Comparing Basal Area Diameter Distributions Estimated by Tree Species and for the Entire Growing Stock in a Mixed Stand

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The purpose of this study was to compare the Weibull distributions estimated for the entire growing stock of a stand and separately for Scots pine (*Pinus sylvestris* L.) and Norway spruce (*Picea abies* Karst.) in describing the basal area diameter distributions in mixed stands. The material for this study was obtained by measuring 553 stands located in eastern Finland. The parameters of the Weibull distribution were estimated using the method of maximum likelihood. The models for these parameters were derived using regression analysis. Also, some parameter models from previous studies were compared with the measured distribution. The obtained distributions were compared using the diameter sums of the entire growing stock, diameter sums by tree species and of the sawtimber part of the growing stock. The results showed that far more accurate results were obtained when the distributions were formed using parameter models separately for the different tree species than when using parameter models for the entire growing stock. This was already true when considering the entire growing stock of the stand and especially when the results were examined by tree species. When the models for the entire growing stock were applied by tree species in relation to basal areas, the results obtained were overestimates for Norway spruce and underestimates for Scots pine. The models from earlier studies, where parameter models were estimated separately for tree species from the National Forest Inventory data, showed good fits also in regard to the data of this study.

Keywords parameter prediction, dbh distribution, Scots pine, Norway spruce
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1 Introduction

In forest inventories, where the empirical diameter distribution of a stand is not measured, diameter distribution models are used to obtain an estimate of tree size distribution. This predicted distribution is needed for the further computation of stand volume characteristics using treewise height and volume models (Bailey and Dell 1973, Päivinen 1980). For example, inventory by compartments in Finland is carried out without measured tally trees and the diameter distribution of stands is predicted using stand characteristics (Kilkki et al. 1989). In most cases, diameter distributions weighted by basal area are used to emphasize larger and more valuable trees (Päivinen 1980).

The most often used theoretical distribution in describing the diameter distribution has frequently been the Weibull distribution (e.g. Bailey and Dell 1973, Kilkki et al. 1989). Most of the studies considering various distribution functions have been concentrated only on pure stands (e.g. Bailey and Dell 1973, Hafley and Schreuder 1977, Rennolls et al. 1985, Magnussen 1986, Borders et al. 1987). The objective of these studies has been to estimate parameters using different methods, to compare different distributions, or to model parameters of the functions.

In mixed stands, diameter distributions can be applied in two ways. First, it is possible to estimate the distribution for the entire growing stock forming a certain stand. This describes the stand quite well if the entire growing stock forms a unimodal distribution. However, in multimodal cases, the distributions for the entire growing stock of the stand may be inadequate (Cao and Burkhardt 1984). The second possibility is to estimate the distributions separately for the different tree species and storeys. In the prediction phase, distributions are formed in the same way: in the case of the entire growing stock, only one distribution is predicted, and then total volumes for different tree species can be derived from the proportions of basal areas by tree species (Siipilehto 1988). In the second case, distributions are predicted separately for the different tree species using the mean characteristics, eg. mean diameter and basal area, of the tree species. The results for the entire growing stock can be obtained by adding the distributions of the different tree species.

Only in a few studies have mixed stands been considered (Little 1983, Tham 1988). In the study by Tham (1988), the structure of mixed stands composed of *Picea abies* Karst., *Betula pendula* Roth. and *Betula pubescens* Ehrh. was investigated using the Johnson S_B distribution. The Johnson S_B distribution was fitted to all three species separately and to the entire stand. In both cases the fits of Johnson S_B were quite good. In the study by Cao and Burkhardt (1984), a segmented distribution approach, i.e. distribution including different functions joined together, was used for irregular thinned stands and it was also proposed to be used for mixed stands.

In Finland, examples of the diameter distribution for the entire growing stock have been studied by Päivinen (1980), Siipilehto (1988), Hökkä et al. (1991) and Maltamo et al. (1995). Although Siipilehto (1988) considered stands as a whole, he also investigated the structure of mixed stands by means of the characteristics of the different tree species. Using the mean diameters of the different tree species, he found that if standwise information is used to estimate volumes separately for different tree species, then volume of Scots pine becomes underestimated and volume of Norway spruce overestimated. However, no test characteristics of the differences by tree species were presented. In the study by Maltamo et al. (1995), mixed stands were classified as pine and spruce stands according to the dominating tree species and the entire growing stock of the stands was considered. In parameter models, the proportions of the different tree species were used as independent variables.

Lönnroth (1925) was the first in Finland to estimate diameter distributions separately for different tree storeys within a stand. He used the normal distribution and the distribution for the entire growing stock was obtained by adding the distributions of the different tree storeys. In studies by Kilkki and Päivinen (1986), Mykkänen (1986) and Kilkki et al. (1989), models were estimated for the Weibull distribution parameters separately for Scots pine and Norway spruce using the data obtained from small relascope sample plots taken in the course of National Forest Inventory. In these studies, the distributions formed by tree species were not compared to the distributions of the entire growing stock.

The objective of this study is to compare the accuracy of the different models to estimate Weibull distribution parameters, especially those where the parameters are estimated separately by tree species and those where overall parameter models are used.

2 Material and Methods

A total of 553 stands were measured in 1991 in central and eastern Finland. Of these, 352 were dominated by Scots pine (*Pinus sylvestris* L.) and 183 by Norway Spruce (*Picea abies* Karst.); the remaining 18 were dominated by broadleaves, i.e. two birch species (*Betula pendula* Roth., *B. pubescens* Ehrh.) and such as alder (*Alnus incana* Moench, Willd.) and aspen (*Populus tremula* L.). All the stands were owned by a private forest enterprise and managed according to usual thinning regimes, such as thinning from below (Vuokila 1987). Six to thirteen relascope (angle-count) sample plots were systematically located in each stand. The diameter at breast height (dbh) in 1 cm classes and tree species were recorded from all the trees included in the relascope plot using a basal area factor of two.

The basal area diameter distributions for Scots pine and Norway spruce were formed by combining the trees tallied as belonging to the measured relascope sample plots within a stand. This was done in order to obtain a representative sample of the tree population in each stand. Also, the problem of small number of trees in maximum likelihood estimation of the basal area diameter distribution (Kilkki and Päivinen 1986, Kilkki et al. 1989) when using relascope sample plots was thereby avoided. A stand was included if there were at least five trees of a certain tree species. The basal area of the stand was the average of the basal areas of relascope sample plots calculated by tree species. Basal area median diameter was determined by tree species from the whole tree tally. In the case of broadleaves, the distribution was calculated for all broadleaves com-

Table 1. Description of the mean, minima and maxima stand characteristics by tree species.

	Scots pine			Norway spruce			Broadleaves		
	min	mean	max	min	mean	max	min	mean	max
Number of sample plots per stand	1	8	13	3	8	12	2	8	12
Number of sampled trees measured per stand	5	48	154	5	47	135	5	16	62
Proportion of basal area of Scots pine, %	3	64	100	0	36	95	0	43	96
Proportion of basal area of Norway spruce, %	0	23	95	3	49	100	0	38	96
Proportion of basal area of broadleaves, %	0	13	84	0	15	86	13	18	86
Basal area of tree species, m²/ha	1	12.2	30	1	11.7	34	1	4.0	17
Basal area median diameter of a tree species, cm	4	20.4	49	5	18.9	49	3	13.8	40
Age of stand, a	16	76	243	19	79	177	19	75	177

bined. After these calculations there were 472 stands including Scots pine, 335 stands including Norway spruce and 342 stands including deciduous tree species (Table 1).

The Weibull parameters were estimated separately for Scots pine and Norway spruce in a stand. Although broadleaves occurred in more stands than Norway spruce, the number of broadleaves measured per stand was so small that their distribution was not estimated separately. Furthermore, the distribution of broadleaves was not representative of any single tree species, because it contained various tree species.

The Weibull function's three-parameter approach was used. The probability density function of the three-parameter Weibull distribution for a random variable x is

$$f(x)=\frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1}\exp\left(-\left(\frac{x-a}{b}\right)^{c}\right) \quad (a \le x < \infty) \qquad (1)$$
$$=0 \quad (x<a)$$

where
a = location parameter
b = scale parameter
c = shape parameter

The Weibull parameters for the different tree species in each stand were estimated by maximising the natural logarithm of the likelihood function of the Weibull density function. The natural logarithm of the likelihood function of the three-parameter approach of the Weibull density function is

$$\ln(L)=(c-1)\sum_{i=1}^{n}\ln(x_i-a)-n(c-1)\ln(b) + n\ln\left(\frac{c}{b}\right)-\sum_{i=1}^{n}\left(\frac{(x_i-a)}{b}\right)^{c} \qquad (2)$$

(Rennolls et al. 1985)

where L = likelihood function and n = number of sample trees in one stand. The maximum likelihood estimation was done using the IMSL library (IMSL... 1984) and also the initial values for the parameters were generated by the library program. For maximum likelihood estimation boundaries for parameter a were calculated according to the following rule

$$0.75(0.3^{1/n}d_{min}) \le a \le 1.001\times 0.75(0.3^{1/n}d_{min})$$

where
d_{min} = minimum diameter of sample trees within a stand
n = number of sample trees within a stand

These boundaries of the parameter a are modifications of the heuristic boundaries developed by Kilkki and Päivinen (1986). The original boundaries allowed parameter a to vary within a certain proportion of the range between 0 and minimum diameter of the stand during the maximum likelihood estimation. However, this led to difficulties when predicting parameter a using stand characteristiscs. In this study the range for parameter a to vary is very limited to obtain better correlation and predictability between parameter a and the stand characteristics (Maltamo et al. 1995).

In estimating, the allowed ranges for parameters b and c were

$$1 \le b \le 80$$
$$0.1 \le c \le 20$$

After the estimation of Weibull distribution was done regression models were constructed specieswise for parameters using stand characteristics (basal area, basal area median diameter, stand age) as explatonary variables. When the parameters of Weibull distribution are predicted, one parameter can be computed if the other parameters and the basal area median diameter are known. Thus, parameters b and c are linked together by the following relations:

$$c=\frac{\ln(-\ln(0.5))}{\ln\left((d_{gM}-a)/b\right)} \qquad (3)$$

$$b=\frac{(d_{gM}-a)}{(-\ln(0.5))^{1/c}} \qquad (4)$$

where d_{gM} = basal area median diameter

The fits of the distributions were compared separately for pine and spruce distributions and also for entire growing stock of the stands. When the results for the entire growing stock were com-

Table 2. Regression models for the 2- and 3-parameter approach of Weibull parameters in the Scots pine and Norway spruce dominated stands (Maltamo et al. 1995).

Weibull approach	Stand	Parameter	Intercept	d_{gM}	d^2_{gM}	$\sqrt{d_{gM}}$	ln(A)	A^2	$\sqrt{A}$	ln(G)	PPC	SPC	s_e	R^2
2-parameter	Scots pine	ln(c)	1.4468		0.00072		–0.1529						0.21	0.31
	Norway spruce	b	–18.3142			8.6228	0.5337						0.83	0.97
3-parameter	Scots pine	a	4.68346		0.00414			0.000078		–1.81226	0.025800		2.35	0.31
		ln(c)	0.43580	0.03143					–0.04351		0.003485		0.21	0.32
	Norway spruce	a	0.72750		0.003988			–0.00008013				0.017670	1.70	0.18
		ln(c)	0.70720	0.03263					–0.15180			0.002367	0.19	0.38

Explanations of the variable codes: a, b, c = parameters of Weibull distribution, d_{gM} = Basal area median diameter of a stand, cm; G = Basal area of a stand, m²/ha; A = Stand age, a; PPC = proportion of basal area of Scots pine in a stand, %, SPC = proportion of basal area of Norway spruce in a stand, %; s_e = Mean square error of model; R^2 = Degree of determination of model.

puted, the distributions of different tree species occurring in the stand were formed separately and summed. The distribution of broadleaves was formed using the parameter models of Scots pine and the mean characteristics of broadleaves.

Some parameter models from earlier studies were also tested (Tables 2 and 3). The models (Table 2) for the entire growing stock (Maltamo et al. 1995) for pine and spruce dominated stands were applied as follows. The models were used according to the dominating tree species in a stand in predicting the distribution of the entire stand. Both two- and three-parameter approaches of the Weibull distribution were tested. The results for the entire growing stock have already been computed in the study by Maltamo et al. (1995), and they are quoted here. The results for Scots pine and Norway spruce distributions, separately, were computed by multiplying the diameter distributions of the entire stand by the proportionate basal areas of the various tree species. The original study material of models of Maltamo et al. (1995) was same as in this study.

Parameter models (Table 3) formed by tree species using the data obtained from the relascope sample plots of National Forest Inventory for Scots pine (Mykkänen 1986) and Norway spruce (Kilkki et al. 1989) were also compared. This was done to test the models, which are representative of Finland as a whole, in certain parts of the country. The study material of these models consists of whole Finland and these models are constructed same way as models made in this study.

Finally, tests were conducted as to the applicability of the parameter models of three-parameter approach of Weibull distribution (Table 2) made originally for the entire growing stock (Maltamo et al. 1995) in predicting diameter distributions by tree species. The purpose of these tests lies in possible applications where only models for diameter distribution of the entire growing stock are available, but distributions are needed by tree species.

The methods using the different parameter models were named as follows:

Method W1: Models estimated separately for Scots pine and Norway spruce diameter distributions in this study (Table 4).

Table 3. Regression models for the Weibull parameters in the Scots pine (Mykkänen 1986) and Norway spruce (Kilkki et al. 1989) distributions based on material of National Forest Inventory data.

Species	Parameter	Intercept	$\ln(d_{gM})$	d_{gM}	G	s_e	R^2
Scots	ln(a)	–1.3065	1.1544			0.340	0.661
pine	ln(c)	0.6479		0.0255	–0.00560	0.354	0.204
Norway	a	0.0014		0.5174		3.780	
spruce	ln(b)	–0.3462	0.9350		–0.00093	0.298	

Explanations of the variable codes: a, b, c = parameters of Weibull distribution, d_{gM} = Basal area median diameter of a certain tree species, cm; G = Basal area of a stand, m²/ha; s_e = Mean square error of model; R^2 = Degree of determination of model.

Method W2: Models for the entire growing stock, two-parameter approach of the Weibull distribution (Maltamo et al. 1995) (Table 2).

Method W3: Models for the entire growing stock, three-parameter approach of the Weibull distribution (Maltamo et al. 1995) (Table 2).

Method W4: Models estimated separately for Scots pine (Mykkänen 1986) and Norway spruce (Kilkki et al. 1989) diameter distributions from National Forest Inventory data (Table 3).

Method W5: Original models for the entire growing stock but applied separately by tree species, three-parameter approach of the Weibull distribution (Maltamo et al. 1995) (Table 2).

The predicted Weibull distributions were scaled to stand level per hectare by multiplying the 1-cm-dbh-class frequencies of the Weibull distribution by the basal area. These 1-cm-dbh-classes were then modified to refer to the stem number per hectare by using the following formula:

$$n_{hec} = \frac{4q}{d^2\pi} \tag{5}$$

(Bitterlich 1984)

where

n_{hec} = number of stems per hectare
d = diameter, m
q = basal area factor, m²/ha

Once the distributions were predicted, they were compared with the empirical distribution of the stand. The method employed in these comparisons was same as in the study by Maltamo et al. (1995): the sums of the first, third and fourth powers of the diameters in each stand were computed. These diameter sums were calculated as follows:

$$D^c = \sum_{i=1}^{n} d_i^c n_{hec} \tag{6}$$

where D^c indicates diameter sum, with c = 1, 3 and 4.

The original idea of comparisons is to study variables such as volume and value of the stand. However, to avoid discrepancies caused by other models (e.g. height and volume), diameter sums were used. The sums of the third and fourth powers of the diameters were used to place more weight on the most valuable part of the distributions: the sum of third powers of the diameters approximates the volume; and the sum of the fourth powers of the diameters approximates the value of the stand. The volume of the stand is commonly considered to be the foremost stand characteristic.

The empirical diameter sums were derived by adding the powers of the diameters of all the trees in a stand. The predicted distributions were divided into 1cm classes and the frequency of the class was computed. The powers of the each class centre were multiplied by the frequency of the class and the obtained values were summed over the distribution. These diameter sums refer to the stand as a whole (Maltamo et al. 1995).

The criteria used in these comparisons were relative root mean square error (RMSE) and bias, and these were computed for the chosen diame-

Table 4. Regression models for the Weibull parameters in the Scots pine and Norway spruce distributions.

Species	Parameter	Intercept	$\ln(d_{gM})$	$\ln(G)$	$\ln(A)$	s_e	R^2
Scots	ln(*a*)	–1.9952	1.2302			0.474	0.463
pine	ln(*c*)	–0.2491	0.3924	0.0057		0.219	0.292
Norway	*a*	–1.9573	2.8419	–0.9677		1.894	0.213
spruce	ln(*b*)	–0.3384	0.9126	0.0888	0.0562	0.133	0.896

Explanations of the variable codes: *a*, *b*, *c* = parameters of Weibull distribution, d_{gM} = Basal area median diameter of a certain tree species, cm; *G* = Basal area of a certain tree species, m^2/ha; *A* = Stand age, a; s_e = Mean square error of model; R^2 = Degree of determination of model.

ter sums of a stand. The statistical significance of the differences between biases of different methods were tested using T-test. The root mean square error for different powers of diameter sums was computed as follows

$$\text{RMSE}^c = \sqrt{\frac{\sum_{i=1}^{N}\left(D_i^c - \hat{D}_i^c\right)^2}{N}} \quad (7)$$

where

D_i^c = the sums of real dbh^c in stand *i*, with $c = 1, 3$ and 4

$\hat{D}_i^c$ = the sums of predicted dbh^c in stand *i*, with $c = 1, 3$ and 4

N = number of stands

Correspondingly, the bias was computed as follows

$$\text{Bias}^c = \sum_{i=1}^{N}\frac{\left(D_i^c - \hat{D}_i^c\right)}{N}$$

The relative RMSE and bias for different diameter sums were obtained by dividing the absolute RMSE by the average diameter sum obtained from the predicted distributions.

A second way to compare the distributions was to compute the corresponding diameter sums for the sawtimber part of the growing stock. The sawtimber proportion represents the most valuable part of a stand and it also describes the fits of models in a certain part of the distribution. The sawtimber part was estimated by considering trees with dbh greater than or equal to 17 cm, and by taking the corresponding part of the predicted distribution.

3 Results

The estimated regression models for predicting the parameters of the three-parameter Weibull function for the basal area diameter distributions of Scots pine and Norway spruce are shown in Table 4. The independent variables for the different parameter models were chosen from modifications of the following characteristics: stand age, basal area and basal area median diameter by tree species. Modifications of the models were compared using root mean square errors, R-squares and residual plots of the models, and also by comparing the diameter sums of the different models. The regression models accounted for only a small percentage of the variation (low R^2 values), except when modelling parameter b for Norway spruce. The low R^2 values for the parameter models were also found in the studies conducted by Little (1983) and Rennolls et al. (1985). When parameter models with logarithmic transformations were applied, a correction factor was made to reduce the bias (Meyer 1941).

In the case of Scots pine, parameters a and c were chosen to be modelled and b was chosen to be computed; in the case of Norway spruce, however, parameters a and b were regressed. This was also the situation in the studies by Kilkki and Päivinen (1986), Kilkki et al. (1989)

Table 5. Correlations between the estimated Weibull parameters and basal area median diameter in the Scots pine and Norway spruce distributions.

	d_{gM}	a	b	c
Scots pine				
d_{gM}	1.00			
a	0.73	1.00		
b	0.82	0.23	1.00	
c	0.53	0.44	0.35	1.00
Norway spruce				
d_{gM}	1.00			
a	0.27	1.00		
b	0.92	–0.07	1.00	
c	0.39	0.32	0.24	1.00

and Maltamo et al. (1995). This was due to the correlations between the different parameters and the basal area median diameter (Table 5). In the case of Norway spruce, the correlation between basal area median diameter and parameter b was very high whereas the other correlations were considerably lower. On examining the correlations of the parameters in the Scots pine distributions, it was observed that the correlation between basal area median diameter and parameter b is still the highest, but that the other correlations are higher than corresponding correlations in the Norway spruce distributions. More accurate results were obtained for the Scots pine distributions when parameter c was modelled instead of parameter b.

The examples of the predicted three-parameter Weibull distributions both for the entire growing stock of the stand (W3) separately for pines and spruces (W1) in a certain stand are presented in Figs. 1 and 2. In Fig. 1, the total number of trees measured in the stand was 62; 32 were spruces and 30 were pines. The basal area and basal area median diameter of pine were 10 m^2/ha and 27 cm respectively the corresponding values for spruce were 11 m^2/ha and 17 cm. For the entire growing stock, the basal area median diameter was 21 cm. For this particular entire stand, the bias of the sums of the third powers of the diameters was –2.1 % for method W1 and –2.7 for method W3. When the bias was consid-

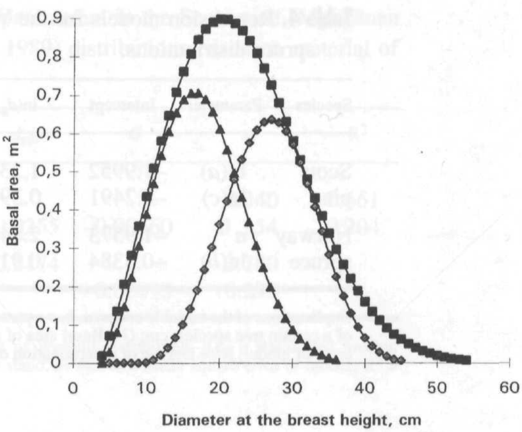

Fig. 1. Comparisons, separately for Scots pine (◆), Norway spruce (▲) and for the entire growing stock (■), of the predicted Weibull distributions in a sample stand. In this stand the basal area and basal area median diameter of pine were 10 m^2/ha and 27 cm, respectively the corresponding values for spruce were 11 m^2/ha and 17 cm. For the entire growing stock, the basal area median diameter was 21 cm.

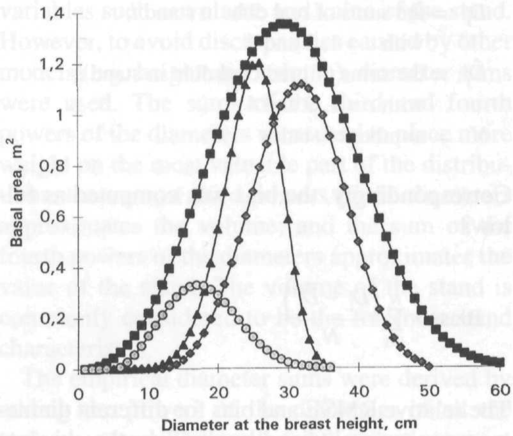

Fig. 2. Comparisons, separately for Scots pine (◆), Norway spruce (▲) and broadleaves (○), and for the entire growing stock (■), of the predicted Weibull distributions in a sample stand. In this stand the basal area and basal area median diameter for pine were 18 m^2/ha and 32 cm, for spruce 12 m^2/ha and 25 cm, and for broadleaves 4 m^2/ha and 18 cm. For the entire growing stock, the basal area median diameter was 29 cm.

Table 6. Relative root mean square error (RMSE) and bias of the different predicted distributions for the entire growing stock of the stands using the models for the entire growing stock (W2 and W3) and the separate models for the Scots pine and Norway spruce distributions (W1, W4, W5). The results for the entire growing stock when using methods W2 and W3 are from the study by Maltamo et al. (1995). Statistical significance of T-test: *** = prob(T < t) < 0.001, ** = 0.001 < prob(T < t) < 0.01, * = 0.01 < prob(T < t) < 0.05.

	RMSE %			Bias %		
	Σd	Σd^3	Σd^4	Σd	Σd^3	Σd^4
Entire growing stock						
W1	12.7	3.6	8.6	7.3 **	–0.7	1.3
W2	13.4	5.4	10.6	3.1	2.0	–2.7
W3	13.6	6.1	12.8	4.3 *	–3.4	–6.9 *
W4	16.1	4.5	8.5	11.2 ***	–2.1	–1.0
W5	9.8	4.3	11.5	0.7	–2.2	–6.9
Scots pine						
W1	11.7	4.3	8.5	5.0	–0.9	–0.7
W2	16.0	9.9	20.3	–10.4 **	4.8	6.7
W3	15.5	8.9	16.4	–5.7	2.3	5.1
W4	13.9	4.3	8.4	7.9 *	–0.9	0.9
W5	10.2	4.5	12.4	–1.2	–1.6	–7.2
Norway spruce						
W1	17.5	4.4	11.2	10.0 *	–0.7	2.8
W2	20.8	11.0	20.2	6.3	–3.3	–3.8
W3	20.2	11.6	22.5	6.6 *	–5.1	6.6 *
W4	23.3	7.3	12.9	16.0 **	–4.4	–6.1
W5	14.8	5.2	10.9	4.9	–2.7	–5.7

ered by tree species, there was a bias of –2.2 % for pine and –2.1 % for spruce when using method W1. With method W3, considerable biases were obtained by tree species: 19.3 % for pine and –23.1 % for spruce. This was due to the large differences in basal area median diameters between pine and spruce.

In Fig. 2, the stand was dominated by pines (92 measured trees), with spruce (61) and broadleaves (22) also in that particular stand. The basal area and basal area median diameter for pine were 18 m²/ha and 32 cm, for spruce 12 m²/ha and 25 cm, and for broadleaves 4 m²/ha and 18 cm. For the entire growing stock, the basal area median diameter was 29 cm. The bias of the sums of the third powers of the diameters for the entire growing stock was –4.6 % for method W1 and –4.5 % for method W3. In the case of tree species, the corresponding biases were –1.0 % for pine and 4.2 % for spruce when using method W1. With method W3, the biases were considerably higher; 5.3 % for pine and –12.5 % for spruce. However, the results were closer to one another than in the first sample stand. In both examples, wider and more right-tailed distributions were obtained when using the models for the entire growing stock of the stand.

The reliability of the compared methods (W1, W2, W3, W4 and W5) was then tested in terms of the chosen diameter sums for the entire growing stock and for the different tree species (Table 6). In almost all cases, method W1 gave the best results; the exception was bias in the sums of the first powers of the diameters, where clear underestimates were obtained. These underestimates were also statistically significant. In the other cases, method W1 was almost unbiased.

For the stands as a whole, the differences between the models by tree species (methods W1 and W4) and for the entire growing stock of the stand (methods W2 and W3) were already clear. Also, the accuracy of method W5 was better than that of methods W2 and W3 for the stands as a whole, although the same models were used in these methods.

When the results were considered by tree species, the differences were marked; when using the models for the entire growing stock (W2 and W3) to compute the results by tree species according to basal areas, the errors were – in most cases – over two times bigger than when using models made separately for the different tree species. The accuracy of method W3 was, on average, a little better than that of method W2 for Norway spruce. However, for Scots pine the result was opposite.

In terms of the relative RMSE, the results obtained seemed to be more accurate for Scots pine than for Norway spruce. The models for the entire growing stock (methods W2 and W3) were biased when results were computed by tree species. These models produced underestimates for Scots pine diameter sums and overestimates for

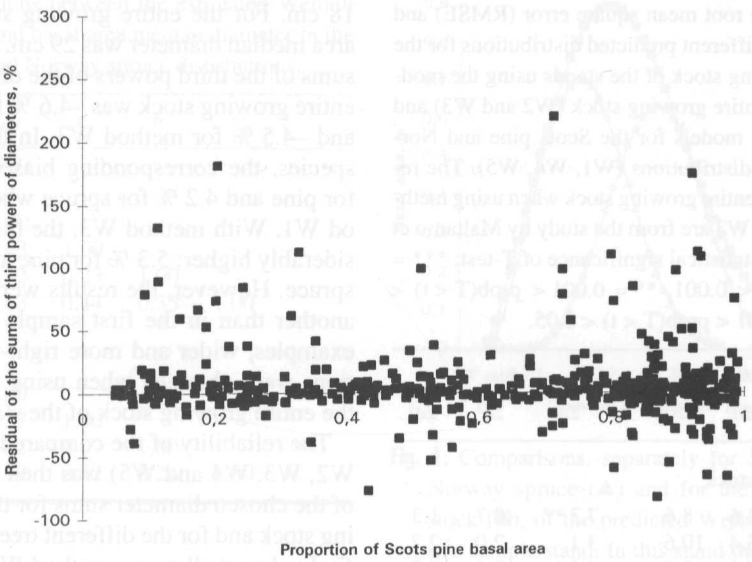

Fig. 3. The residuals of the sums of diameters' third powers for the entire growing stock with respect to the basal area proportion of Scots pine. The parameter models used are estimated separately for the different tree species.

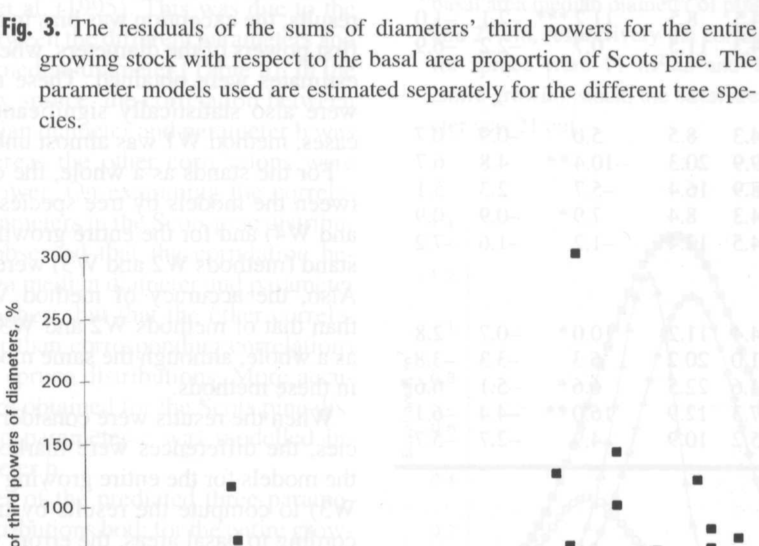

Fig. 4. The residuals of the sums of diameters' third powers for the entire growing stock with respect to the basal area proportion of Scots pine. The parameter models used are estimated for the entire growing stock of the stand.

Table 7. Relative root mean square error (RMSE) and bias of the different predicted distributions for the sawtimber fraction of the growing stock using the models for the entire growing stock (W2 and W3) and the separate models for the Scots pine and Norway spruce distributions (W1). Statistical significance of T-test: *** = prob(T < t) < 0.001, ** = 0.001 < prob(T < t) < 0.01 , * = 0.01 < prob(T < t) < 0.05.

	RMSE %			Bias %		
	Σd	Σd^3	Σd^4	Σd	Σd^3	Σd^4
Entire growing stock of sawtimber						
W1	14.2	8.6	11.1	–0.3	1.5	3.3
W2	17.0	10.5	12.2	1.9	–1.0	–1.4
W3	17.1	9.9	14.8	5.2	–2.2	–7.0
W4	15.0	9.7	11.1	–0.8	0.0	1.1
W5	18.6	8.0	12.0	8.8 *	–0.4	–6.6
Scots pine sawtimber						
W1	14.5	10.3	11.5	2.4	1.2	0.9
W2	26.3	21.3	25.9	13.4 **	11.1	9.8
W3	28.6	20.3	20.8	16.5 ***	10.3	5.4
W4	15.6	11.3	12.1	0.7	1.5	3.1
W5	23.1	10.3	13.2	12.3 **	1.3	–6.6
Norway spruce sawtimber						
W1	18.6	9.7	12.9	–4.0	0.5	4.8
W2	21.2	19.9	24.4	–1.4	–4.2	–3.8
W3	21.2	19.5	26.1	3.1	–5.3	–10.2
W4	18.6	12.0	14.3	–3.5	–4.0	–4.9
W5	18.4	8.9	11.2	5.2	–1.1	–4.9

Norway spruce diameter sums. However, the bias was statistically significant only for the sums of the first powers of diameters.

The accuracy of the models estimated by tree species when using the National Forest Inventory data (method W4) was, in the case of Scots pine, very close to method W1; however, for Norway spruce, the said accuracy was inferior. The bias of method W4 was statistically significant for the sums of first powers of diameters. In the case of method W5, where models for the entire growing stock were applied by tree species, the accuracy was surprisingly good for the entire growing stock. When considering the results by tree species the method W5 was even the most accurate method in some cases.

The results concerning the sawtimber fraction of the growing stock were also computed (Table 7). These results showed the same trend as the results for the diameter sums for the entire distribution: specieswise methods W1, W4 and W5 were the most accurate alternatives and the results obtained with methods W2 and W3 were close to one another. The results for the sawtimber fraction of the stand were more inaccurate when compared to the results for the entire stand. For example, the RMSE of method W1 increased from 3.6 % for the entire stand to 8.6 % when the sawtimber part of the growing stock was considered. All methods were relatively unbiased in the case of the sawtimber fraction of the entire growing stock and also in the case of Norway spruce. Consequently, the fit of the distributions seemed to be small biased also with respect to certain parts of the growing stock.

Finally, a graphic assessment was made of whether there were differences in the residuals of the mixed stands in the different kinds of tree species mixtures. The residuals of the sums of the third powers of the diameters of methods (W1 and W3) were plotted against the basal area, the basal area median diameter and the proportions of tree species. Examples of these residuals are given in Figs. 3 and 4 with respect to proportion of basal area of Scots pine. The plots showed no clear differences and it seemed that, on average, the different kinds of tree species mixtures had no influence on the level of the residuals.

4 Discussion

The parameter models estimated in this study separately for Scots pine and Norway spruce were compared to models for the entire growing stock, and these were applied according to the dominating tree species. The chosen theoretical distribution function was the Weibull distribution. The material was representative of the two conifers. Broadleaves were not modelled separately, but in the application phase they were predicted using the parameter models of Scots pine.

The use of diameter distribution models for the entire growing stock has still been the practice in Finnish forestry in connection with standwise inventories when forecasting different products obtainable from a stand. This study revealed considerable differences in accuracy among the different models for the tree species and models for the entire growing stock. For the entire growing stock, or the sawtimber fraction of the growing stock, the results were relatively close to one another. In most of the stands forming the study material, the diameter distribution was unimodal and one distribution model was able to describe them properly. But were results needed by tree species, the models for the entire stock turned out to be far too inaccurate and also biased in some cases. Also, the models for the entire growing stock produced excessively wide distributions, and these produced some over-large predicted trees.

The results for the different tree species were underestimates for pine and overestimates for spruce when using the models for the entire growing stock by tree species. This was due to the fact that pine in mixed stands commonly exceeds spruce in size. These results are in line with those obtained in the study by Siipilehto (1988). In the study by Maltamo et al. (1995), the results obtained were far better in the case of the two-parameter approach of the Weibull distribution than in the case of the three-parameter approach. This was no longer true in this study, where these models were applied to different tree species by their basal areas.

In the studies by Mykkänen (1986) and Kilkki et al. (1989) the number of sampled trees from a stand was very small when estimating the diameter distributions. A small number of sampled trees in National Forest Inventory plots has been considered as a problem of these models for applications of the stock of the whole stand. Furthermore, in maximun likelihood -estimation small number of observations (< 10) may lead to inaccurate estimates. However, the parameter models obtained from the studies by Mykkänen (1986) and Kilkki et al. (1989) showed quite good fits in the stands used in this study, especially when considering Scots pine distributions.

The applicability of the models used in the study by Maltamo et al (1995) was better when these models were used separately for the tree species than when used for the entire growing stock of the stand by dominating tree species (as they were originally estimated). It is obvious that when applying the models it is more important to predict the distributions separately for the different tree species even though the parameter models were not made for this purpose.

The use of very narrow boundaries of parameter a in the estimation phase may have improved the predictability of this parameter and it may also have, in the end, influenced the test results. Consequently, the models estimated in this study and the models for the entire growing stock obtained from the study by Maltamo et al. (1995), where no fixing was made, are not entirely comparable. However, the two-parameter approach of the Weibull distribution was also used in the same way, and the results were on same level as in the case of the three-parameter approach. Therefore, the effect of fixing parameter a was probably a minor one.

Diameter distributions were estimated separately for the different tree species in this study. More accurate results could have been obtained had the different tree storeys of a particular tree species been estimated separately. Especially in the case of Norway spruce, the tree size distribution in a certain stand can be composed of more than one tree storey. This may also have influenced the accuracy of the models estimated in this study with respect to the entire Norway spruce distribution within a stand.

The estimation of different tree species and storeys requires relatively large samples of trees from the one stand. The material used in this study is adequate for this purpose, but it covers only a part of eastern Finland. The collecting of corresponding data for the whole country requires a vast amount of field work and funds. The only existing parameter models representative of the whole of Finland, and made separately for the different tree species, are those provided in the studies by Mykkänen (1986) for pine and Kilkki et al. (1989) for spruce.

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