0	0	0	0	0	0	0.35	0.51	

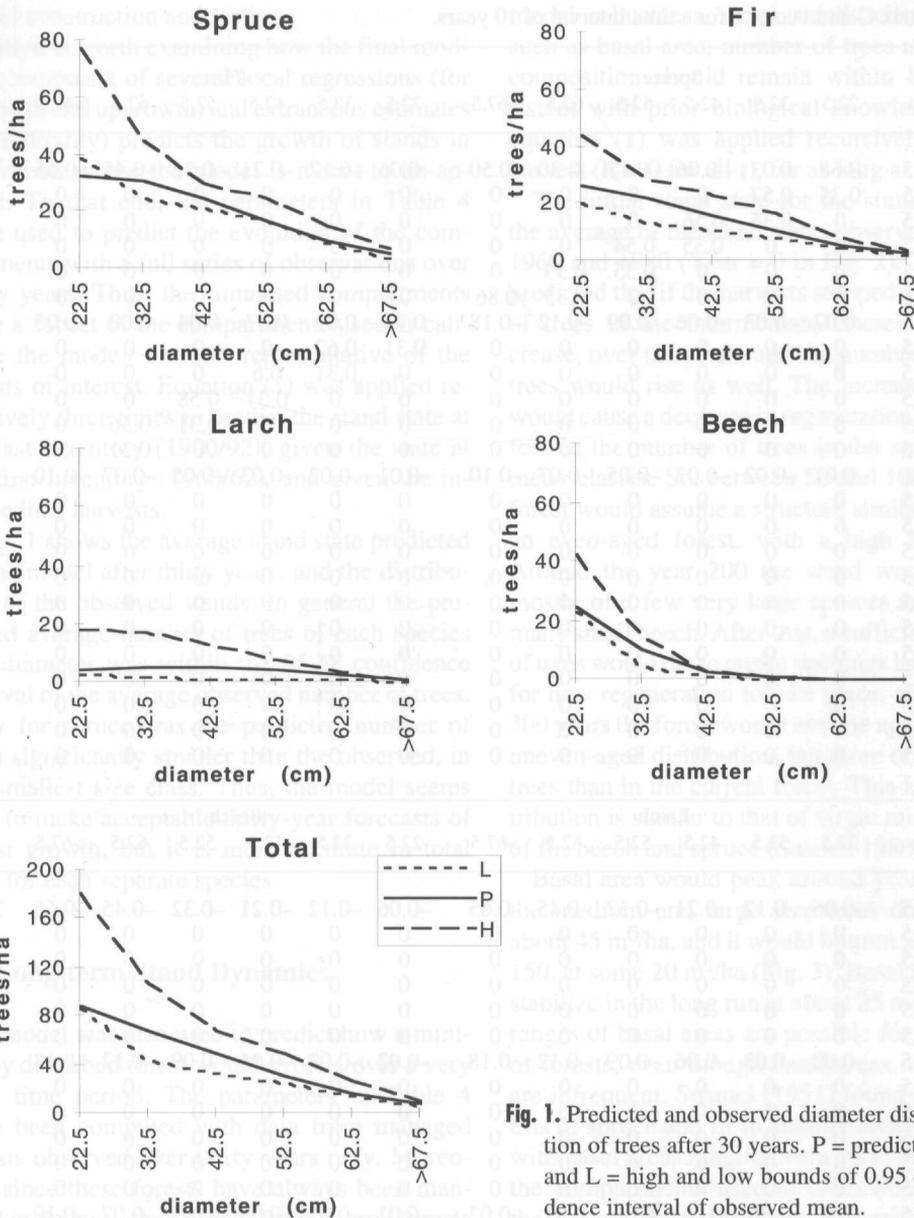


Fig. 1. Predicted and observed diameter distribution of trees after 30 years. P = predicted, H and L = high and low bounds of 0.95 confidence interval of observed mean.

the compartments that gave the data, the number of trees ranged between 115 trees/ha and 422 trees/ha. But, the simulations describe an unmanaged situation, where there are more trees in the largest diameter classes than in typical managed forests. As a result basal area remains always quite high, which tends to limit regeneration, and thus the number of trees in the stand.

This general pattern, of long, damped oscillations, both in basal area and number of trees, is similar to results of previous studies, for different forest types (Borman and Likens 1979, Buongiorno and Michie 1980), and for similar forests in other regions (Buongiorno et al. 1995).

Other data provided by this simulation concern the dynamics of the species composition in

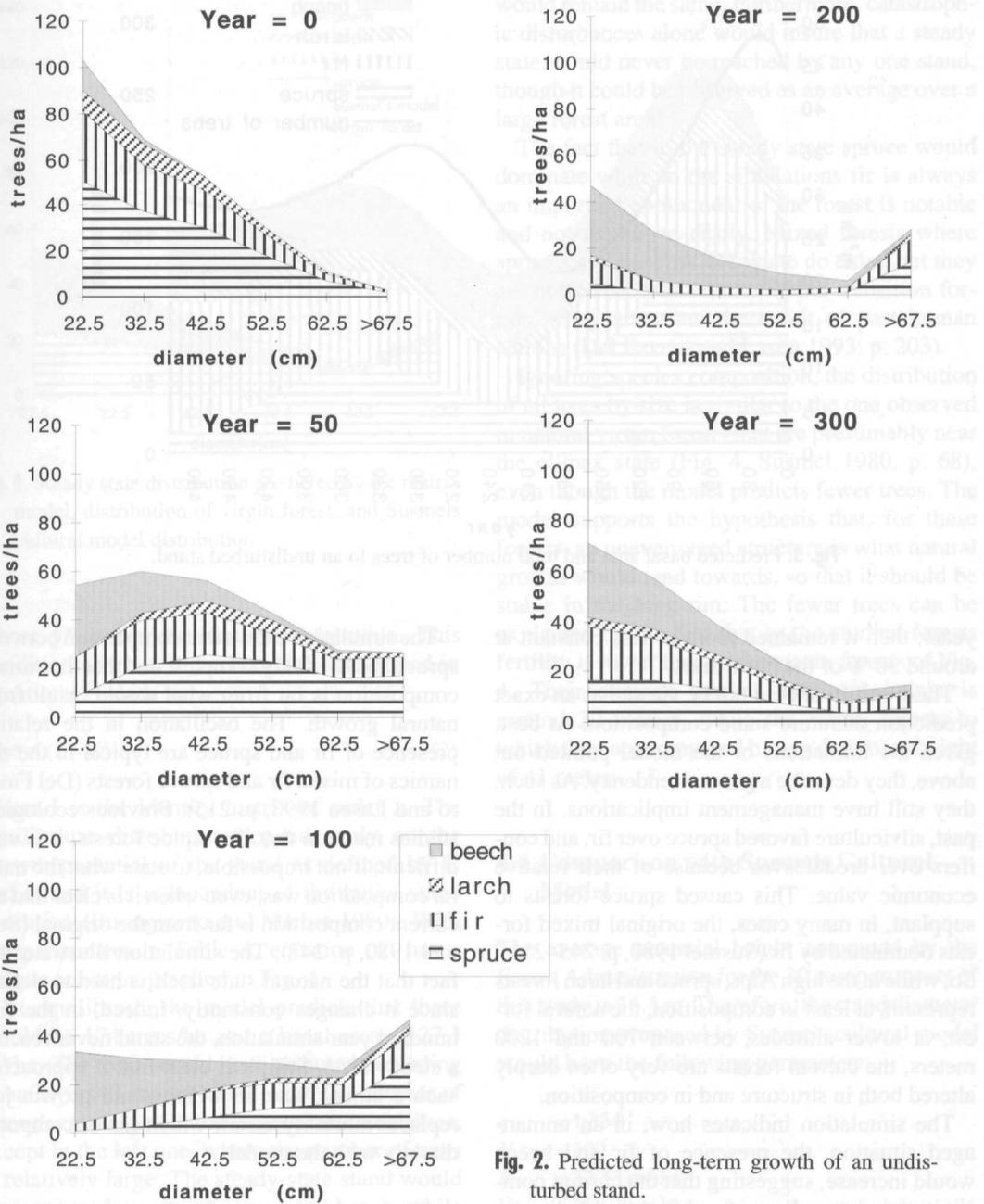


Fig. 2. Predicted long-term growth of an undisturbed stand.

a naturally growing stand (Fig. 3). Spruce which, at present, dominates the stands, both as number of trees and basal area, had large oscillations, with a decreasing presence over 500 years. The species never reached the current percentage in

500 years. Fir maintained its share of basal area, and even increased it to near 50 % by the year 500. Larch always stayed a secondary species, never more than 10 % of the trees. Beech increased very quickly in the first one hundred

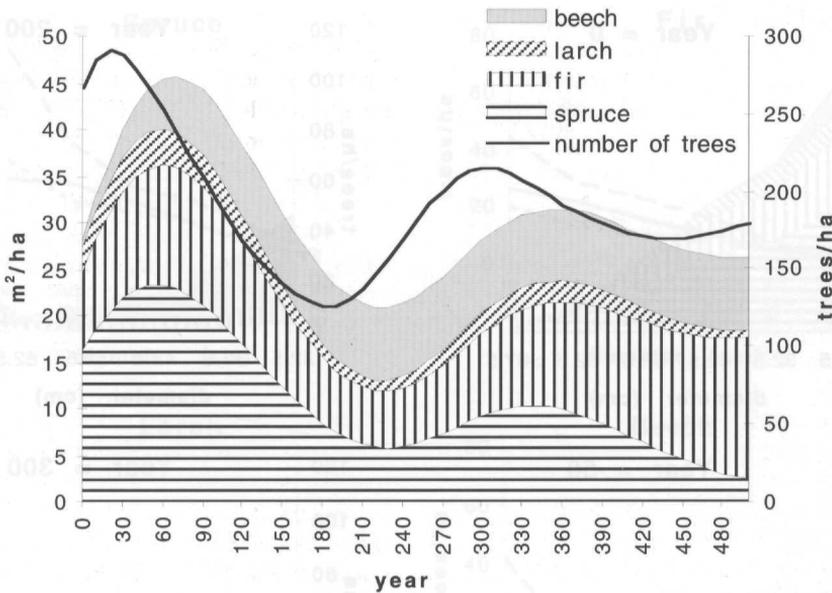


Fig. 3. Predicted basal area and total number of trees in an undisturbed stand.

years, then it remained more or less constant at around 30 % of the basal area.

These results should not be viewed as an exact prediction of future stand composition. At best, given the limitations of the model pointed out above, they describe a general tendency. As such, they still have management implications. In the past, silviculture favored spruce over fir, and conifers over broadleaves because of their relative economic value. This caused spruce forests to supplant, in many cases, the original mixed forests dominated by fir (Susmel 1980, p. 245–251). So, while in the high Alps, spruce and larch forests represent, at least in composition, the natural forest, at lower altitudes, between 700 and 1200 meters, the current forests are very often deeply altered both in structure and in composition.

The simulation indicates how, in an unmanaged situation, the presence of fir and beech would increase, suggesting that the current composition is not a climax for this site and climate, but mainly the result of human action. The evolution of beech is particularly difficult to project, since it is now present as coppice, but over a long period regeneration by seeds should start and substitute the coppice. Therefore, an increase in this species is probable, as predicted by the model.

The simulation shows that competition between spruce and fir is very complex, and that the current composition is far from what would result from natural growth. The oscillation in the relative presence of fir and spruce are typical in the dynamics of mixed fir and spruce forests (Del Favero and Lasen 1993, p. 215). Previous ecological studies mention that, for alpine forests, it is very difficult, if not impossible, to state what the natural composition was, even when it is clear that the current composition is far from the original (Susmel 1980, p. 247). The simulation illustrates the fact that the natural state itself is hard to define, since it changes constantly. Indeed, in the five hundred year simulation, the stand never reaches a steady state. Still, it is clear that it approaches such a climax state in which stand growth just replaces mortality, a state which can be computed directly with the model.

4.3 Steady State

Since the growth model is linear, its steady state, if it exists, can be computed exactly. At equilibrium, regardless of the value of t , one must have:

$$y_{t+1} = y_t = y^*$$

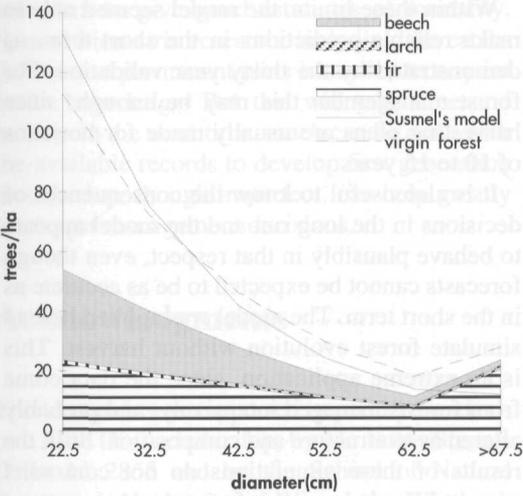


Fig. 4. Steady state distribution predicted by the matrix model, distribution of virgin forest, and Sussmel's cultural model distribution.

where y^* is the equilibrium distribution. This condition and $h_t = 0$ (no harvest), substituted in equation (1) leads to:

$$y^* = (I - G)^{-1} c \tag{2}$$

where I is the identity matrix of order n . The equilibrium distribution depends only on the growth potential of the stand as defined by G and c , and it is independent of the initial stand condition (Buongiorno and Michie 1980). With the parameters in Table 4, equation (2) has a unique solution, graphed in Fig. 4.

At equilibrium the model predicts that there would be 174 trees/ha and a basal area of 27.3 m²/ha. The trees would be distributed according to an inverse J shaped curve, with the number of trees decreasing with the larger diameter classes, except in the last one, where the number of trees is relatively large. The steady-state stand would be composed mostly of spruce and beech, while fir and larch would be secondary.

This state is mostly of theoretical interest. It supports the plausibility of the model, even at the limit. But, as noted above, even in the simulation of 500 years the forest never got to this steady state. Over such a long time it is unrealistic to assume that the forest growth parameters

would remain the same. Furthermore, catastrophic disturbances alone would insure that a steady state would never be reached by any one stand, though it could be observed as an average over a large forest area.

The fact that in the steady state spruce would dominate while in the simulations fir is always an important component of the forest is notable and not explained easily. Mixed forests where spruces and beeches dominate do exist, but they are not considered climax, rather transition forests. They are often the result of past human actions (Del Favero and Lasen 1993, p. 203).

Ignoring species composition, the distribution of all trees by size is similar to the one observed in natural virgin forests that are presumably near the climax state (Fig. 4, Sussmel 1980, p. 68), even though the model predicts fewer trees. The model supports the hypothesis that, for these forests, an uneven-aged structure is what natural growth would tend towards, so that it should be stable in the long run. The fewer trees can be explained by the fact that in the studied forests fertility is lower than in the virgin forests of Fig. 4. Their average current dominant height is around 30 meters, while the virgin forests to which they are compared have a dominant height of 41 meters.

4.4 Comparison with Sussmel's Cultural Model

The average potential height computed by the Forest Administration for the 30 compartments of this study is 33.4 m. Therefore, the stand diameter distribution proposed by Sussmel's cultural model would have the following parameters:

- $q = 1.334$
- $N = 300$
- $B = 32.43 \text{ m}^2/\text{ha}$
- $V = 368.79 \text{ m}^3/\text{ha}$
- $D_{max} = 88 \text{ cm}$

But, since the current average dominant height is 30 m, an intermediate model would be used for management, based on a height of 31.7 m. Fig. 4 shows the corresponding tree distribution, and how it compares with the steady state predicted

by the matrix growth model. The cultural model has more trees in all size classes, except the largest. The implication is that the steady state assumed by the cultural model may not be reachable given the growth potential of the forests under study. The potential is embedded in the parameters of the matrix model, calibrated with data of the forests under consideration.

Still, this does not mean that the cultural model, as a goal, is without interest, even if it could not be reached. Indeed, other research on forests of the Dolomites suggests that a policy that tries to approach the cultural model (which could not be reached there, either) was very attractive from the point of view of both ecological and economic criteria (Volin and Buongiorno 1996). But such an inference would not be possible without a growth model to predict the full dynamic consequences of policies based on the cultural model, or other silvicultural principles.

5 Summary and Conclusion

The matrix model applied in this paper appeared suitable for the Trentino situation. It can be calibrated with the data generally collected by the Forest Administration for normal management, and it offers the possibility of predicting forest growth with a degree of detail sufficient for management purposes.

A specific matrix model was built with the management data collected on 30 stands during the past 30 years from three uneven-aged forests located in the Valsugana valley of the Trentino. With these data, regression equations could be developed to predict ingrowth and upgrowth, but mortality rates had to be estimated from other forests.

The model describes average stand growth over the entire forests from which the data were originated. Thus, it cannot predict accurately the growth of each individual stand. Predictions that deviate considerably from the current stand structures should also be viewed cautiously, since they are the results of extrapolations beyond the range of observations. For the same reason, the model should not be applied to other forests without ascertaining that sites and ecological conditions are similar.

Within these limits, the model seemed able to make reliable predictions in the short term, as demonstrated by the thirty year validation. For forest management this may be enough, since harvesting plans are usually made for horizons of 10 to 15 years.

It is also useful to know the consequences of decisions in the long run and the model appears to behave plausibly in that respect, even though forecasts cannot be expected to be as accurate as in the short term. The model was applied here to simulate forest evolution without harvest. This is an extreme application, since the data come from forests managed intensively, and probably altered as to structure and composition. Still, the results of these simulations do not contradict present knowledge about forest dynamics.

One of the most difficult variable to predict was the species composition of the stand. The composition predicted by the model in the steady state was different from that observed in the 500 year simulation, when the steady-state composition was never reached. Nevertheless, the model predictions support the view that the current forests are deeply altered as to composition, relative to what would obtain from natural growth. The model also supports the concept of long cyclic changes in natural stands, gradually approaching a climax state.

Since the model seems able to make accurate short term forecasts and it gives reasonable results in long term simulations, it can be used to study the effects of management guidelines. Although the model simplifies a reality that is very complex and cannot pretend to represent the full richness of forest succession, it has sufficient detail to help understand the dynamics of natural and managed forests. Recognition of four species of trees should help also in economic assessments, an aspect of forestry that remains critical for Alpine communities (Merlo 1995).

In sum, it is possible to build practical matrix models with the management records available for the uneven-aged forests in the Italian Alps. This is a cheap source of data, available now, that can lead quickly to a workable model, adapted to a specific forest. However, these data have clear shortcomings: they do not come from a rigorous experiment design and they are not always accurate. In particular, the salvage data do

not seem to give good estimates of mortality. Formal biometric research, including the establishment of permanent plots is needed to acquire better knowledge. But this will take time and money. In the meantime, much can be done with the available records to develop first generation models that, though imperfect, can help greatly in current management decisions.

Acknowledgments

This research was supported in part by Fondazione Aldo Gini scholarship, by Mc Intire-Stennis Grant M 2855 and by the School of Natural Resources, University of Wisconsin. We thank Valeria C. Volin, Maurizio Merlo and Luca Cesaro for their collaboration, and Servizi Forestali of Trentino Province that helped to collect the data.

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