

# Optimization Bias in Forest Management Planning Solutions Due to Errors in Forest Variables

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The yield of various forest variables is predicted by means of a simulation system to provide information for forest management planning. These predictions contain many kinds of uncertainty, for example, prediction and measurement errors. Inevitably, this has an effect on forest management planning. It is well known that uncertainty in the forest yields causes optimistic bias in the observed values of the objective function. This bias increases with the error variances. The amount of bias, however, also depends on the error structure and the relations between the objective variables. In this paper, the effect of uncertainty in forest yields on optimization is studied by simulation. The effect of two different sources of error, the correlation structure of these errors and relations among the objective variables are considered, as well as the effect of two different optimization approaches. The relations between the objective variables and the error structure had a notable effect on the optimization results.

**Keywords** decision analysis, forest planning, prediction, uncertainty

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## 1 Introduction

In forest management planning, the land area is divided into homogeneous units (here called forest stands), for which various management schedules can be applied. These schedules may vary with respect to the treatments (e.g. clearcutting or thinning), timing (different years or periods) of these treatments, or both. The goal of forest

management planning is to select a combination of standwise management schedules that is optimal with respect to chosen criterion variables at the forest holding or area level. The optimization is based on predicted yields of various forest products under different schedules. The predictions are obtained with the aid of a simulation system consisting of several models, such as growth and mortality models.

The yields predicted with statistical models include uncertainty which has four main sources: (i) model misspecification, (ii) random estimation error of the model coefficients, (iii) residual variation of the models, and (iv) errors in the independent variables of the models (e.g. Kangas 1997). The independent variables may include sampling error, measurement error or prediction error, if the independent variables of the model are predicted with another model (e.g. Gertner 1991). In addition to the uncertainty about forest yields, there are other sources of uncertainty; the future prices of timber, for example.

The structure of the errors varies with respect to the error source. For example, additive measurement errors in a standwise inventory are perfectly correlated among the management schedules of any particular stand (within-stand correlation). Instead, the measurement errors in different stands may be independent (between-stand correlation). It may be assumed that the prediction errors are positively correlated among the management schedules of a particular stand: if stand growth is overestimated in connection with one treatment alternative, this will most likely apply to other treatment alternatives available for the same stand as well. However, these correlations cannot be observed, as only one treatment can be applied in a stand at time. The prediction errors may also be positively correlated among the stands and among the planning periods considered. For example, in Hof et al. (1995) the product levels were assumed to be spatially correlated.

The errors for different forest products may be correlated in a given stand. For example, if the growth of a stand in a certain planning period is overestimated, then the value of the growing stock at the end of this period will also be overestimated. The errors can either be independent of the value of the measured/predicted variable or they can depend on it. For example, the measurement error for tree height is often the larger the higher the tree. In the former case the errors can be described with an additive error model, in the latter case with a multiplicative error model.

Since the sources of uncertainty differ in their characteristics, also their effects on optimization differ. The interpretation of the effects of uncertainty can also differ. For example, if the price of

timber is above the expected value, the decision maker can adapt to the new situation by selling more timber. If the growth of a stand has been over- or underestimated, it is not possible to adapt to the situation since the true values usually remain unknown until the stand is cut (Pickens et al. 1991).

Numerical optimization is finding increasingly wider use in forest management planning. Usually, uncertainty about forest development is ignored and optimization is done as if under certainty. This can lead to two kinds of undesirable results. Firstly, a non-optimal alternative may be chosen. This leads to a smaller realized objective function value than the optimal objective function value. This loss is called "regret" (Bell 1982). In the case of constrained optimization the non-optimal alternative may also be infeasible. Secondly, the true worth of the optimal solution may be overestimated. This kind of loss is called "disappointment" (Bell 1985).

The effect of uncertainty has been studied fairly widely in the context of linear programming. It has been noted that uncertainty about the coefficients of the objective function causes optimization to be optimistically biased. This means that the value of the optimal solution will be overestimated if there are random errors in the objective function coefficients (Hobbs and Hepenstal 1989). Errors on the right hand sides of the constraints have the opposite effect (Itami 1974). Uncertainty about the forest yields also causes optimistic bias in the observed value of the optimal solution, and the obtained solutions may be infeasible (Pickens and Dress 1988, Pickens et al. 1991). The aforementioned problems apply also to other optimization approaches, not only to linear programming. The effect of uncertainty on decision making has also been studied from the perspective of deciding the optimal inventory method and timing (Ståhl 1994, Ståhl et al. 1994).

The effect of uncertainty about the forest yields on the feasibility of the optimization problem have been accounted for by chance-constrained optimization (e.g. Hof et al. 1992, 1996, see also Weintraub and Abramovich 1995). In these studies the expected values and the variances of the uncertain coefficients were assumed to be known. It is more realistic, however, to assume only an

unbiased estimate of the forest yields to be known (e.g. Pickens and Dress 1988, Pickens et al. 1991). In this case, an estimate of the optimization bias can be obtained using simulation: with an additional simulation level including generated random variation, the optimization bias can be modelled (Pickens et al. 1991). The obtained estimate of the bias can then be used to aid decision making in the original situation. The simulation approach can also be used to ensure the feasibility of the chosen solution (Pickens et al. 1991).

The effect of uncertainty about the forest yields on the expected value of the objective function has been studied by generating random variation from some distribution to forest variables of interest in the LP-matrix (e.g., Pickens and Dress 1988, Pickens et al. 1991). However, this way it is difficult to consider the varying sources of uncertainty with varying characteristics. A realistic error structure may be difficult to incorporate. When the forest yields under different treatment alternatives are projected by a simulation system in which the errors from different sources are propagated (e.g. Gertner 1987, Mowrer 1991, Kangas 1997), it is easier to account for the error structure.

In this study, the effect of uncertainty is first considered theoretically. Then, the effect of uncertainty about forest yields is studied by simulation. The aim of this study is to examine the effect of the error structure on the optimization results. Managing the uncertainty is beyond the scope of this study. However, it is shown that the error structure need to be taken into account in order to properly account for the uncertainty in planning.

The simulation study was carried out as follows. Firstly, the expected values of forest yields under a set of management schedules were simulated for given (simulated) stands. For the sake of simplicity, each management schedule was assumed to consist of only one treatment alternative. The expected yields were treated as if they were the true values of the forest yields. "True" optimal alternatives for each stand under different objective functions and using two different approaches were found. After this, the forest yields under the same treatment alternatives were simulated so that there were (correlated) random errors in each criterion variable.

These random errors were a combination of unbiased random errors from different sources, propagating through the simulation system.

The sources of uncertainty considered were prediction errors of different models used in simulation, and measurement errors of the initial stand variables. The initial stand variables were assumed to be from a compartmentwise inventory, and thus they only included measurement errors, not sampling errors. Random errors were generated with different assumptions about the correlation structure among these errors.

The optimal solution for each stand with each set of simulated criterion variables was then chosen and compared to the "true" optimal solution. Optimization was carried out using either unconstrained optimization, in which an additive utility function was maximized (see Kangas 1992, Pukkala and Kangas 1993, 1996) or constrained optimization, which was solved using an LP-formulation. The utility function was obtained from the dual solution of the LP-problem, in order to obtain comparable results with these two approaches. In a more general case, the utility function could be a nonlinear function of the criterion variables (e.g. Kangas et al. 1998).

## 2 Methods

### 2.1 The Effect of Random Errors on the Expected Value of the Utility Function

In a simple case it is possible to examine the bias due to optimization under uncertainty analytically. The decision maker is assumed to maximize his or her utility  $U$ . Let there be  $n$  alternative management options, with utilities  $U_i$ ,  $i = 1, \dots, n$  that are not known exactly. The observed utilities are  $u_i = U_i + e_i$ . Then, the optimization bias is defined as

$$bias = \max(U_i) - E(\max(u_i)) \quad (1)$$

If  $n = 2$  and the joint distribution of the observed utilities  $u_i$  is known, then the expected maximum utility can be presented as

$$E(\max(u_i)) = \int_{u_2=-\infty}^{\infty} \int_{u_1=-\infty}^{\infty} u_1 f(u_1 u_2) du_2 du_1 + \int_{-\infty}^{\infty} \int_{u_1}^{\infty} u_2 f(u_1 u_2) du_2 du_1 \quad (2)$$

If the joint distribution of the errors  $e_i$  is known, so is that of the observed utilities  $u_i$ . The optimization bias (1) increases with increasing variance of the errors  $e_i$  (e.g. Pickens et al. 1988).

In a more general case, the utility of the alternative options  $i, i = 1, \dots, n$  may consist of several criterion variables  $S_{ij}$

$$U_i = \sum_{j=1}^p \alpha_j S_{ij} \quad (3)$$

where  $\alpha_j, j = 1, \dots, p$  are the weights given to the criterion variables  $S_j$ . The observed value of each criterion variable under each alternative  $i, s_{ij}$ , may contain random errors as  $s_{ij} = S_{ij} + e_{ij}$ . In this case, the error variance of observed utility  $u_i$  can be calculated from the error variances and covariances of the observed criterion variables  $s_{ij}$

$$\text{var}(u_i) = \sum_{j=1}^p \alpha_j^2 \text{var}(e_{ij}) + 2 \sum_{j=1}^p \sum_{k=j+1}^p \alpha_j \alpha_k \text{cov}(e_{ij}, e_{ik}) \quad (4)$$

Thus, when the errors in the criterion variables are negatively correlated, the error variance of  $u_i$  is reduced and optimization bias is reduced. When they are positively correlated, the error variance of  $u_i$  increases and the optimization bias also increases. However, these relationships may be confused by the correlation of the objective variables  $S_j$  themselves. If the errors are multiplicative, i.e.  $s_{ij} = S_{ij} e_{ij}$ , the problem can be re-formulated as to a problem of additive errors using logarithms, in order to obtain an analytical presentation of the bias.

## 2.2 Forest Management Planning Situation

In a forest management planning situation, we have  $m, k = 1, \dots, m$  separate stands with  $n_k$  treatment alternatives. The formulation of the management problem, using an unconstrained approach with a utility function, is then to maximize the utility of a plan by maximizing the sum

of utilities over the  $m$  stands,  $n_k$  treatment alternatives and  $p$  objective variables as

$$\max(U_i) = \max \left( \sum_{k=1}^m \sum_{i=1}^{n_k} x_{ki} U_{ki} \right) = \max \left( \sum_{k=1}^m \sum_{i=1}^{n_k} x_{ki} \sum_{j=1}^p \alpha_j S_{kij} \right) \quad (5)$$

where  $x_{ki}$  is the proportion of stand  $k$  treated according to alternative  $i$  and  $\sum_{i=1}^{n_k} x_{ki} = 1$  for each  $k$ .

In the unconstrained optimization approach, the whole stand will be treated according to the same treatment, i.e.  $x_{ki}$  is either 1 or 0.

The expected loss due to choosing a non-optimal alternative is defined as the expected value of the difference between the true objective function value of a chosen alternative and the true optimum as

$$\text{loss} = E \left\{ \sum_{k=1}^m \sum_{i=1}^{n_k} x_{ki}^{opt} U_{ki} - \sum_{k=1}^m \sum_{i=1}^{n_k} x_{ki}^{cho} U_{ki} \right\} \quad (6)$$

where  $x_{ki}^{opt}$  is the proportion of stand  $k$  treated with alternative  $i$  in optimal solution, and  $x_{ki}^{cho}$  the proportion of stand  $k$  treated with alternative  $i$  in the chosen solution.

In the constrained optimization approach, the corresponding problem is formulated by using one of the criterion variables (say  $p$ ) as an objective variable and the others (say  $1, \dots, p - 1$ ) as constraints in

$$\max(U_i) = \max \left( \sum_{k=1}^m \sum_{i=1}^{n_k} S_{kip} x_{ki} \right) \quad (7)$$

with

$$\sum_{k=1}^m \sum_{i=1}^{n_k} S_{kij} x_{ki} \geq b_j, \quad j = 1, \dots, p - 1$$

where  $n_k$  is the number of treatment alternatives,  $p - 1$  is the number of constraints,  $b_j$  is the value of constraint for criterion variable  $S_j$ , and  $x_{ki}$  is the proportion of the area of stand  $k$  on which treatment alternative  $i$  is applied. The problem can be solved using linear programming.

The LP-problem can also be presented as a (casewise) additive utility function by using the

dual solution in determining the weights  $\alpha_j$  (Seo 1980, Kilkki 1985). In this study, the weights in the utility function were obtained from the dual solution of a corresponding LP-problem, so that the approaches are comparable.

When the error structure becomes complicated, an analytical presentation of the optimization biases is difficult to obtain. This is especially true with forest management planning where the errors in the criterion variables may contain errors from several sources (e.g. sampling error, prediction error, measurement error) and some of these errors may be additive and some multiplicative, some independent and some correlated. This being the case, simulation is the easiest way to study the effects of different assumptions. Using simulation, the expected value of maximum utility can be estimated as

$$E(\max(u_i)) \approx \frac{1}{R} \sum_{r=1}^R \max(u_{ir}), \quad i = 1, \dots, n \quad (8)$$

where  $R$  is the number of simulation realizations. The expected loss is estimated respectively.

### 2.3 The Simulation Experiment

The simulation was carried out for a set of twelve simulated stands, which were, for simplicity's sake, assumed to be equal in area. The ages of the stands varied from 15 to 85 years (mean 50), mean diameter from 7 to 30 cm (mean 17.8), and basal area from 10 to 33 m<sup>2</sup>/ha (mean 21.1). A set of management schedules, each consisting of one treatment, was simulated for each stand. The treatment alternatives included twelve thinning alternatives, which were combinations of the timing of thinning (after 5 years, 10 years or 15 years), the proportion of stems removed (30 %, and 60 %) and the treewise probabilities for removal. These probabilities were due to the primary thinning principles chosen: low thinning (the probability of removal being the highest for small trees) and high thinning (the probability of removal being the highest for large trees). In addition, one alternative was always to let the stand grow and yet another to clear-cut the stand at the end of the period. There were altogether fourteen management schedules for each stand.

In the unconstrained optimization case, the op-

timal solution was obtained using a linear additive utility function (Eq. 5). The expected value of the maximum utility and the corresponding values of the criterion variables were calculated from simulations and compared to the true optimum (Eqs. 8 and 1). Also, the expected loss due to choosing a non-optimal alternative was calculated (Eq. 6).

In the constrained optimization case, the problem was solved using an LP-formulation. This meant maximizing one of the variables as the objective, the other criterion variables being treated as constraints. As in the unconstrained case, the expected value of the maximum objective function value was calculated, as well as the expected loss with respect to objective variable. Also, the proportion of solutions that were infeasible was calculated.

The criterion variables used in this study were (i) the present discounted value of net incomes from cuttings (Finnish marks, FIM), (ii) the monetary value of the growing stock after 20 years (FIM), (iii) the amount of dead and decaying wood (m<sup>3</sup>/ha), and (iv) the growth of the stand (m<sup>3</sup>/ha). The discount rate applied in the calculations was 3 %. The stumpage price for pine saw logs was FIM 238/m<sup>3</sup> and that for pine pulpwood FIM 84/m<sup>3</sup>, which were the mean prices in Finland for the 1994/1995 logging year (Statistical Yearbook of Forestry 1996).

The above mentioned criterion variables were chosen since they represent both the common variables for planning and variables which are frequently used, for example, in biodiversity considerations. They also represent different kinds of prediction uncertainty. In the simulation experiment, the weights of these variables were varied, giving several different problems.

### 2.4 Prediction of the Criterion Variables

The initial stand table for each simulated stand was obtained using the Weibull distribution, the parameters for which were predicted using basal area and mean diameter (e.g. Kilkki et al. 1989). The sources of error in the initial stand table were the measurement errors for the basal area and mean diameter. The measurement errors were assumed to be additive, normally distributed and

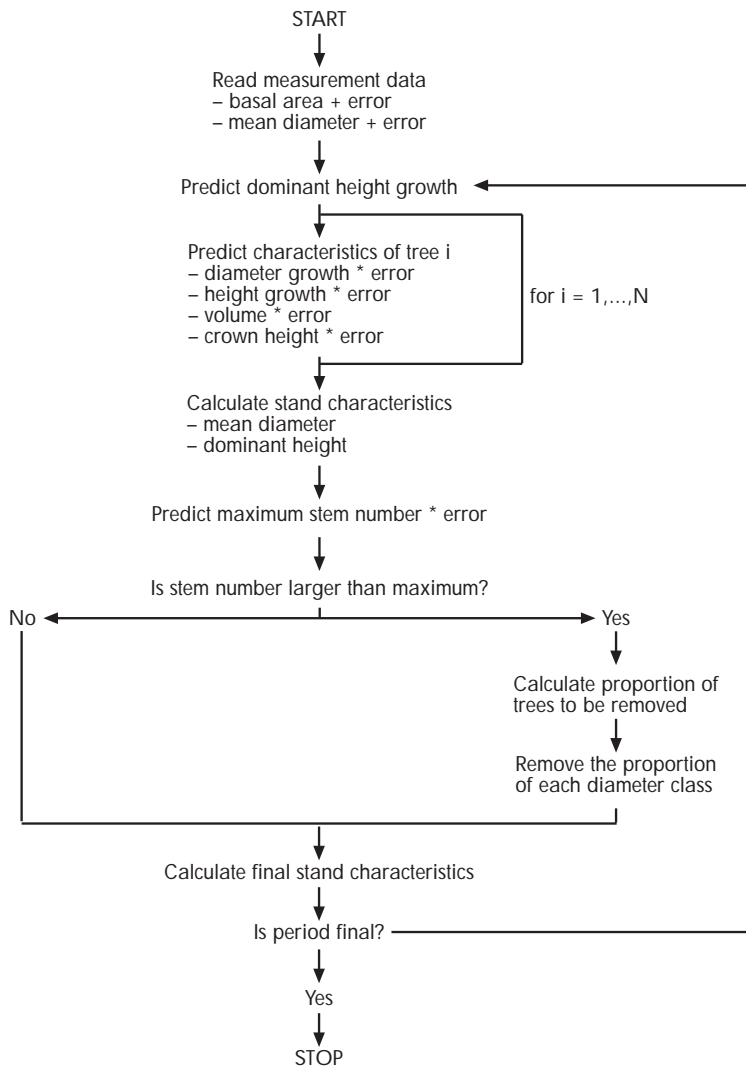


Fig. 1. Flow chart of the simulation in one stand.

mutually independent between stands.

The simulation of the development of the stands was based on models for diameter growth and height growth of single trees (Hynynen 1995a). In addition to height and diameter growth models, models for crown ratio (Hynynen 1995b), volume (Laasasenaho 1982), dominant tree growth (Vuokila and Väliäho 1980), and natural mortality (Hynynen 1993) were also used. Natural mortality was modeled as the maximum number of trees in a stand with given stand characteristics. Surplus trees were removed from the

tree list. The flow chart of the simulation for one stand is presented in Fig. 1. A more detailed description of the simulation system can be found in Kangas (1997). However, unlike in Kangas (1997), the trees to be removed due to natural mortality in the present study were not chosen randomly but by removing a constant proportion from each diameter class.

In addition to the measurement errors, the uncertainty in the criterion variables was due to the residual errors of the above mentioned models. The prediction errors in the criterion variables

**Table 1.** The relative standard errors of the used models. The RMSE of diameter and height growth models (Hynynen 1995a) and crown ratio model (Hynynen 1995b) is related to the predicted mean value, and that of volume model (Laasasenaho 1982) and maximum number of stems model (Hynynen 1993) is obtained from model fit.

Model	Relative RMSE
Diameter growth	35.2 %
Height growth	30.4 %
Crown ratio	14.1 %
Tree volume	7.1 %
Number of stems	17.1 %

are a result from the errors in tree characteristics, such as diameter and height, and from the errors in stem number. The development of each stand under the different treatment regimes was simulated  $R$  ( $R = 100$ ) times over four 5-year planning periods, altogether 20 years. In each realization  $r$ , random errors were generated to the predicted tree characteristics. In this study, the prediction errors were assumed to be multiplicative lognormal errors. Similarly, multiplicative random errors were generated for the maximum number of stems in a stand. Each time any model was used, the result was multiplied with a generated value from a lognormal distribution. The variances for the lognormal distributions were obtained from the residual variances of the used models (Table 1).

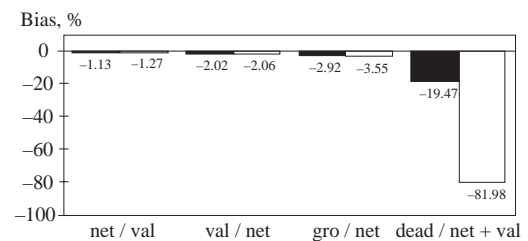
Since all the alternatives for a given stand were predicted with the same models, the prediction errors for a given tree most likely are positively correlated among the treatment alternatives of that stand. Most of the simulations were carried out assuming this inter-alternative correlation to be either zero or one in logarithmic scale. The prediction errors in the adjacent periods were assumed to be independent for the height growth. The autocorrelation between the adjacent 5-year periods was assumed to be 0.5 for the diameter growth, 0.9 for the volume and crown ratio and 0.81 for the maximum stem number (see Kangas 1997). Prediction errors were also assumed to be mutually independent between the stands.

### 3 Results

#### 3.1 The Effect of Multiplicative Prediction Errors on the Observed Objective Function Value

In the simulation study, the constrained optimization approach was carried out first. Four different problems, with different objective variable and constraint variable(s) were considered. The problems considered were 1) net income as objective with value of growing stock as constraint, 2) value of growing stock as objective with net income as constraint, 3) growth as objective with net income as constraint, and 4) volume of dead and decaying wood as objective with net income and value of growing stock as constraints. For each of these problems, several values for the constraint (right hand side, RHS) were tried. However, the value of the RHS did not markedly affect the results in the range studied.

The prediction errors in each stand were first assumed to be uncorrelated among the treatment alternatives. The bias in the observed objective function value of the LP-problem varied from  $-1.27\%$  to  $-81.98\%$ , depending on the prediction variance of considered variables (Fig. 2). When the correlation of prediction errors among the treatment alternatives was assumed to be one, the corresponding values varied from  $-1.13\%$  to  $-19.47\%$ . The effect of correlation assumption



**Fig. 2.** Relative bias of the objective function value in four different LP-problems, assuming independent □ or perfectly correlated ■ prediction errors (in logarithmic scale) among treatments. The problems vary with respect to the objective variable / constraint variable(s). Net stands for net income, val the value of growing stock, gro the stand growth and dead for the volume of dead and decaying wood.

was greatest for the case with dead and decaying wood as objective. For this variable the prediction variance is the greatest.

The effect of correlation on the optimization bias was more closely examined in a case with dead and decaying wood as objective and growth as constraint (Fig 3), as the correlation had the greatest effect on problems with largest error variances. The bias increased almost linearly as a function of the correlation.

The problems 1–4 were then solved again using an unconstrained approach with additive utility function, in which the weights of the criterion variables were obtained from the dual solutions of the aforementioned LP-problems. The bias of the observed utility function value varied from  $-0.73\%$  to  $-58.57\%$ , if the prediction errors were assumed to be uncorrelated among the alternatives, and from  $-0.76\%$  to  $-17.76\%$ , if the correlation was equal to one. The biases had smaller absolute values than in the constrained case. This is due to the problem formulation: the utility function was constructed of two (or three) criterion variables, in which the errors could be to opposite directions (see also eq. 4), while the objective function in the constrained approach included only one variable.

On the other hand, the bias in each of the criterion variables had greater absolute value in the unconstrained than in the constrained case. For example, when net income was used as an objective variable and growing stock was used as a constraint, the net income in the LP-solution was overestimated by  $1.1\%$ – $1.3\%$ , depending on the assumed correlation and value of the constraint. The value of the growing stock was, on the other hand, underestimated by  $0.3\%$ – $1.0\%$ . In a respective unconstrained problem, the utility function value was overestimated by  $0.5\%$ – $0.8\%$ . However, the net income was at the same time overestimated by  $2.5\%$ – $7.7\%$ , and the value of the growing stock was underestimated by  $2.4\%$ – $13.4\%$ . In the chosen solution, an underestimation in one variable was compensated for by an overestimation in another with both approaches. In the constrained optimization case, however, such compensation is only possible when the constraint is met. Thus, possibilities for compensating are larger in the unconstrained case. The larger relative underestimation in the

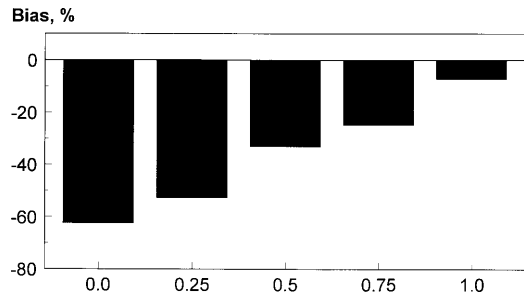


Fig. 3. Relative bias of the objective value in LP-problem with dead and decaying wood as objective and stand growth as constraint, with different assumptions of the correlation of prediction errors among the treatments alternatives of one stand.

value of growing stock, compared to the overestimation of net income, is due to the larger weight of net income in the utility function.

The expected loss (Eq. 7) generally had smaller absolute values than the bias. In the aforementioned problems with constrained approach, the expected loss in net income varied from  $1.0\%$  to  $1.3\%$ . Using an unconstrained approach, the expected loss varied from  $0.4\%$  to  $0.5\%$  of the utility function value. In the latter case the loss, as well as the bias, was constructed from two (or three) variables, the prediction errors of which could be to opposite directions. Thus, the expected loss for each criterion variable could be much greater. For example, the expected loss in net income varied from  $-6.1\%$  to  $1.8\%$  (i.e. the net income was in some cases greater in the chosen than in the optimal solution). On the other hand, the expected loss in the value of the growing stock was  $3.5\%$ – $13.4\%$ , which was more than could be compensated by the increased net income.

However, when the unconstrained approach was used with only one variable as an objective variable (the weight of this variable was 1 in the utility function), the observed biases and losses had similar (or smaller) absolute values than those in the constrained case (Fig. 4). Thus, the greater absolute values of bias for the criterion variables in the unconstrained case are due to combining the criterion variables, not to inferior performance of the approach as such.



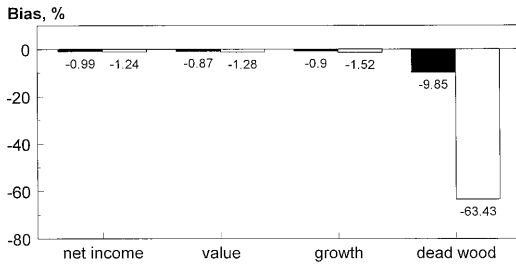


Fig. 4. Relative bias of the objective function value with four different objective variables in the utility function, assuming independent □ or perfectly correlated ■ prediction errors (in logarithmic scale) among treatments.

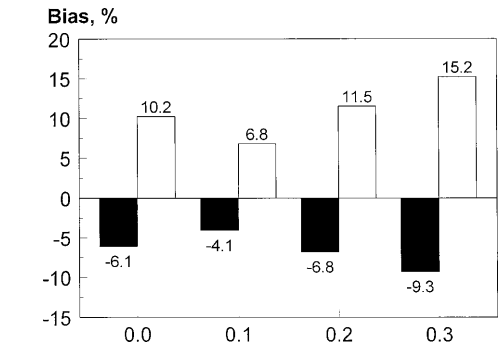
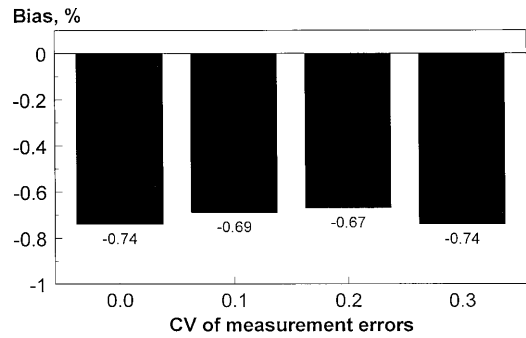


Fig. 6. Effect of measurement errors with different CV on the relative bias of (a) utility function value and (b) the values of the criterion variables, with value of net income ■ and value of growing stock □ as objectives. In each case, the prediction errors are assumed independent errors among treatments.

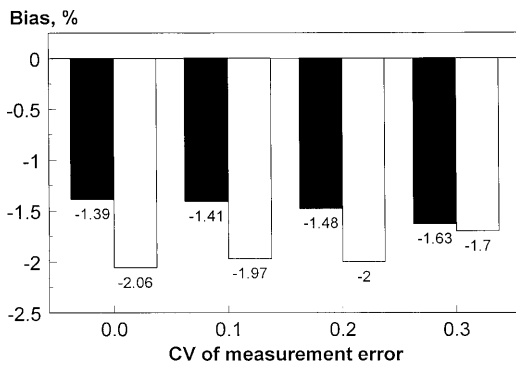


Fig. 5. Effect of measurement errors with different CV (coefficient of variation) on the relative bias of the objective function value, with growth as objective and value of growing stock as constraint ■, and with value of growing stock as objective and net income as constraint □. In each case, the prediction errors are assumed independent errors among treatments.

### 3.2 The Effect of Additive Measurement Errors on the Observed Objective Function Value

The effect of additive measurement errors of the stand variables (mean diameter and basal area), from which the original growing stock was predicted, was quite small. In some cases, the measurement errors increased the absolute value of the optimization bias (growth as an objective variable and growing stock as a constraint), and in some cases they reduced it (growing stock as an objective variable, net income as a constraint) (Fig. 5).

When an unconstrained approach was used for the problem with net income and value of growing stock as criterion variables, increasing the CV of measurement errors had a very small effect on the bias of the utility function value (Fig. 6a). On the other hand, the effect of the measurement errors on the biases of the two criterion variables was notable (Fig. 6b). This can be explained by the interactions between the criterion variables in the simulation. For example, overestimation of the basal area may lead to underestimation of the growth. Then, an underestimation of one variable in the optimal solution can be compensated for by an overestimation in the other.

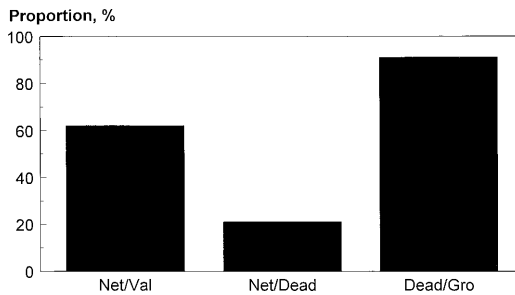


Fig. 7. Proportion of feasible solutions in three LP-problems with net income as objective and value of growing stock as constraint, with net income as objective and volume of dead and decaying wood as constraint and with volume of dead wood as an objective and stand growth as a constraint. In each case, the prediction errors are assumed independent errors among treatments.

### 3.3 The Effect of Prediction Errors on the Feasibility of the Obtained Solution

The variance of the prediction errors of the constraint variable had a notable effect on the proportion of the LP-solutions that were truly feasible. The biggest proportion of feasible solutions was obtained with the amount of dead wood as an objective and stand growth as a constraint, the constraint was met in 90 % of the realizations. When the net income was set as an objective variable and the growing stock as a constraint, the constraint was met in 62 % of the realizations. If the amount of dead wood was set as a constraint, the constraint was met only in 21 % of the realizations. In each case, independent prediction errors among the treatments were assumed. (Fig. 7). If both the growing stock and the amount of dead wood were used as constraints, the dead wood constraint was feasible in 56 % of realizations. This is due to the fact that the constraint variables are positively correlated and the growing stock constraint was more restrictive, thus the dead wood constraint was met when the other constraint was also met.

## 4 Discussion

Tactical forestry planning is a complex planning problem. There may be tens of alternative management schedules for each stand, amounting to thousands of possible solutions at the forest holding level. The number of possible criterion variables may be several hundreds. Predictions concerning the future development of these variables, under alternative choices of action, contain much uncertainty. Taking the uncertainty in these variables into account makes planning even more complex. The effects of uncertainty on the optimal solution are twofold: first, the true value of the optimal solution may be overestimated (optimization bias) and second, the realized value of the chosen solution may be smaller than that of the optimal solution (expected loss). In constrained optimization case, the solution may also be infeasible.

So far, many simplifying assumptions have been made in studies examining the effects of uncertainty on the choice of a forest plan. Often, the errors in the criterion variables are assumed to be additive, mutually independent and normally distributed (e.g. Pickens et al. 1991). Since the errors may be from several sources with different characteristics, such assumption may simplify the situation too much. In this study, the effects of two different sources of error, the correlation structure of the prediction errors and the relations between the objective variables in the optimization problem were considered. The Monte Carlo simulation with treewise growth and yield models requires much computer resources, as does the optimization algorithm. Thus, the computations made in this study only included twelve stands. In spite of the small number of stands, the results indicate clearly the effect of prediction variance and correlation assumptions on the planning problems.

The correlation among the errors of treatment alternatives proved to be important: the larger the correlation among the treatments, the smaller the absolute value of bias. The effect of multiplicative errors was greater than that of additive errors. Since the measurement errors were assumed to be additive and perfectly correlated among treatment alternatives, their effect on the observed utility function value was negligible.

Increasing CV of measurement errors in stand variables could in some cases even lead to decreasing absolute value of bias in the optimization. The prediction errors were assumed multiplicative and thus, the effect of prediction errors proved to be greater. If also the measurement errors were assumed multiplicative, the results would have been different.

Yet, increasing the CV of measurement errors increased the absolute value of the biases for different criterion variables. The phenomenon is due to the interactions of the variables in the simulation system, and the possibility of compensating the loss in one criterion variable by a gain in the other. Although our simulation system consisted of only six models, the effects of interactions were surprising. With a more complicated system, for example with the natural regeneration and development of plants included, the effects of interactions could be difficult to understand.

The constrained approach and the unconstrained approach behaved similarly in many respects. The unconstrained approach with additive utility function seemed to be more vulnerable to errors, as the underestimation of one criterion variable can be fully compensated by a larger overestimation in the others. The effect of uncertainty on the utility function value may be small, but the biases of the criterion variables may be considerable at the same time. However, this holds true also for LP-problem, if the objective function is a combination of two or more variables.

In the studied case, the utility function was obtained from the dual solution of the LP-problem. Generally, the utility function may be a nonlinear function of the criterion variables (e.g. Kangas et al. 1998). In such case, the effect of uncertainty on the observed utility function value may be more profound, similarly as the effect of measurement errors in independent variables depends on the curvature of the models (Gertner 1991, Ducey and Larson 1999). This aspect remains to be studied.

It was assumed that the true expected values of the yields of forest variables were known so that the expected values of the optimization bias and loss could be estimated. However, in real applications of optimization we only have an un-

biased estimate of the forest yields available. In such case, estimates of the bias and loss could be obtained with the approach of Pickens et al. (1991). Only the simulation of the LP-matrix coefficients containing errors would be more complicated. In the best case, the optimization bias is taken into account in the estimates given to the decision maker, as well as the probability of the constraints being met. Such an approach, however, requires accurate information about the error sources, the level of uncertainty, and the correlation structure of errors.

If the uncertainty is not taken into account in the results given to the decision maker, it is especially important to give the decision maker understandable information about the bias and loss that are due to the nature of optimization procedure (see also Pickens and Dress 1988). Even simplified considerations of uncertainty are valuable. For example, private forest owners are mostly persons who have no or only little prior knowledge about planning or optimization. These decision makers may rely too much on seemingly accurate optimization results, if they are not enlightened about the uncertainty. Then, the errors inevitably taking place in reality may lead to disappointment on planning in general. Especially, the information should be given to the expert planners, who could then advise the decision makers in the decision making process. For example, using input variables as constraints is more reliable than using output variables (Pickens and Dress 1988).

Management of uncertainty in optimization calculations was not included in this study. However, different decision-makers have different attitudes towards risk and uncertainty. Consequently, they follow different decision rules. When risk and uncertainty are involved in decision alternatives, rational decision-makers with different risk taking behavior choose different alternatives (e.g., Pukkala and Kangas 1996). There are several ways to operationalise risks and risk attitudes in optimization and in investment calculations. For example, the well known Capital Asset Pricing Model (CAPM) has been applied in forestry risk management. According to CAPM, the expected return of an asset should be risk-free plus the case-wisely determined risk adjustment (the risk adjustment is the total amount

of risk multiplied by "the price" of risk) (e.g., Varian 1990). In numerical optimization, decision rules reflecting the attitude towards risk, often also called as decision criteria, can be applied. Then, the risks involved in decision alternatives might be described, e.g., by distributions of possible outcomes for each alternative.

The variables dealt with in this study were wood-production-oriented ones, although some of them have interest also from the ecological viewpoint. The greatest uncertainty among these variables was involved in the estimates of the volume of dead wood, which is not an important variable in pure timber management planning. The measures related to non-wood forest products and benefits can be more complicated than wood-production-oriented ones. They are often constructed as indirect indices, containing different combinations of variables describing the forest. Obviously, such indices may include remarkable uncertainty.

In this study, all the errors were assumed to be unbiased. However, it is possible that the growth models over- or underestimate the reaction of tree growth to treatments. Such biases may be due to model misspecification, but they can also be due to the measurement errors in the data set from which the parameters of the models are estimated (e.g. Kangas 1998). The more the effect of any one treatment is overestimated, the more frequently a planning system recommends such a treatment. This kind of bias may further complicate the problem.

The time horizon used in this study was 20 years, which is a time horizon rather commonly used in forest management planning. With longer time horizon, further complications are involved. The longer the time horizon, the more uncertainty the predictions contain (Kangas 1997). In addition, the longer the time horizon, the more complicated the error structures, due to the increasing number of treatments in each management schedule. Consequently, the results of optimization should not be interpreted as forecasts of future development, in particular concerning results related to objective variables and to a long time period. More appropriate forecasts could be obtained from models or systems estimated for such use.

In this study, the uncertainty considered was

random variation. In order to produce practical decision support considering uncertainty involved in planning process as a whole, also other types of uncertainty, such as ignorance or ambiguity, should be considered. For example, the decision-makers may not be able to express their preferences exactly. In such a case, uncertainties in the problem formulation can successively be reduced by applying interactive optimization procedures (e.g. Kangas et al. 1996), or fuzzy optimization (e.g. Mendoza et al. 1993).

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