ON THE PRECISION OF SOME METHODS OF FOREST INVENTORY

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SELOSTE:
ERÄIDEN METSÄNARVIOIMISMENETELMIEN TARKKUUDESTA

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Foreword

This paper is concerned with studies of methods of forest inventory made at the Institute of Forest Mensuration and Management, University of Helsinki. The project has been financed in part by a grant from the United States Department of Agriculture, Agricultural Research Service.

The three authors acted in close cooperation during the course of the project by engaging in detailed discussion of various analyses. NYYSSÖNEN was responsible for the overall planning and arrangement of the work effected both in the field and in the office, and wrote the paper in its final form. The junior authors, KILKKI and MIKKOLA, carried out the calculations with electronic computers; MIKKOLA was also the advisor in some statistical problems, and wrote section 61 after preparing a thesis on the subject. KILKKI also took part in field work as a survey crew leader, and wrote drafts for some parts of the report, especially for those concerning regression analyses.

The different forms of assistance provided by a number of persons are greatly appreciated. On the suggestion of Dr F. C. HUMMEL, material measured at the Forest Research Institute of Mexico was made available by permission of Mr R. VILLASENOR ANGELES, Director General. Discussions with Professor Dr BERTIL MATÉRN (Swedish School of Forestry), Dr LARS STRAND (Forest Research Institute of Norway), and several officials of the U.S. Forest Service, along with the comments received from them, have certainly contributed to the avoidance of errors. Mr TERO OLLILA, B. F., assisted in field and calculation work during the first year of study, Mr C. E. M. KEIL, M. F., made several suggestions after reviewing the manuscript, Mrs M. MÄÄTTÄ and Mr L. E. REHNSTRÖM took part in various phases of office work, and the figures were drawn by Mrs I. NYLANDER. The English text was checked by Mr F. A. FEWSTER.

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Contents

1. Introduction

Information on forest inventory methods has increased substantially during recent decades. The advantages of using strata have been studied, the variable (relascope) plot method has been invented and developed, and other advances have been made. Thus a larger variety of methods than before is available if it is necessary to select the procedure for a given task.

Against this, the need for more intensive management planning, and other tasks based upon forest inventory, impose great demands as regards the knowledge of sampling methods. We need to know which is the most efficient method, that is, which method will supply the necessary information at the lowest cost, or alternatively, the best information at a given cost.

When the present situation is faced from this point of view, it becomes clear that a great deal of research still needs to be done. Although theoretical studies have been made, and practical surveys carried out, the lack of studies based upon empirical material in particular leaves many important questions open. This is exemplified, for instance, by the varying practices concerned with the size of sample plots (cf. NYYSSÖNEN and VUOKILA 1963).

The purpose of the present study is that of providing a basis for the establishment of optimum field work procedures in a forest inventory, by making a comparison of the precision of alternative methods. At this stage, the main aim is that of discovering the number of sample plots of different types needed for given levels of precision. Instead of making various types of forest survey to this end, the work plan included detailed studies of the variation in the growing stock. Treatment of the material leads to the comparison of different principles of sampling, a discussion of error calculations, and other related aspects of sampling.

2. Test areas

The study material was collected from five areas.

Area 1, the principal area of study, is called *Evo*, situated 61°15′ N.lat. and 25°10′ E.long. It comprises 100 hectares of forest land. The main tree species is Norway spruce, but the Scotch pine is also important; in addition, broadleaved species, particularly birch, are represented. The mean cubic volume, solid measure, is 151 cu.m./ha. The number of trees averages 688/ha. and the mean diameter, weighted by basal areas, is equivalent to 24.9 cm.

The area was divided into squares of 10 metres by 10 metres; thus the total sample amounted to 10 000 plots of 100 square metres each. In every square, the D.B.H. of all the trees exceeding 10 cm. was measured; the relascope was applied to check the ocular estimation of smaller trees. In addition, variable (relascope) plots were measured at every 20 metres on lines 100 m. apart in both N-S and E-W directions, which made the total number of 860 plots. The BAF (basal area factor) used was 4 for sq.m./ha. (equals 17.424 for sq.ft./acre). All the uncertain borderline trees were checked by caliper and measuring tape. All the trees counted on these variable plots became simultaneously the sample trees for use in volume calculations both on these and fixed-area plots. This meant that the height and taper of about 3 400 trees were measured to permit of finding unit volumes by D.B.H. classes from ILVESSALO's (1947) tables.

Stands were classified, for example, by site quality and their phase of development. As regards the former, use was made of forest site types, and in the latter, treatment classes. The following treatment classes, applied also within Areas 2—4, were distinguished:

0. Open areas and seed-tree stands. Basal area of seed trees or standards not more than 4 sq.m./ha. in general.
1. Seedling and sapling stands. In general, no merchantable timber obtainable as yet. Standards can be present (symbol 1y).
2. Stands in the thinning stage. In cuttings, as a rule the main product is pulpwood.
3. Stands in the preparatory cutting stage. In addition to pulpwood, cuttings generally yield saw-timber.
4. Mature stands to be regenerated. The main purpose of treatment is regeneration.
5. Shelterwood stands.
6. Low-yielding stands. For various reasons (inadequate growing stock, unsuitable tree species for the site, uneconomic tree species, overmaturity, etc.) in need of immediate regeneration.

The dominating treatment class in Evo is No. 4, comprising 40 per cent of the area. The more precise proportions of different treatment classes will be given later.

Fig. 1 presents the stand map of Evo; the quarter areas used in analysing the material have also been indicated in the figure. Fig. 1 is augmented by Table 1, which indicates the treatment classes of stands. Fig. 2 indicates the cubic volumes in the 100 sq.m. sample plots of Evo [1].

Area 2 is *Toivala*, situated 63° N.lat. and 27°40′ E.long. The area is somewhat over 400 ha. Again, the main tree species is Norway spruce, but in addition, birch and other broadleaved trees, along with the pine, are also important. The mean cubic volume on forested lands is 103 cu.m./ha., the number of trees,

[1] The material is available with 3 digits on punched cards at the Institute of Forest Mensuration and Management, University of Helsinki.

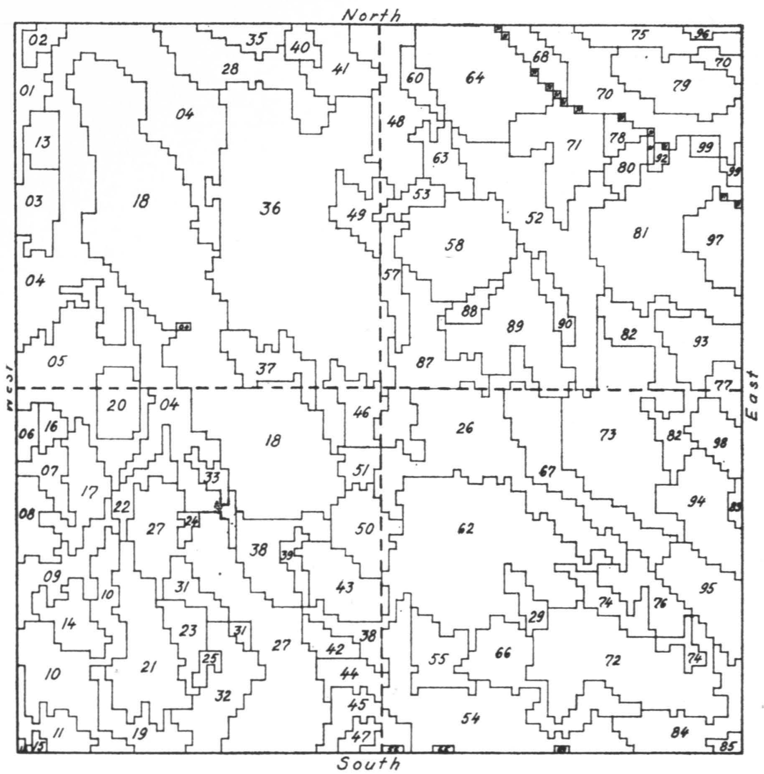

Fig. 1. Stand map of Area 1, Evo. The four sub-areas are indicated by broken lines. Scale about 1: 10 000. Cf. Table 1.

1107/ha., and the mean diameter, 22.5 cm. The dominating treatment class is No. 3, which covers more than a half of the area.

The survey pattern for this area consisted of equidistant lines put at right angles to each other, one set of them following the topographical configuration of the terrain. The distance between lines was 120 m. Fig. 3 shows the location and numbering of the plots in Toivala; the site of each plot was indicated by a system of coordinates. The number of survey lines was 13 in the N-S direction, and 25 in the E-W direction; the total number of plots on the forest land was 7 000. The size of the sample plot was 100 sq.m.

The D.B.H. of all the trees over 10 cm. was measured on these whole areas, and that of the trees between 2 and 10 cm. on an area of 25 sq.m. in one corner of each plot. Variable plots were measured in the centres of the line intersections. The BAF was 1 for sq.m./ha. (4.356 for sq.ft./acre), and another count was taken with BAF 2. It was assumed that all the uncertain trees were checked by caliper and measuring tape. In alternate variable plots, the height and taper of all the trees counted were measured for employment in calculation of the

Table 1. Treatment classes in Area 1, Evo. Cf. Fig. 1 and p. 6.

Stand No.	Class No.	Stand No.	Class No.	Stand No.	Class No.	Stand No.	Class No.
01	3	25	2	51	6	77	ly
02	ly	26	4	52	3	78	4
03	3	27	1	53	4	79	ly
04	4	28	4	54	3	80	3
05	3	29	4	55	3	81	4
06	3	31	2	56	1	82	3
07	4	32	6	57	2	83	3
08	1	33	4	58	1	84	3
09	3	35	3	60	4	85	3
10	2	36	4	62	4	87	5
11	4	37	4	63	0	88	4
12	0	38	4	64	1	89	4
13	6	39	3	65	3	90	5
14	3	40	5	66	4	91	non-forest
15	4	41	3	67	4	92	non-forest
16	1	42	3	68	1	93	2
17	6	43	1	69	4	94	4
18	ly	44	2	70	3	95	0
19	6	45	3	71	3	96	1
20	3	46	3	72	3	97	4
21	1	47	4	73	4	98	2
22	2	48	3	74	4	99	4
23	2	49	1	75	2		
24	2	50	2	76	3		

volumes. The total number of the sample trees on Area 2 and Area 3 combined was about 3 400.

The borderlines between various stands were drawn on the map. Fig. 4 shows the stands. During the treatment of the material, the whole area was divided into four equal parts by three E-W lines.

Area 3 is *Meltaus*, located 67° N.lat. and 25°20′ E.long. It is about 900 ha. in size. By far the most important tree species is Scotch pine; broadleaved species and spruce are significant only in occasional spots. The mean cubic volume on forested lands is 77 cu.m./ha., the number of trees 1095/ha., and the mean diameter, 18.5 cm. The most important treatment class is No. 3, covering 38 % of the area.

Squares were located in Meltaus in the same basic way as Toivala, but they were not measured along continuous lines. Rather, clusters of 9 squares were placed at 144-metre distances. In Meltaus, 16 survey lines were run in the E-W direction and 28 lines in the N-S direction. The total number of plots on forested land was 4 500.

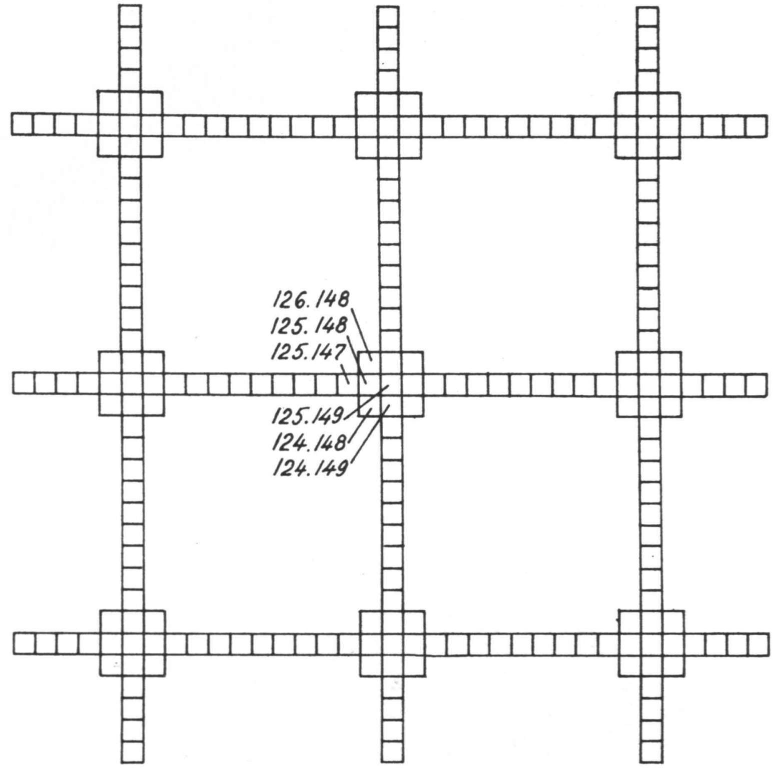

Fig. 3. Location and numbering of sample plots in Area 2, Toivala.

The size of the basic plot was 144 sq.m. (12 m. by 12 m.) in Meltaus; the trees between 2 and 10 cm. were counted on an area of 25 sq.m. A variable plot, using BAF 1, was measured in the centre of each cluster of 9 squares.

Fig. 5 is the stand map of the Meltaus area.

Area 4, here called *Ruotsinkylä*, comprises two 10 ha. woodlots a short distance apart in the south of Finland. The data are based on a 100 per cent count of trees separately by plots, each of 49 sq.m. (7 m. by 7 m.). Thus the two areas include a total of approximately 4 000 plots. The experimental material from this area has been more closely described in the study published by Nyyssönen and Vuokila (1963). In the present investigation, this material has been re-used, but for no more than certain preliminary calculations and comparisons.

Area 5 is an experimental area located in *Durango*, Mexico, at approximately 24° N.lat., 105° W.long., and of 108.8 ha. The measurements were made by the Forest Research Institute of Mexico. The main tree species are pines, which exceed 80 per cent of the total cubic volume. In addition, there are some oaks, firs, and other species. The mean cubic volume is 116 cu.m./ha., and the number

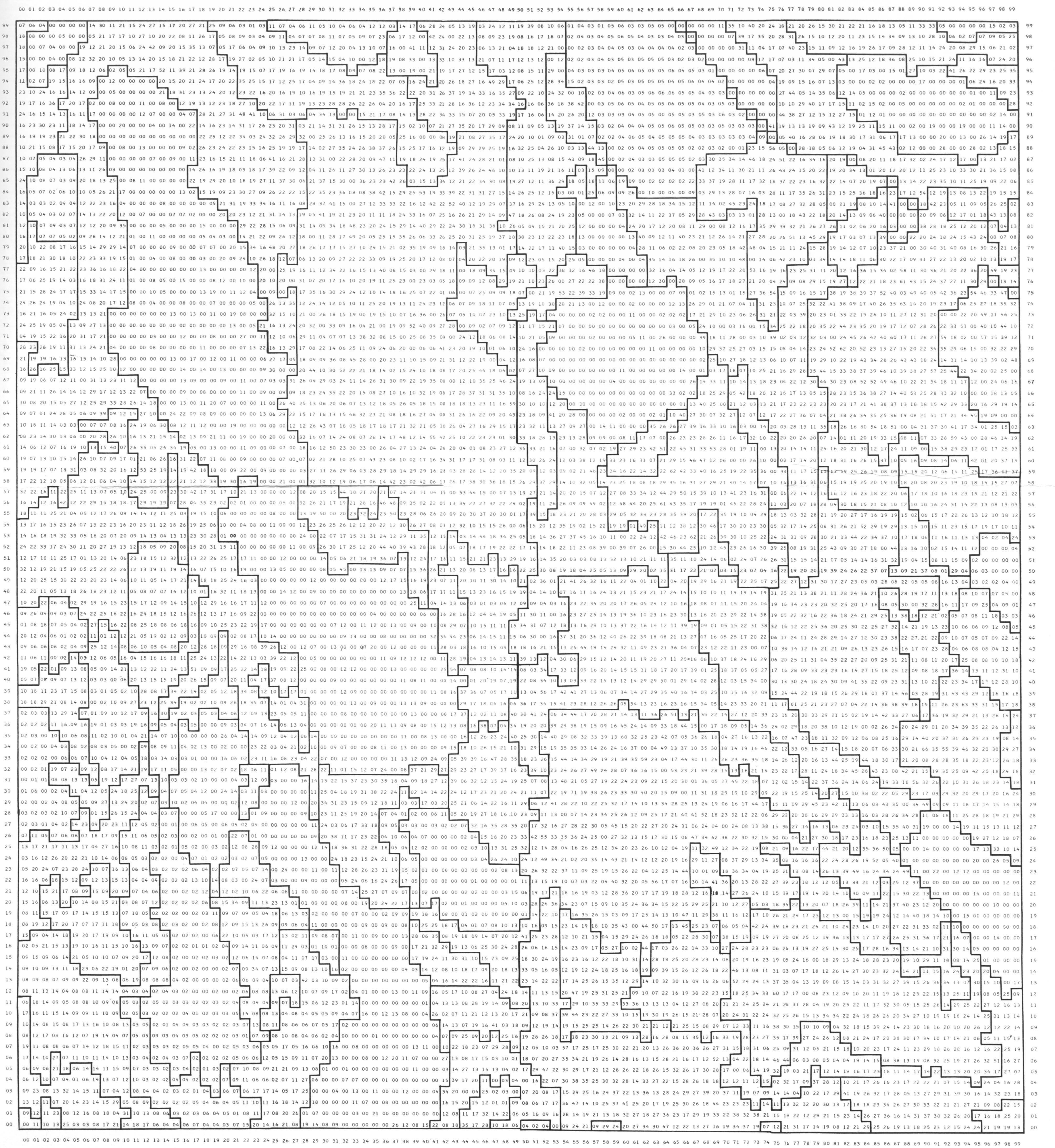

Fig. 2. Growing stock volumes in 10 cu.m./ha. on the 100 sq.m. plots of Area 1, Evo.

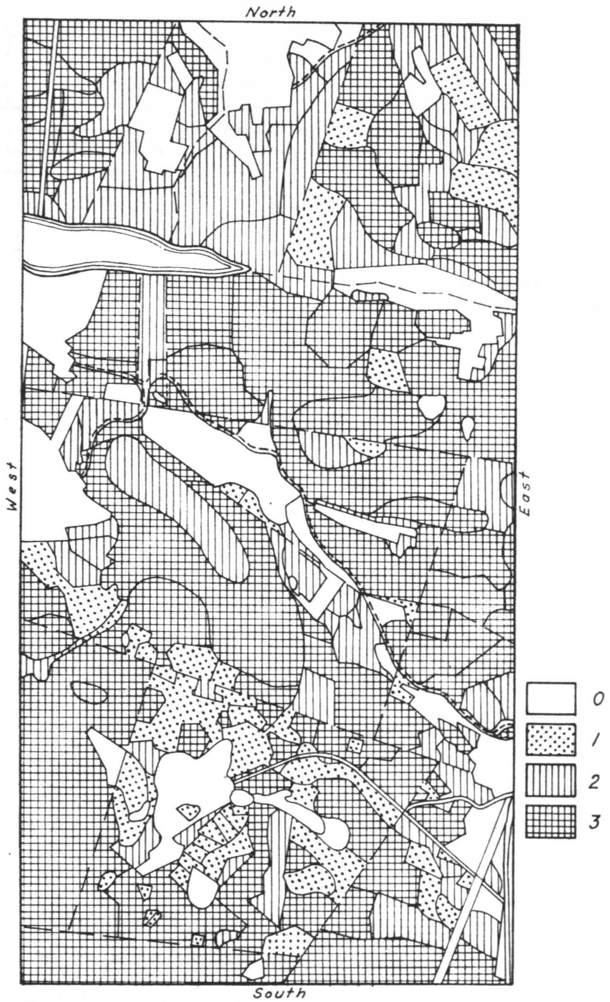

Fig. 4. Stand map of Area 2, Toivala. Scale 1: 20 000.

Stratum 0 = non-forest
» 1 = treatment classes 0 and 1
» 2 = » » 2 and 6
» 3 = » » 3, 4 and 5

of trees 212/ha.; the timber is thus of rather larger dimensions than the tree sizes in the other areas of the present study.

The area was measured in squares of 1 600 sq.m., 40 m. by 40 m. in dimensions, totalling in all 680 sample plots. In addition, as is indicated by the arrangement in Fig. 6, several different size and type of plots were measured at 98 locations.

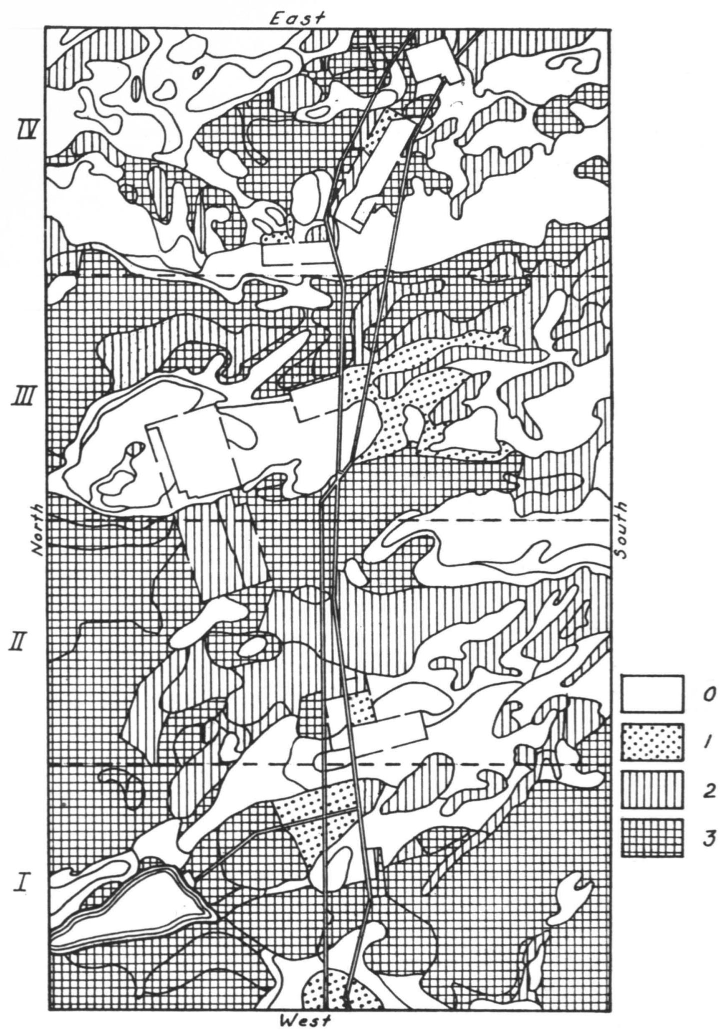

Fig. 5. Stand map of Area 3, Meltaus. Scale 1: 30 000. The Roman numbers at left refer to quarter-areas.

Stratum 0 = non-forest
» 1 = treatment classes 0 and 1
» 2 = » » 2 and 6
» 3 = » » 3, 4 and 5

NORTH

WEST / EAST

	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	
34	77	191	183	172	111	101	80	187	181	163	138	42	86	134	95	56	31	118	133	118	34
33	101	203	172	118	126	96	114	98	161	114	89	114	87	131	122	74	28	25	113	151	33
32	78	176	136	39	122	141	99	106	78	113	63	56	58	71	86	106	132	40	24	199	32
31	88	137	136	248	152	139	154	68	130	101	112	92	81	95	80	102	108	103	70	35	31
30	96	102	100	117	125	119	96	87	99	63	72	116	46	72	204	108	22	89	79	76	30
29	77	76	92	158	55	133	102	104	152	142	80	30	9	-	12	14	-	5	59	58	29
28	100	46	35	76	41	102	58	54	157	75	1	26	-	-	-	-	6	-	-	53	28
27	52	36	111	74	74	32	69	62	88	16	121	78	44	-	-	8	26	18	-	-	27
26	100	113	86	88	56	30	58	23	-	9	102	74	36	15	24	10	130	94	132	49	26
25	137	62	52	129	42	-	1	42	29	14	188	80	187	124	75	56	96	135	109	64	25
24	111	61	57	102	94	15	6	146	66	32	90	89	89	99	84	76	119	53	73	61	24
23	76	59	56	38	-	9	47	48	65	79	106	63	80	56	56	114	52	78	64	47	23
22	56	115	134	41	18	2	-	56	166	125	89	79	131	47	78	62	86	59	52	64	22
21	26	9	-	-	5	40	31	14	59	89	77	47	110	76	92	83	53	19	28	81	21
20	24	43	42	15	41	23	102	69	86	84	51	50	115	118	114	141	69	79	38	62	20
19	26	111	88	88	70	98	116	144	96	55	37	141	114	102	77	88	101	114	72	64	19
18	12	154	60	39	67	61	68	91	162	34	28	69	56	58	39	138	158	91	28	39	18
17	18	134	87	44	49	71	81	48	119	28	29	81	74	100	103	106	96	82	92	98	17
16	18	96	141	218	206	126	111	141	158	139	123	181	179	46	148	137	143	162	112	150	16
15	31	140	99	112	107	76	114	106	89	148	111	122	166	147	69	140	122	207	167	106	15
14	112	82	221	102	147	54	52	107	109	128	171	194	172	102	185	92	136	139	197	106	14
13	192	172	202	98	69	33	74	80	92	136	201	124	309	110	140	135	164	98	167	135	13
12	176	108	130	159	149	128	133	126	147	177	228	164	73	53	218	137	102	66	144	75	12
11	114	152	168	119	84	129	102	177	121	75	90	122	77	24	43	135	124	98	133	47	11
10	76	125	121	71	154	114	103	163	84	78	88	125	134	86	34	129	98	148	111	108	10
09	130	112	114	146	129	143	274	263	305	236	221	138	192	158	110	171	284	126	144	152	09
08	124	198	189	188	101	121	175	174	228	126	154	96	182	93	128	134	104	106	135	159	08
07	102	122	198	159	127	139	261	256	250	245	230	171	159	216	102	130	149	161	124	111	07
06	144	172	150	174	218	136	191	216	228	161	208	153	256	197	114	224	158	172	116	66	06
05	144	132	177	150	148	162	186	173	278	283	137	168	143	224	149	126	148	151	194	162	05
04	154	317	419	305	226	216	226	156	168	281	166	302	344	202	249	194	198	162	124	229	04
03	181	257	141	181	203	169	244	168	233	156	161	241	213	269	173	201	246	157	187	192	03
02	258	116	272	244	151	192	134	159	144	369	157	248	267	128	181	310	196	170	140	204	02
01	168	180	179	189	179	133	216	268	298	281	239	252	289	227	262	228	116	170	168	200	01
	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	

SOUTH

Fig. 6. Growing stock volumes in cu.m./ha. on the 1 600 sq.m. plots of Area 5, Durango. Nine other types of plots were measured in each of the 98 locations marked in the figure.

3. Considerations in treatment of the material

31. Plot types to be compared

Within different research areas, the following plot types are compared:

Area 1, Evo

No. 1. 10 × 10 m. = 100 sq.m.
No. 2. and 3. 10 × 20 m. = 200 sq.m. rectangle in both directions
No. 4. 20 × 20 m. = 400 sq.m.
No. 5. 30 × 30 m. = 900 sq.m.
No. 6. Combination of two squares: trees exceeding 20 cm. on 900 sq.m. and those below 20 cm. in 100 sq.m. in the centre.
No. 7. Variable BAF 4.

Area 2, Toivala

No. 1. to 6. as in Evo
No. 7. Variable BAF 1.
No. 8. » BAF 2.

Area 3, Meltaus

No. 1. 12 × 12 m. = 144 sq.m.
No. 2. and 3. 12 × 24 m. = 288 sq.m. rectangle in both directions
No. 4. 24 × 24 m. = 576 sq.m.
No. 5. 36 × 36 m. = 1 296 sq.m.
No. 6. Combination of two squares: trees exceeding 20 cm. on 1 296 sq.m. and those below 20 cm. in 144 sq.m. in the centre.
No. 7. Variable BAF 1.

Area 4, Ruotsinkylä

No. 1. 7 × 7 m. = 49 sq.m.
No. 2. 7 × 14 m. = 98 sq.m.
No. 3. 14 × 14 m. = 196 sq.m.
No. 4. 14 × 28 m. = 392 sq.m.
No. 5. 28 × 28 m. = 784 sq.m.

Area 5, Durango

No. 1. 40 × 40 m. = 1 600 sq.m.
No. 2. Circular 500 sq.m. (r = 12.62 m.)
No. 3. » 1 000 » (r = 17.84 »)
No. 4. » 1 500 » (r = 21.85 »)
No. 5. Variable BAF 1
No. 6. » BAF 2
No. 7. » BAF 2.3
No. 8. » BAF 3
No. 9. » BAF 5
No.10. » BAF 6

32. Characteristics to be used

From the very beginning, it was recognized that the cubic volume is the most important characteristic to be estimated, and that in view of this, the development of methods should be carried out with primary emphasis on volume. It is correlated with many important characteristics, growth being one of them. For instance, the future allowable cut has been found to be highly dependent on the cubic volume (cf. SEIP 1964).

However, to avoid calculation work, basal area and not cubic volume was used as a characteristic in the first analyses of this study. The basal area had earlier been applied by some writers, for example by PRODAN (1958), who indicated that the coefficient of variation for the basal area (C_g) was about 80 per cent of the corresponding coefficient for the volume (C_v). GROSSMANN (1961, p. 328) found that on the average the basal area percentage was 88 of the volume coefficient (C_v). In this study, by taking 116 sample plots of 100 sq.m. along one line in Area 2, Toivala, the percentage was found to be 81. Meanwhile, material from Area 4, Ruotsinkylä, provided an opportunity for comparison of both characteristics in greater detail, for different plot sizes and treatment classes. The results are given in the following tabulation, which lists the ratios of C_g/C_v expressed as percentages.

Plot size in sq.m.	98	196	392	784
		Cg/Cv per cent		
Area 4, total	82	81	79	80
Treatment class 2	76	73	68	68
Treatment classes 4 and 5 combined	96	92	89	87

In this material, the average percentage is about 80, but the extremes are 68 and 96, and the percentage decreases with increasing plot size, and appreciably so among different classes of forest. This means that the plot numbers necessary for a standard error of say 5 per cent of the mean basal area may vary between 46 to 92 per cent of the corresponding number of plots in estimating the volume. Consequently, the basal area does not seem to provide an adequate criterion. For this reason it was decided, notwithstanding the additional cost involved, also to calculate the volumes of all the plots in the different types of material.

For all the plots, there were made available both the volume and basal area of all the trees and with a D.B.H. of more than 20 centimetres. From these, the entire volume in cu.m./ha. will be the main characteristic, the volume of trees exceeding 20 cm. D.B.H. being an additional characteristic. The latter proportion of growing stock is generally economically most important, as in the main it consists of saw-timber. At the same time, the separate treatment of a growing stock portion of this sort illustrates in more general terms the estimation of a certain part of the stock.

33. Reliability of plot volumes

From the aspect of the reliability of results, it is important to know the dependence which can be placed on the calculation of the cubic volumes of individual plots. In these calculations, different kinds of systematic and observation errors may occur by reason of careless work, such as incorrect determination of the area of squares, erroneous measurement of diameters and heights, equipment maladjustments, and so on. The effect of these sources of error has been minimized as much as possible by careful field work, but it is impossible at this point to ascertain the number of errors which may nevertheless have crept in. Two other possible sources of error should be mentioned, the effect of fittings employed in the calculation of plot volumes, and the reliability in defining perimeter trees of variable plots.

The source of error from fitting may originate in the necessary procedural simplifications. The starting point for computation of the numbers of different types of sample plots is the computation of plot volumes. In this study, where thousands of plots were in question, the computation had to be simplified, applying the same unit volumes by D.B.H. classes for each stratum on Areas 1—3. The common sample trees were thus measured for each of those areas by species and age-class groups (deriving 8 series of unit volumes, i.e., 4 for pine, 2 for spruce and 2 for hardwood species). However, this short-cut simultaneously introduces some plot volume adjustment which results in smaller variation. To determine the importance of this point, the volumes were calculated for the same plot types both by the use of adjusted volumes, and by taking stem volumes separately for each tree from volume tables. This test involved samples from Area 1 and Area 4.

Area 1, Evo: Eight hundred and sixty variable plots with BAF 4 in sq.m./ha. (17.424 in sq.ft./acre) gave an average 151.5 cu.m./ha. and C_V 65.2 per cent by the use of adjusted volumes by strata, and an average of 151.5 cu.m./ha. and C_V 67.5 per cent, when volumes were taken for each tree from volume tables.

Area 4, Ruotsinkylä: Eight hundred and seventy seven plots of 49 sq.m. each gave an average volume of 0.644 cu.m. per plot and a coefficient of variation of 68.1 per cent, when adjusted volumes by stands were used. On the application of volume tables for each single tree, the average per plot was 0.641 cu.m. and C_V 70.1 per cent.

Thus the use of adjusted volumes means a slight diminution in the coefficient of variation as compared with the application of volume tables for each single tree. It is noticeable that there exists an effect of fitting even in the use of tree volume tables. The error created is also affected to some extent by the size of the sample plot. However, in this investigation the results will be dealt with unchanged, that is as given by the calculations referred to above. Consequently, by this means the numbers of sample plots pertaining to certain standards of precision are somewhat reduced, but it does not exercise a major influence on the comparison of different plot types which forms one important aspect of the present study.

Next follows an assessment of the comparison of the mean cubic volumes of different plot types, with special reference to variable plots.

Table 2 gives the mean cubic volumes of growing stock by different plot types within Areas 2 and 3, Toivala and Meltaus. The numbers of comparable plots are 240 in Toivala and 230 in Meltaus. It can be remarked that the results from plots of fixed areas are very close to each other. Nonetheless, the variable plot method has resulted in a considerable underestimation of volume in these cases, despite the instruction given to field crews to make checks of doubtful border trees.

Table 2. Mean volumes of growing stock on different plot types.

Test area	Plot type	Entire stock	D.B.H.more than 20 cm
		cu.m./ha.	
Area 2, Toivala	100 sq.m.	103.5	76.3
	200 »	103.1	75.5
	400 »	104.1	76.6
	900 »	103.0	75.5
	BAF 1	98.2	66.9
	BAF 2	93.8	65.7
Area 3, Meltaus	144 sq.m.	75.7	41.6
	288 »	76.7	41.8
	576 »	76.5	41.5
	1 296 »	77.2	42.0
	BAF 1	68.4	39.1

The significance of differences can be judged from Table 3. The variable plots are compared with the largest sample plots of each cluster, in Toivala 900 sq.m. and in Meltaus 1 296 sq.m.; these are the best comparative objects for variable plots with BAF 1 in particular. Since $t_{.05}$ for 200 degrees of freedom is about 1.97, the t-values of Table 3 are higher. Thus the mean volumes of variable plots are significantly different from the plots with fixed areas.

On Area 1, Evo, the comparison was made between fixed area plots of 100 sq.m. and variable plots with BAF 4, in number totalling 860. The mean of the former was 154.12 cu.m./ha. and that of the latter 151.51 cu.m./ha., the difference thus being 2.61. Since $t = \bar{d}/s_{\bar{d}} = 1.052$, the difference is not significant. The standard error of difference ($s_{\bar{d}}$) has been calculated as above, based on the differences of sample plot pairs.

Table 3. Difference of mean volumes ($\bar{d}$) divided by the standard error of difference ($s_{\bar{d}}$); variable plots and fixed area plots (Toivala 900, Meltaus 1 296 sq.m.).

Test area	Growing stock	BAF	Number of plot pairs	$\bar{d}/s_{\bar{d}} = t$
Area 2, Toivala	entire	1	240	2.53
		2	212	4.81
	D.B.H. more than 20 cm.	1	117	3.33
		2	105	3.79
Area 3, Meltaus	entire	1	230	5.35
	D.B.H. more than 20 cm.	1	127	3.48

The mean volume of the Area 5, Durango, is 116 cu.m./ha. but the mean volume of the 98 sample plots (size 1 600 sq.m.) on which 9 other types of plots were measured (cf. p. 13) is 114.8 cu.m./ha. The following list indicates the mean volumes of different plot types:

No. 2	Circular		500 sq.m.	109.0 cu.m./ha.
No. 3	»		1 000 »	117.6 »
No. 4	»		1 500 »	123.0 »
No. 5	Variable	BAF	1	126.6 »
No. 6	»	»	2	144.1 »
No. 7	»	»	2.3	141.6 »
No. 8	»	»	3	133.1 »
No. 9	»	»	5	134.2 »
No. 10	»	»	6	127.9 »

Other plot types have in general resulted in higher mean volume estimates. The results obtained by variable plots Nos. 6 to 9, especially, are larger than those of the 1 600 sq.m. plots. It is not known whether border trees in Durango were checked or not.

Major differences were thus found in many instances between the fixed area and variable plots, for example in Toivala and Meltaus. The obligation to check border trees with caliper and measuring tape was included in the work instructions, but since the inclusion of a tree in a sample plot also brought about the measurement of a sample tree and thus some additional work, the crews may unintentionally have effected the underestimation on variable plots.

In an earlier comparative report on circular and variable plots Nyyssönen (1954) found differences, but these were not necessarily attributable to the erroneous results of the variable plot method. Vuokila (1959) and Mäkinen (1964) could not detect any major differences between the variable and fixed area plots in their respective comparisons.

At any rate, it can be concluded that there exists a possibility of systematic error in the variable plot method. The check of doubtful border trees must be arranged with full reliability, say by checking a certain constant percentage of trees on each plot. The need for checking is least on a small variable plot, that is on plots with a large BAF.

In consequence of the major underestimation of cubic volume on variable plots, the variation coefficients of plot volumes have in certain cases (cf. Table 7 p. 23) been corrected by the ratio of mean cubic volumes with a view to enhancing the comparability of variable and fixed area plots.

4. Variations in plot volume

The material can be further described, and a foundation laid for the application of sampling methods, by a presentation of the variation in growing stock apparent in different types of sample plots.

41. Total

The coefficient of variation, in percentage form, as a function of plot size on different areas is shown in Fig. 7. The coefficient was arrived at in the usual way, by division of the standard deviation of the population by the mean volume, and multiplication of the result by 100. The values of different plot types for each material have been connected by curves.

It is observable that the different compilations of material, despite divergences in size, are relatively close to each other as regards *the entire volume*, although the Meltaus material is lower than the rest. The curve which represents Area 5, Durango, is located higher than those of other areas, the number of trees being one of the explanatory reasons for the levels found. The result published by KUUSELA (1960) for pine stands in northernmost Finland, with the designation »Inari», fits in relatively well with the present findings. For the sake of comparison, the variation coefficients for average Norwegian forests, with the designation »Norway», published by SEIP (1964), have also been included; these are somewhat higher than the values now under examination. In addition, the coefficient of variation of 138.5 sq.m. plots varied by districts in an inventory made in the forests of a timber company (between 61° and 64° N.lat.) from 77 to 97 per cent (mean volume 69.5 to 87.5 cu.m./ha.), and in the forests of another company (about 64° and 65° N.lat.) from 111 to 154 per cent (mean volume as low as 52.3 to 65.0 cu.m./ha.).

The curves could also be shown as functions by the application of »FAIRFIELD SMITH's law» (1938), as was done by KINASHI (1953), who expressed his results in the equation (cf. STRAND 1957)

$$C_v = k A^{-b}$$

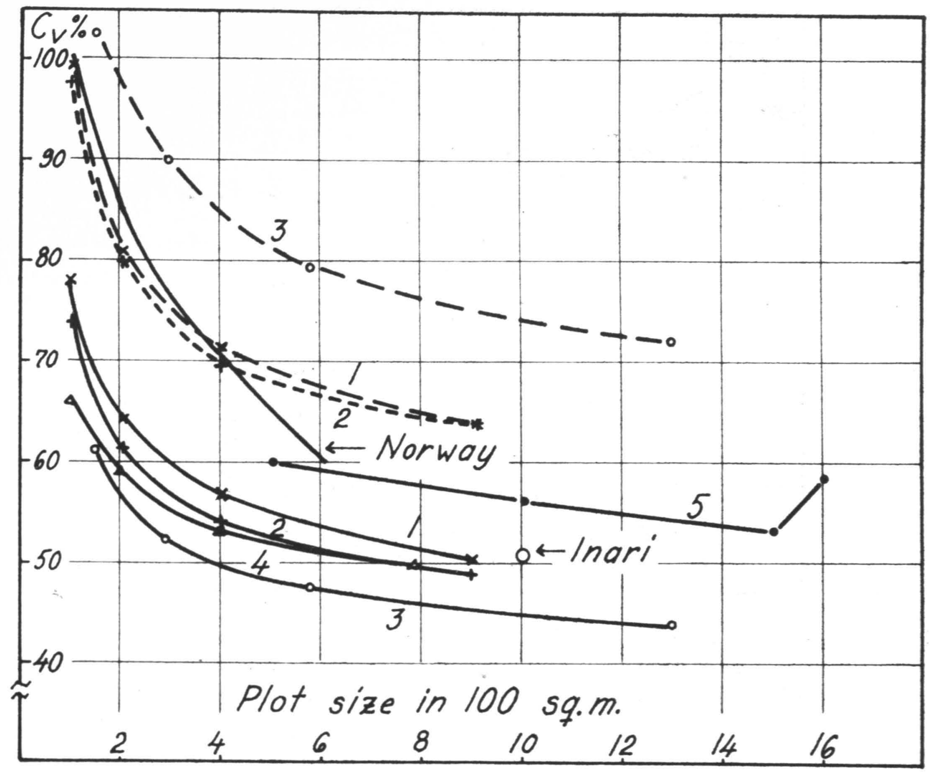

Fig. 7. Coefficient of variation as a percentage of the mean volume as a function of plot size on different areas. Solid lines: entire volume; broken lines: volume of trees exceeding 20 cm. D.B.H. The numbers refer to different study areas.

where C_v = coefficient of variation,
A = plot area,
k = constant,
b = coefficient with certain values.

In a survey of growing stock say by means of circular plots, a realistic alternative is that of counting large trees from a larger area than small ones. In this connection, there should be mentioned the sizes of sample plots in Fig. 7, which are equivalent as to variation with the plot combinations employed on different areas, where trees of less than 20 cm. in D.B.H. were tallied within a small area, and trees over this size within a comparatively large area (plot type 6 p. 13). In addition, it need be remembered that in each basic square the trees less than 10 cm. D.B.H. were estimated mainly ocularly in Area 1, and measured on an area of 25 sq.m. in Areas 2 and 3.

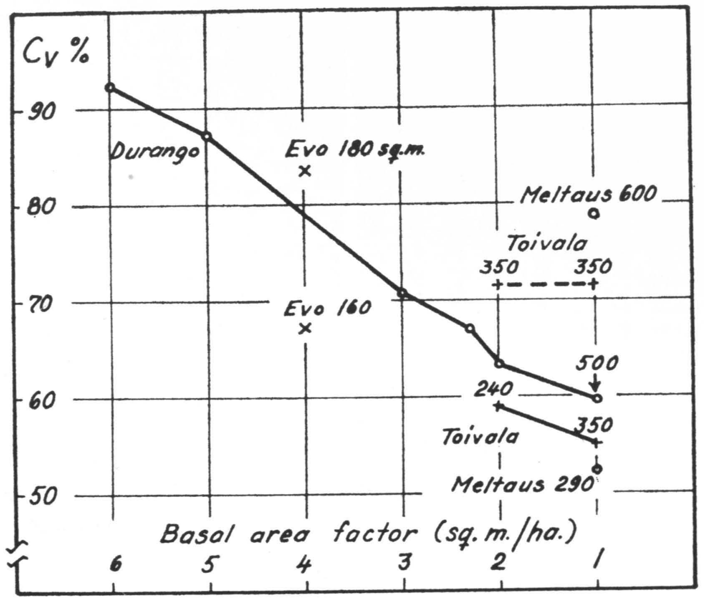

Fig. 8. Coefficient of variation as a function of BAF on different areas. Numbers give the sizes of fixed area plots equivalent as regards variation on each area. Above the Durango curve is shown the variation in volume of trees exceeding 20 cm. D.B.H.

Area	Plot combination sq.m.	C_v per cent	Plot size in sq.m. if all trees are measured
1	100 + 900	52.6	690
2	100 + 900	53.1	440
3	144 + 1 296	46.8	650

On examination of *the volume of trees exceeding 20 cm. D.B.H.* in Fig. 7, a greater relative variation is noticeable. Within Area 3, Meltaus, there is a large difference between the growing stock as a whole and the saw-timber part of it, but the proportion of saw-timber trees is there also least, about 55 per cent, against corresponding percentages within Area 1, Evo, of 80, and within Area 2, Toivala, of 73.

The coefficient of variation resulting from the study of variable plots is illustrated in Fig. 8. With each coefficient of variation there is a figure which indicates the size of fixed area plots corresponding to the respective relascope plot as regards variation.

For the entire volume, the BAF 1 plots correspond to fixed-area plots of 290 to 350 sq.m. in Toivala and Meltaus, and of 500 sq.m. for the larger-sized timber in Durango. BAF 2 is equivalent to 240 sq.m. in Toivala, BAF 4 to 160 sq.m. in Evo. On separate examination of the trees which exceed 20 cm.

D.B.H., it is observable that BAF 1 corresponds in Toivala to 350 sq.m., in Meltaus to as much as 600 sq.m., BAF 2 to 350 sq.m. in Toivala and BAF 4 to 180 sq.m. in Evo. Thus as regards the saw-timber certain variable plots correspond to somewhat larger fixed-area plots; this is rather natural.

42. By strata

The variation by strata is interesting, particularly from the aspect of stratified random sampling. In this sampling method, the units of the population are grouped on the basis of the similarity of some characteristic. Each group or stratum is then sampled, and the group estimates combined to give a population estimate. If the variation among units within strata is substantially less than the total variation, the population estimate will be more precise than if sampling had been effected at random throughout the entire population (c.f. FREESE 1962).

The study included experiments in several different ways of stratification; this task was connected with the separate study on photo interpretation.

The first method of stratification is based upon the use of treatment classes. The classification has been described previously (p. 6). Table 4 presents the proportions and mean volumes of treatment classes on Areas 1 to 3. In analysis of the material, the treatment classes have been combined as follows: 0 + 1 comprise Stratum 1; 2 + 6 Stratum 2 (these two treatment classes have been combined mainly by virtue of the similarity in their cubic volumes), and classes 3 + 4 + 5 Stratum 3. In addition, non-forest areas form Stratum 0. In Figs. 4 and 5 (pp. 10 and 11), the presentation of results was based upon this same grouping of material.

Table 4. Proportions and mean volumes in the various treatment classes.

Treatment class	Area 1, Evo		Area 2, Toivala		Area 3, Meltaus	
	Area percentage	cu.m./ha.	Area percentage	cu.m./ha.	Area percentage	cu.m./ha.
0	2	46	1	8	0	—
1	22	52	11	42	5	33
2	7	129	13	102	13	66
3	24	179	53	131	38	92
4	40	199	4	86	24	88
5	2	161	5	73	2	63
6	3	86	13	61	18	52

The other method of stratification employs the cubic volume. On different areas, three strata were used; the limits and proportions of strata are given in Table 5.

Table 5. Volume classes and their proportions.

Area 1, Evo		Area 2, Toivala		Area 3, Meltaus	
Class cu.m./ha.	Area percentage	Class cu.m./ha.	Area percentage	Class cu.m./ha.	Area percentage
10 and less	0	10 and less	4	10 and less	1
10.1 to 60	23	10.1 to 60	14	10.1 to 50	19
60.1 and more	77	60.1 and more	82	60.1 and more	80

The third method is based upon dominant height, and in more advanced stands on cubic volume as well. The classes and their proportions appear in Table 6.

Fig. 9 indicates the standard deviation of the growing stock as a whole and for the treatment-class strata on Area 1, Evo. For instance, for 900 sq.m. plots the central square has been decisive in determination of the stratum of the plot. It can be seen distinctly that the standard deviation by strata is less than that for the total area.

To get an idea of the average effect of stratification, the standard deviations of strata were weighted by their numbers of observations, to arrive at a mean denoted in Fig. 9 as »average within strata». Table 7 presents the effect of the different stratification methods described above on the reduction of standard deviation. Consequently Table 7 indicates the efficiency of stratification methods.

Initially, it can be concluded that no major differences exist between the various methods of stratification. Stratification has brought about increasing utility of larger plots, i.e. the reduction of standard deviation resulting from stratification is most pronounced for large plots. Where a forest area does not introduce classes clearly distinguishable, as exemplified by the Meltaus area, the importance of stratification is weaker.

Table 6. Classes based on dominant height and volume, and their proportions.

Area 1, Evo		Area 2, Toivala		Area 3, Meltaus	
Dominant height	Area percentage	Dominant height	Area percentage	Dominant height	Area percentage
0	0	0	4	0	0
1— 8 m.	2	1— 8 m.	11	1— 8 m.	2
9—16 m.	10	9—16 m.	13	9—16 m.	51
17 m. and more a)	17	17 m. and more a)	1	17 m. and more c)	3
17 » » » b)	71	17 » » » b)	71	17 » » » d)	44

a) = 60 cu.m./ha. and less
b) = more than 60 cu.m./ha.
c) = 50 cu.m./ha. and less
d) = more than 50 cu.m./ha.

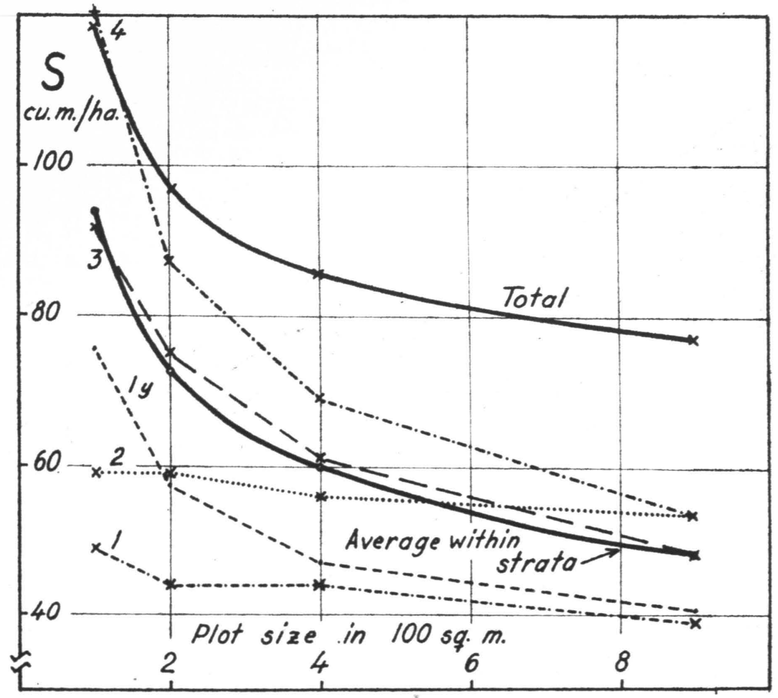

Fig. 9. Standard deviation (*s*) as a function of plot size in Area 1, Evo, for total and within strata. Mean volume of the growing stock 151 cu.m./ha.

Table 7. Effect of stratification on the standard deviation of the growing stock shown for both total and average within strata.

Test area	Plot type	Total	Average within strata: Treatment class	Average within strata: Volume	Average within strata: Height and volume
		Standard deviation, cu.m./ha.			
Area 1, Evo	100 sq.m.	117.8	93.7	98.8	96.9
151 cu.m./ha.	200 »	96.7	72.2	76.2	74.3
	400 »	85.7	59.4	63.1	61.0
	900 »	76.3	47.6	51.9	48.9
	BAF 4	101.2	75.1	79.3	77.4
Area 2, Toivala	100 sq.m.	76.2	64.8	64.7	64.4
103 cu.m./ha.	200 »	63.1	50.9	50.7	50.5
	400 »	55.5	41.6	40.4	42.7
	900 »	50.3	35.4	37.1	36.8
	BAF 2	62.1	49.8	50.3	50.1
	BAF 1	56.7	45.8	44.6	43.6
Area 3, Meltaus	144 sq.m.	47.2	43.3	41.9	44.5
77 cu.m./ha.	288 »	40.2	34.6	33.4	36.3
	576 »	36.5	30.4	29.6	32.8
	1 296 »	33.9	27.1	26.3	29.7
	BAF 1	40.4	33.5	33.9	34.8

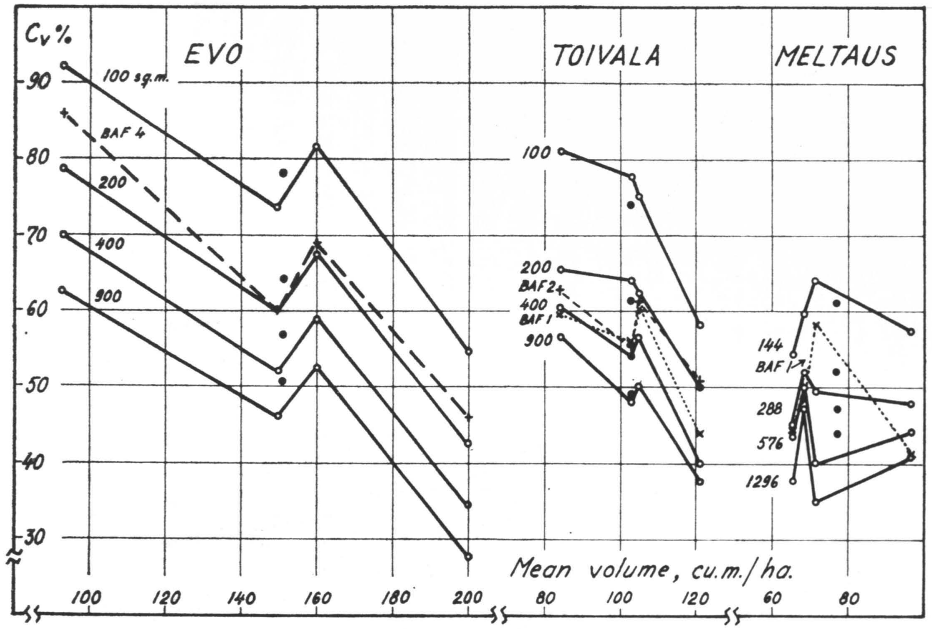

Fig. 10. Coefficient of variation as a function of the mean volumes of quarter-areas in Area 1, Evo. Circles with dots show C_v on each total area.

Combinations of treatment classes, to be dealt with later, and described earlier in this report but not included in Table 7, have yielded results approximately similar to those from the treatment classes taken individually. Thus the classification into three strata on the Evo area has meant a relatively efficient method of stratification.

In particular, Fig. 10 gives an idea of the effect exercised by mean volume on the relative variation. The various areas represent clearly differing levels, but in Evo and Toivala at least, the trend is this: the relative variation is reduced by the increasing mean volume in general. The effect of this conclusion is traceable in the later equations. Furthermore, it is noticeable that the variation within sub-areas is approximately of the same level as that within total areas, perhaps with the exclusion of Meltaus.

5. Comparison between different sampling methods

51. The methods to be compared

In this investigation, the comparisons to be made will concern the following methods in particular:

1. simple random sampling;
2. stratified random sampling;
3. simple uniform systematic plot sampling (plot distances the same in both directions);
4. stratified uniform systematic plot sampling
   a) stratum areas by planimeter from map,
   b) stratum areas measured on lines;
5. systematic strip sampling.

As a measure of the precision of survey, there is applied the relative standard error of the mean, obtained by dividing the standard error of the mean by the mean, and multiplying this by 100. For Methods 1 and 2, the standard error can be calculated by the application of generally known formulae, since the standard deviations dealt with earlier are available. For other sampling methods, empirical calculations must be made, since no general formulae of standard error are available. These calculations, a particular characteristic of the present study, will be described subsequently in greater detail than the other computations made.

52. Calculation of standard error

521. Simple random sampling

In simple random sampling without replacement, in which the same unit does not enter the sample more than once, the standard error has been calculated as follows:

$$s_{\bar{x}}^2 = \frac{s^2}{n}\left(1 - \frac{n}{N}\right) \tag{1}$$

where $s_{\bar{x}}$ = standard error of the mean
s = standard deviation of the population
n = number of units in the sample
N = total number of sampling units in the entire population.

522. Stratified random sampling

The following equation enables calculation of the standard error of the mean in stratified random sampling, presupposing that the stratum areas are free from error, i.e. based on complete delineation (e.g. Freese 1962):

$$s_{\bar{x}}^2 = \frac{1}{N^2}\sum_{h=1}^{L}\left[\frac{N_h^2 s_h^2}{n_h}\left(1 - \frac{n_h}{N_h}\right)\right] \tag{2}$$

where L = number of strata
N_h = total number of units of stratum h
n_h = number of units observed in stratum h
s_h = standard deviation of stratum h
$s_{\bar{x}}$, n and N are the same as above.

Proportional allocation, i.e. the division of sample plots among strata in relation to their superficial areas, entails that the ratio n_h/N_h is constant in all the strata. Consequently, the equation is simplified to:

$$s_{\bar{x}}^2 = \frac{N-n}{N n} \sum_{h=1}^{L} r_h s_h^2 \qquad (3)$$

where $r_h = N_h/N$ = relative size of stratum h.

523. Simple uniform systematic plot sampling

In random sampling, the standard error of the mean indicates the standard deviation of sample means. Also for systematic sampling the standard error can be calculated as a standard deviation of sample means. Thus, the basic equation for calculation of the standard error of the mean from systematic samples ($s_{\bar{x}\,sys}$) has here been:

$$s_{\bar{x}\,sys}^2 = \frac{\sum_{i=1}^{m} \bar{x}_i^2 - \frac{\left(\sum_{i=1}^{m} \bar{x}_i\right)^2}{m}}{m} \qquad (4)$$

where $\bar{x}_i$ = mean of sample i
m = number of all the possible samples taken; as the total population is the basis of calculations, m instead of $m-1$ has been used in the denominator.

The following is a description of the method of calculations made with a computer in Area 1, Evo.

The 100 sq.m. plot volumes form a 100×100 matrix. From this, several systematie samples of the same type can be taken by changing the starting point, and thus in this casc the variance of sample means can be computed exactly. A computer program for the IBM 1620 was devised to take the samples and perform the computations. By reason of the small memory capacity and low speed of the computer, the program had to be effected with a symbolic programming system instead of FORTRAN, and even then the plot volumes could be stored with only three numbers. The program is of such interpretive type that the following information on the sampling is given in a special parameter card (see Fig. 11).

A. To define the type of sample there must be given:
1. the type of sample plot; use was made here of the five types and notations given on p. 13,

2. the coordinates of the first plot in the sample, denoted by AX and AY,
3. the step lengths in both directions, denoted by SX 1 and SY 1,
4. the numbers of sampling units taken in a row in both directions, denoted by NX 1 and NY 1.

B. When different samples are taken by moving the starting point, there must be known:
1. the step lengths in both directions, SX 2 and SY 2,
2. the numbers of starting points used in a row in both directions, NX 2 and NY 2.

C. As the samples have been so located that every 100 sq.m. plot belongs to at most one sample of the given type, the number of samples is small when the sample plot is large. Accordingly the program was given the ability to repeat the steps mentioned in B by moving the starting point by smaller steps. Thus there must be given again:
1. the step lengths in both directions, SX 3 and SY 3,

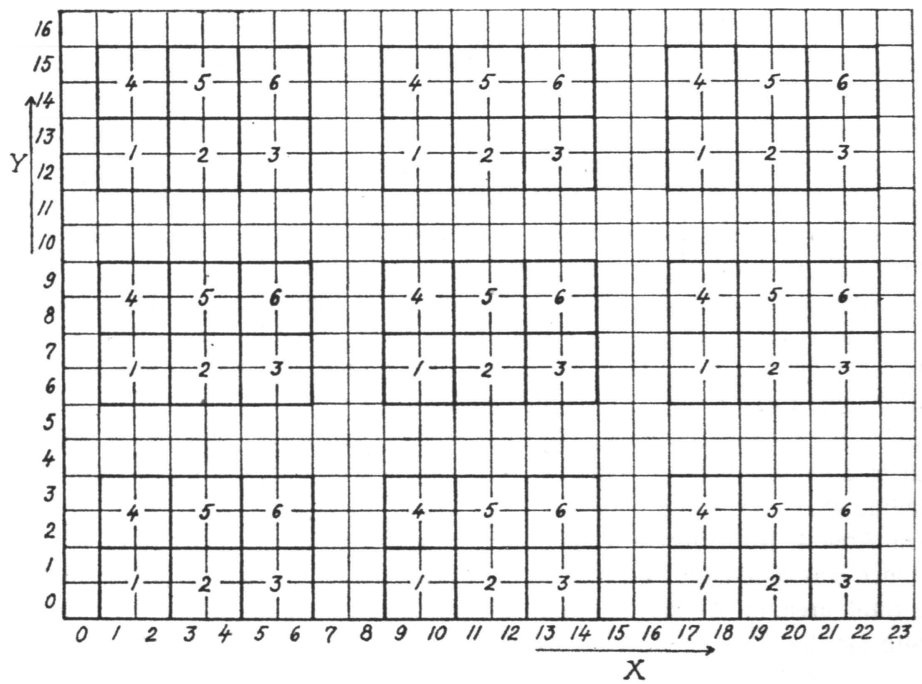

Fig. 11. An illustration of the coding used on parameter cards. For this hypothetical set of 6 samples we have:

Plot type = 4	
AX = 1	AY = 0
SX 1 = 8	SY 1 = 6
NX 1 = 3	NY 1 = 3
SX 2 = 2	SY 2 = 2
NX 2 = 3	NY 2 = 2

The plots belonging to one systematic sample have been denoted by the same number. If the foregoing, with new starting points (2.0), (1.1) and (2.1) is repeated, then

SX 3 = 1	SY 3 = 1
NX 3 = 2	NY 3 = 2

2. the number of starting points used in a row in both directions, NX 3 and NY 3.[1]

The program gives the following output on cards at the end of each section:

A. sample mean, sample standard deviation (computed as if the sample were random) and the coefficient of variation; this output can be suppressed by a program switch,

B. the mean of sample means, the standard deviation of sample means, and the corresponding coefficient of variation,

C. the means of the statistics given in B above.

In Evo, calculations were made for the entire volume, and for the volume of trees exceeding 20 cm. D.B.H. Systematic equidistant samples were taken, using plot types 1 to 5 (p. 13), and the distances between plots increasing by 10 m. intervals from 60 to 240 m., and then after that at 20 m. intervals up to 320 m. For plot types 1 to 3 (100 and 200 sq.m.), samples were taken at 40 m. intervals as well. One sample contained from 9 to 576 plots. For preference, the samples were taken from as large an area as possible in the middle of the 100 ha. area.

Sample means were employed for calculations of the standard deviations of the sample means, by the application of Equation (4).

As regards the entire volume, standard errors were also computed for quarter areas (cf. p. 6), use being made of the distances 60, 90 and 120 m. for plot types 1 to 5 and, in addition, 40 m. intervals for plot types 1 to 3.

On Area 2, Toivala, calculations were made for the entire volume, separately for the total area and for the 4 sub-areas of about 100 ha. each. The distance between plots was a constant 120 m., and the plot sizes were 100 and 200 sq.m. Within the total area, 27 samples were taken with 100-sq.m. plots, and 12 samples with 200-sq.m. plots; there were 250 plots in each sample. From each quarter area were taken 27 samples, 108 in all, of 100-sq.m. plots, and 12 samples, totalling 48, of 200-sq.m. plots.

On Area 4, Ruotsinkylä, calculations concerned the entire volume. The plot distance was a constant 84 m. On the average, each sample had 27.5 plots in the total area of 20 ha. The numbers of samples are stated in the following set-up for the plots of different sizes:

98 sq.m.	72 samples
196 »	36 »
392 »	18 »
784 »	9 »

On Area 5, Durango, calculations were made in respect of the entire volume. Here also, the sampling areas were located in the middle of the area proper. The distances between lines and plots varied from 120 to 280 m. increasing by 40-m. intervals. The number of plots in samples varied from 15 to 66, and the number of samples for different cases from 9 to 36.

[1] Instead of the procedure used, it would be better to move starting points with shorter steps and leave stage C out.

524. Stratified uniform systematic plot sampling

The stratification applied in taking systematic samples was the same as that in the corresponding random sampling. The areas of strata were arrived at in two ways. In the first alternative, there were used the stratum areas based on complete delineation; these were free from error as in Equations (2) and (3) of the stratified random sampling. In the second, the areas of strata were determined on the assumption of a survey being a line-plot survey, in which the areas of strata were obtained separately for each sample from the distribution of line lengths among strata. The mean volume of the strata was calculated as a mean of the plot volumes in each sample.

The material for study comprised the entire volume in the whole area of Evo. Samples were taken for plot types 1 to 5 (p. 13).

As regards the actual areas of strata, the variance of the mean was derived by means of the following equation:

$$s^2_{\bar{x}\,sys} = \frac{\sum_{i=1}^{m}\left(\sum_{h=1}^{L} r_h \bar{x}_{hi}\right)^2 - \frac{\left(\sum_{i=1}^{m}\sum_{h=1}^{L} r_h \bar{x}_{hi}\right)^2}{m}}{m} \qquad (5)$$

where $\bar{x}_{hi}$ = the mean volume of stratum h in sample i.

As regards the area proportions of strata determined by means of line lengths in connection with the sampling, the equation was as follows:

$$s^2_{\bar{x}\,sys} = \frac{\sum_{i=1}^{m}\left(\sum_{h=1}^{L} r_{hi} \bar{x}_{hi}\right)^2 - \frac{\left(\sum_{i=1}^{m}\sum_{h=1}^{L} r_{hi} \bar{x}_{hi}\right)^2}{m}}{m} \qquad (6)$$

where r_{hi} = relative size of stratum h in the sample i.

It should be noted that by taking the equal plot distances on the whole area, no independent determinations were made of the starting points within each stratum. Thus, the term stratified systematic employed here does not possess the same meaning as that applied by COCHRAN (1963, p. 227). The present system is easier in practical application.

525. Systematic strip sampling

On Area 1, Evo, systematic strip sampling was effected separately for the entire volume, and separately for the volume of trees exceeding 20 cm. D.B.H.

over the whole area, with the distances between strips increasing by 10-m. intervals from 40 to 330 m. Strips 10 m. in width, each constituting one sampling unit, were taken in both N-S and E-W directions. To get square areas for strip surveys, the length of strips equalled the product of the number of strips and the distance between strips. The standard error of the strip survey was calculated by application of the same equation as that relating to systematic plot sampling (No. 4, p. 26).

Within the quarter areas of Evo, strips were run at distances of 40-, 60-, 90-, 120- and 240-m. The survey related to the entire volume.

On Area 5, Durango, strips were 40 m. in width. The distances between strips increased at 40 m. intervals from 120 to 240 m. in the N-S direction, and from 40 to 440 m. in the E-W direction.

53. Discussion of the results of comparison

531. Plot sampling

In comparison of the precision of results based on different plot surveys, the same plot size and the same number of plots have always been applied. Furthermore, as a rule the same area is used for a better comparability of the results.

The comparative series for the entire volume in Area 1, Evo, includes simple random sampling, stratified random sampling, simple uniform systematic plot sampling, and stratified uniform systematic plot sampling in two ways. Plot types 1 to 5 (p. 13) are represented; both plot types of 200 sq.m. were combined by computation of the means of the pairs of standard errors. All the systematic surveys made with plot distances of 40, 60, 90, 120, 150, 180 and 240 m. are represented as is the simple uniform systematic plot sampling with distances of 260, 280, 300 and 320 m. By the application of Equations (1) and (3) (pp. 25 and 26), it was possible to calculate the standard errors as regards the corresponding numbers of plots in simple and stratified random sampling.

Fig. 12 presents the results for different size plots as a function of the number of plots, and additionally as a function of the distance between plots for those plot surveys in which the distances between lines and plots are equal. The distance between plots increases, and the number of plots diminishes to the right; the consequence of this is an increase in standard error. (Fig. 20 p. 50 provides an idea of the increase of standard error in simple uniform systematic plot sampling.)

Without exception, the relative standard error of simple random sampling has been given the value of 1 in Fig. 12. The result of stratified random sampling based on a corresponding number of plots is considerably more precise. With increasing size of sample plot, the relative difference rises, i.e. in a strati-

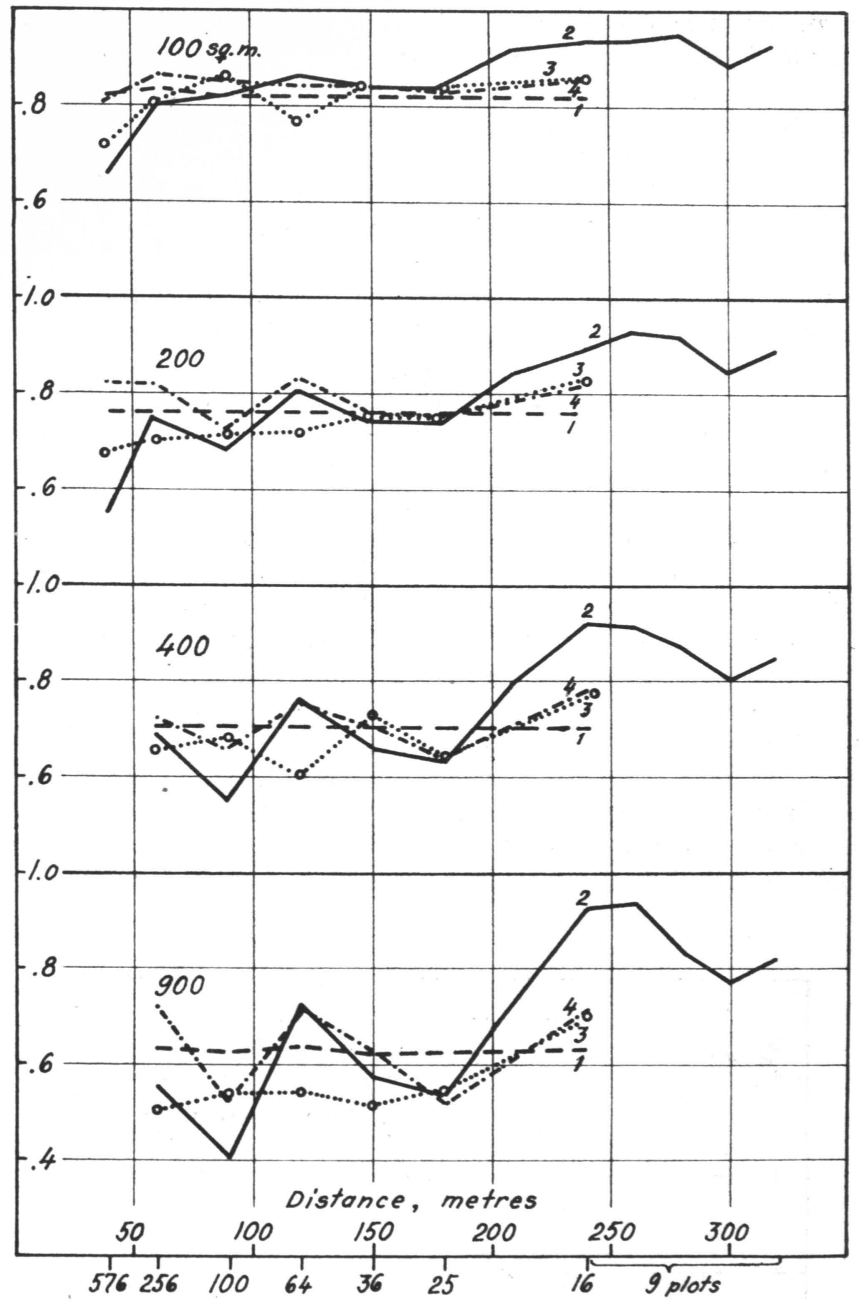

Fig. 12. Ratio of standard errors in different plot sampling methods in Area 1, Evo. Entire volume. The values for simple random sampling are equal to 1.0.

Curve 1: stratified random
» 2: simple systematic
» 3: stratified systematic, strata-areas from map
» 4: stratified systematic, strata-areas from line measurement.

fied sampling, large plots become relatively more efficient. This is in accordance with earlier findings (STRAND 1957; cf. NYYSSÖNEN and VUOKILA 1963).

To some extent, results vary in simple systematic surveys. This is attributable to the periodicity noticeable in the material at intervals of about 250 m., as is brought out below. However, a systematic survey is always more advantageous, as it provides a standard error which is less than that in a simple random sampling. The advantageous nature of systematic surveys diminishes with the fall in the number of sample plots, but within the numerical range likely to occur in practice, the relationship of standard errors does not change appreciably. Moreover, in systematic surveys large plots can be used to greater relative advantage.

Stratification has added to the precision of systematic surveys only when the plots are comparatively small (less than 25) in number. With large numbers of plots, and consequently when an attempt is made to achieve a high standard of precision, the relationship may even be reversed. The actual areas of strata, such as those measured from maps, are helpful in surveys with large plots, as in this way the effect of variation attributable to the periodicity of the growing stock is diminished. The results of systematic surveys are with few exceptions at least as good as those of stratified random sampling, but if provision is made for the actual areas of strata and a relatively large plot size, the results of systematic surveys seem to be better. In fact, the methods with error-free strata areas are best comparable with each other.

Fig. 13 illustrates some of the findings as regards Areas 2, 4 and 5. Attention is paid here to the relationship of simple uniform systematic plot sampling and simple random sampling. The results are rather similar to those of Fig. 12, although a certain irregularity exists by reason of the small number of sample plots.

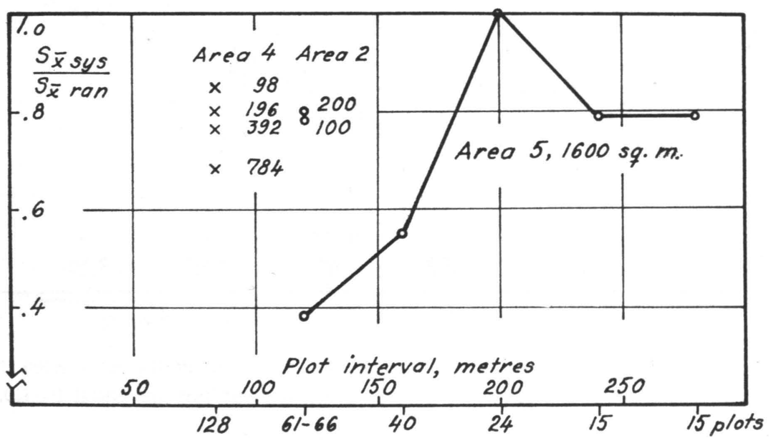

Fig. 13. Ratio of $s_{\bar{x}\,sys}/s_{\bar{x}\,ran}$ in plot sampling on Area 2, 4, and 5.

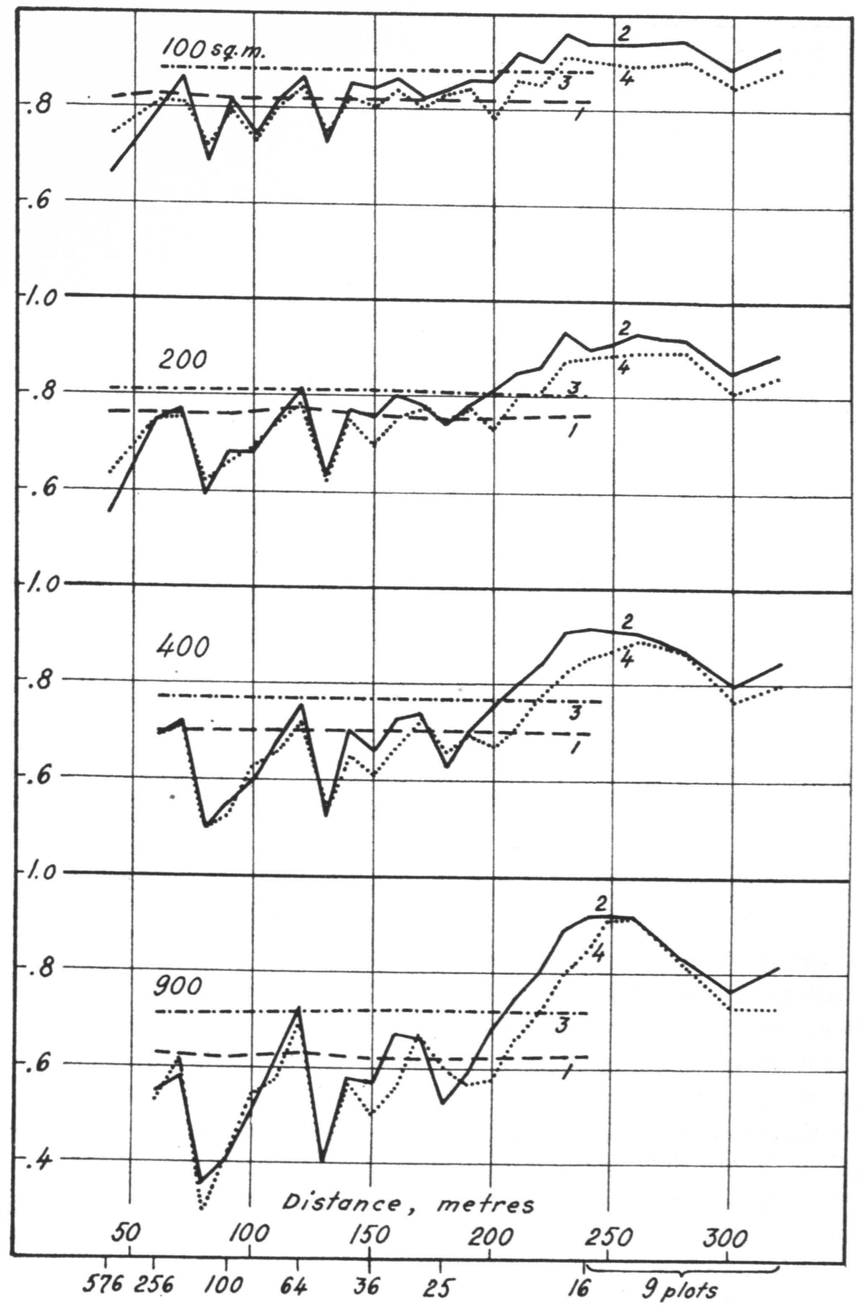

Fig. 14. Ratio of standard errors in different plot sampling methods in Area 1, Evo. The values for simple random sampling are equal to 1.0.

Curve 1: entire volume; stratified random
» 2: entire volume; simple systematic
» 3: volume of trees exceeding 20 cm. D.B.H.; stratified random
» 4: volume of trees exceeding 20 cm. D.B.H.; simple systematic.

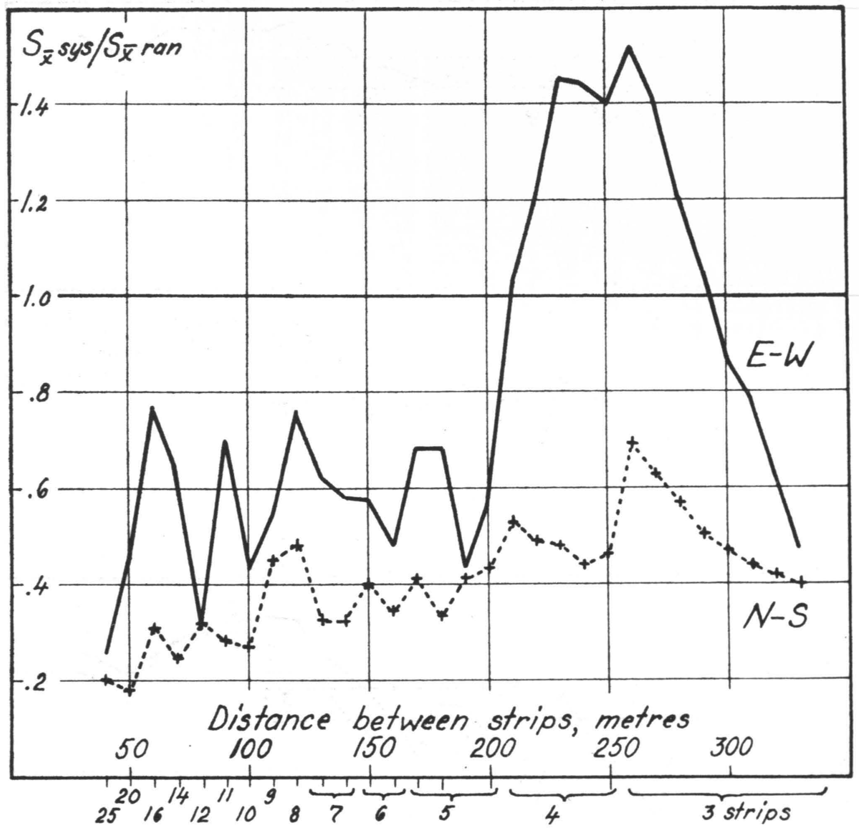

Fig. 15. Ratio of $s_{\bar{x}sys}/s_{\bar{x}ran}$ in strip sampling in Area 1, Evo.

As regards the volume of trees exceeding 20 cm. D.B.H., and comparison of this with the entire volume, reference is made to Fig. 14, which presents simple random, stratified random, and simple uniform systematic sampling. For the sake of comparison, the results relating to the Figure also include entire volume.

The systematic arrangement of sample plots improves the results in respect of saw-timber rather than those of the entire volume on comparison with random sampling. In stratified random sampling, the volumes arrived at assume different aspects: for the volume of trees exceeding 20 cm. D.B.H., stratified random sampling is relatively less efficient than that concerned with the entire volume. The way of stratification applied here is obviously not of advantage for large-sized timber.

532. Strip sampling

Fig. 15 indicates the ratio between a systematic strip survey and a corresponding random survey on strips of varying directions in Area 1, Evo. The

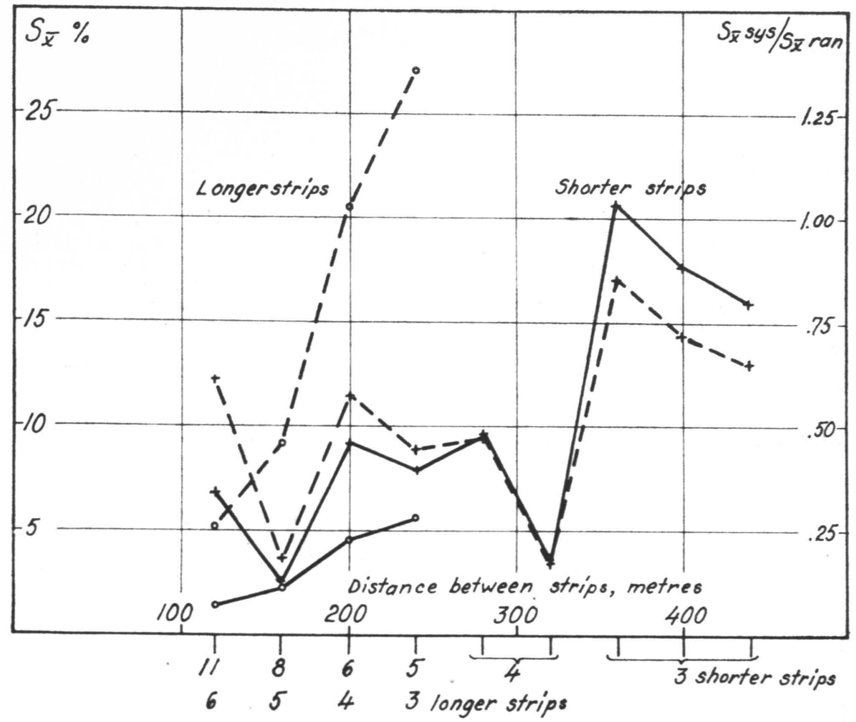

Fig. 16. Standard error as percentage (solid lines) and ratio of $s_{\bar{x}sys}/s_{\bar{x}ran}$ (broken lines) in Area 5, Durango.

material from Area 5, Durango, is illustrated in Fig. 16, in which broken lines and the scale on the right indicate the relationship concerned; solid lines with the left-hand scale give the standard error. It is to be concluded that as a rule the ratio is considerably below 1, although two distinct exceptions are apparent, the strips in Evo in the E-W direction at distances of about 250 m., and the longer (N-S) strips in Durango at distances of 200 and 240 m.

In this connection, interest is attached to study of the cubic volumes of individual strips within both research areas. These are shown in Figs. 17 and 18. On the N-S strips in Evo, and on the shorter (E-W) strips in Durango, a clear trend is discernible, and the standard deviation of strip volumes is accordingly great. The ratio of standard errors arrived at by means of systematic and random samplings is in these cases always less than 1. Conversely, within the E-W strips in Evo and longer (N-S) strips in Durango, no clear trend is noticeable, although in Evo in particular the periods of about 250 m. in length are rather apparent. In both the last-mentioned cases, the standard deviation from the mean is relatively small. The ratios between systematic and random surveys which exceed 1, given in Figs. 15 and 16, need to be seen in the light of the

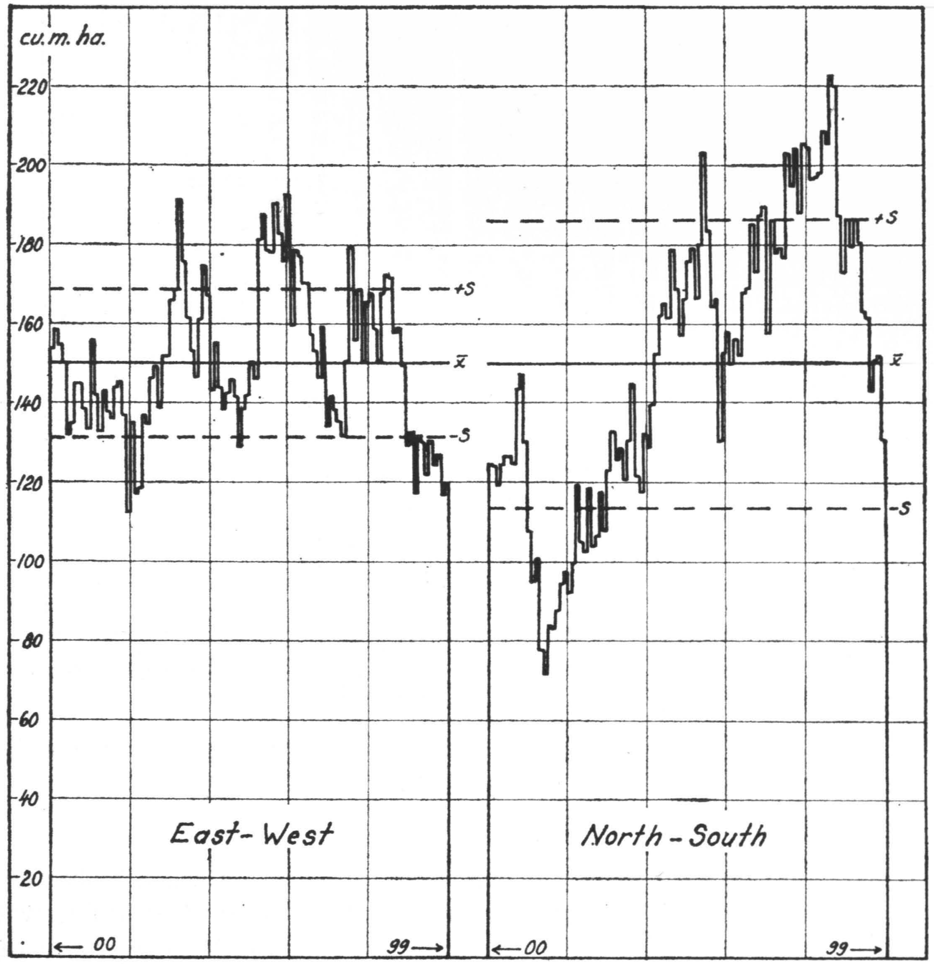

Fig. 17. Mean volumes on different strips in Area 1, Evo.

periodicity of growing stock (cf. FINNEY 1950). The same phenomenon also makes itself felt in comparisons of plot sampling (cf. Fig. 12).

533. Conclusion

In examination of the relationships of different survey methods, many interesting features have been discovered. In this connection, special attention is due to the finding that only in exceptional cases has a systematic survey given less precise results than other methods (cf. COCHRAN 1963, pp. 221—224). In addition, when strata exist which deviate from each other, ascertainment of

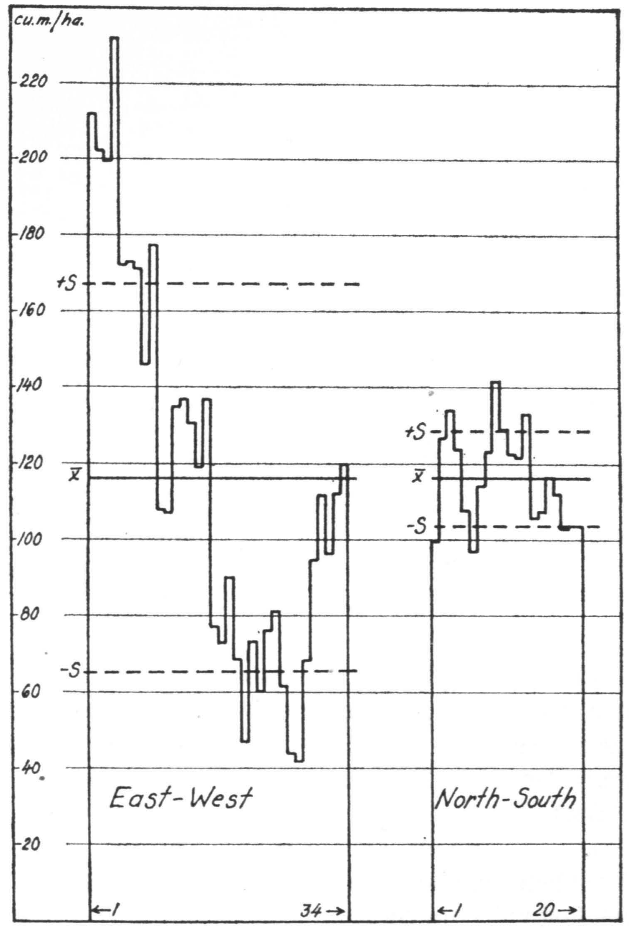

Fig. 18. Mean volumes on different strips in Area 5, Durango.

the relative size of their areas in systematic sampling often improves the precision of the results. Moreover, if the simplicity of field work is borne in mind, the use of systematic survey is motivated from many points of view. In a later part of this investigation, good reason is given for concentration on the discussion of systematic sampling.

6. Systematic sampling

61. Some possibilities for calculation of the precision

In simple random sampling, the standard error $\sigma_{\bar{x}}$ and the variance $\sigma_{\bar{x}}^2$ of the sample mean $\bar{x}$ can be calculated very easily from the population variance σ^2.

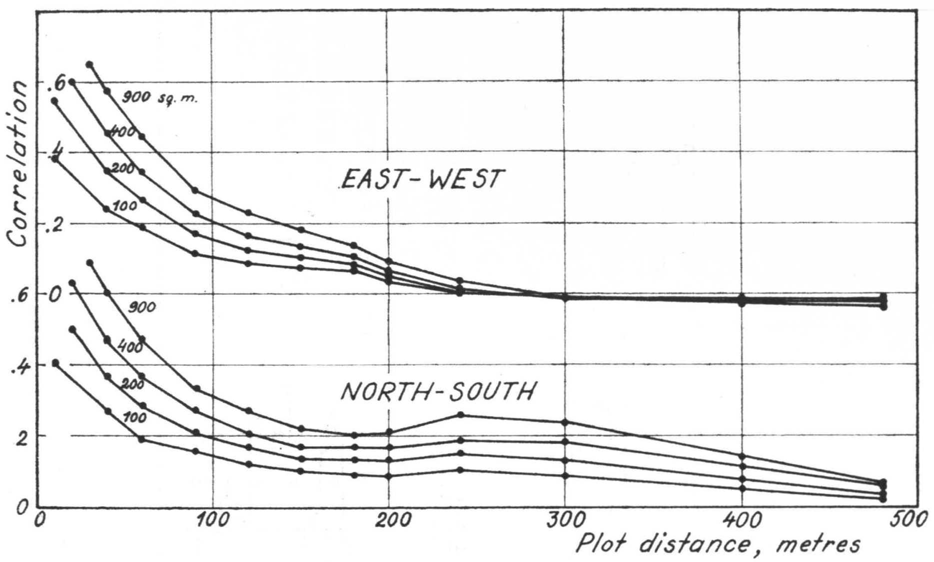

Fig. 19. Correlograms of plot volumes in Area 1, Evo.

If, as usual, σ^2 is not known, it can be estimated without bias from the sample itself, and simple calculations provide an unbiased estimator $s_{\bar{x}}^2$ of $\sigma_{\bar{x}}^2$ as well[1]. All this holds good regardless of the distribution law of the population, or in other words, we prefer to think of the values of sampling units as random variables (cf., e.g. WILKS 1962, p. 195), regardless of the probability distribution of the population.

In stratified random sampling, the corresponding results hold, provided that at least two sampling units are chosen in each stratum.

The mathematical methods used in derivation of the standard error and variance of the mean, and as proof of the validity of variance estimation formula in simple and stratified random sampling, do not generally apply in systematic sampling (cf. p. 25). This is because of the fixed mutual positions of sampling units, which can bring about a distance-dependent correlation between the values of sampling units in the same systematic sample. This often happens in natural populations. As an example from Area 1, Evo, Fig. 19 presents the estimates of correlation functions in the E-W and the N-S direction for different plot types, calculated on the basis of all the possible combinations. It is observable that the correlation depends on both distance and direction; the effect of some periodicity is also traceable in the figure.

It is generally known that when the variance estimation equations for simple random sampling are applied in systematic sampling of natural populations,

[1] For what follows below in this section, it should be borne in mind that as a rule the standard deviation $\sigma_{\bar{x}}$ cannot be estimated without bias.

they tend to overestimate the variance of the sample mean; for references concerned with forest inventory, see e.g. SPURR (1952, p. 383); LOETSCH and HALLER (1964, p. 165). It has been shown that this holds good for the present data (section 53), in which the equations for stratified random sampling would also have meant overestimation of the error.

In what follows, brief consideration is first given to the possibilities of theoretical determination of the precision of systematic sampling, that is by the application of a mathematical model of the forest and sampling, and not by some equations applied to the sample values. Secondly, there are discussed the possibilities of developing an equation for estimation from the sample itself of the variance of sample mean, and an empirical study is made of the behaviour of some formulae of this nature given in the literature.

Furthermore, to determine the precision of systematic sampling, and its dependence on various factors, use can be made of regression analysis if the results of complete enumeration of a forest are available. In the analysis, the true variance of sample mean (or some function of it, such as the coefficient of variation) is the dependent variable, and the factors are the independent variables. The application of this method in the present material is considered in detail in section 62.

611. Theoretical methods

For theoretical studies, there is needed a mathematical model of the population to be sampled. Following MATÉRN (1947; 1960), the forest in question is regarded as a realization of a second order stationary stochastic process. The term »second order» means that this model enables study of only those properties which are expressible with the means, variances and covariances of the process. »Stationary» means that the covariance[1] between two plot volumes depends only on the mutual but not on the absolute positions of the plots. This assumption seems intuitively realistic if the forest concerned is of the same type throughout. For determination of the structure of the stochastic process, there must then be known only the mean and the covariance function of the process.

With this model, a study could be made of the precisions of all sampling types in this population, but this approach is especially necessary for systematic sampling. Consideration is given to the general case, and that best applicable in forest inventory practice, where the size, form and mutual positions of sampling units are predetermined, but the sample is located at random, i.e. where systematic sampling with a random start is in question.

[1] To avoid repetition, it is recalled that for a second order stationary process, covariance at zero distance equals variance, and the correlation function is obtained by dividing covariance function by variance. Thus »covariance function» can be replaced in what follows by »variance and correlation function», and conversely.

Let us first consider a certain forest, corresponding in our model to one fixed realization of the process. By reason of the random location, the sample mean can have different values, and thus has a certain variance. In another forest of the same form, the sample mean would also have a variance, although its value would in general be different. Thus, when consideration is given to different forests of the same form, the variance of the systematic sample mean is a random variable. In our mathematical model, therefore, we examine quantity $E\sigma_{\bar{x}}^2$, i.e. the variance of the systematic sample mean, averaged over all the realizations of the process. Thus, the result given by Equation 4 (p. 26) can be considered as an estimate of $E\sigma_{\bar{x}}^2$ in this particular case.

This quantity $E\sigma_{\bar{x}}^2$ can be calculated if the covariance function of the process is known, although in practice the computations of numerical values can be very complicated. Zubrzycki (1958) has given the formulae for simple random, stratified random, and simple systematic sampling in the two-dimensional case. Although these expected variances do not necessarily reflect the merits of the various sampling methods in any given single forest, they nevertheless give a good indication of their average precision in the long run. Consequently any theoretical comparisons between various sampling methods must be based on these expected variances; Matérn (1960) has done just this for some types of covariance function.

For effective use of the mathematical model, we must know

(1) the true covariance function in forest,
(2) how it differs in different types of forest, and
(3) how strongly this affects $E\sigma_{\bar{x}}^2$.

To answer the first two questions, empirical estimation of the covariance function is necessary. Here there is encountered the drawback of covariance and correlation functions, that the estimation is biased, and the amount of bias depends on the area in respect of which estimation is made (cf. e.g. Matérn 1947, pp. 63—64). To avoid these drawbacks, Jowett (1952) has proposed the use of a serial variation function; for the exact definition, see Jowett (1955) or Matérn (1960, p. 51). It is possible to express $E\sigma_{\bar{x}}^2$ with this function.

Nevertheless, $E\sigma_{\bar{x}}^2$ depends only on the »correlational properties» of the population, expressible by (a) covariance function or (b) correlation function and variance or (c) serial variation function. Thus the measure applied for those properties should be easy to compare in different populations. Here, the correlation function has an advantage by virtue of its norm being established between the values + 1 and — 1. Thus, to settle the question of use of the serial variation function, an examination is due of how superior it is to the others in estimation of the precision of systematic sampling in forest inventory practice.

If, for instance, the covariance function in a forest is known with sufficient accuracy (or its different forms in different types of forest) a theoretical determination is possible of the precision of systematic sampling, and its dependence on plot distances, arrangements, and so on. Nonetheless, the model applies to one size and form of sampling unit alone with which the estimation of covariance function has been made. To make a general study of the effect of plot size and form, the model must incorporate the alterations given in the next section.

In the model Matérn (1960) applied for forest sampling, the basic random variable of the process is a plot volume. One could also think of a model in which the basic random variable is the height of the tree above a point; this type of model has also been discussed by Matérn (1947, pp. 61—63). In this case, even the plot volumes would be stochastic integrals of the basic process. The formula for $E\sigma_{\bar{x}}^2$ can also be derived for this case, and the effect of plot size and form can be studied theoretically. However, a serious drawback of the latter model is that the estimation of covariance function (or serial variation function) requires a great deal more field work, as the position and height of every single tree must be measured.

So far, examination has been confined to the quantity $E\sigma_{\bar{x}}^2$. Of course, it would be advantageous to know more of the distribution of $\sigma_{\bar{x}}^2$, e.g. the variance $D^2\sigma_{\bar{x}}^2$, although such a study would call for a more complicated model than the second order stationary process.

In conclusion, it must be admitted that a great deal remains to be done before full use can be made of theoretical methods.

612. Variance estimation formulae by the use of sample

Today, no generally applicable formulae exist for estimation of the variance of the systematic sample mean from the sample itself and it seems very unlikely that there will ever be any. Nevertheless, it is necessary to study whether some formulae would be acceptable in forest sampling.

If the covariance function of the process can be estimated from the systematic sample itself, it can be introduced to the equation for $E\sigma_{\bar{x}}^2$, which could be used as an estimate for $\sigma_{\bar{x}}^2$. Cochran (1946) did this for a one-dimensional case when the covariance function is of exponential type. Yates (1948, p. 362) studied the effect of deviations from this model on the equation, and showed that even a slight additional superimposed random variation induces a serious underestimation of variance. Thus, if we wish to employ this approach, we must first find out whether the covariance function can be estimated from the sample with sufficient accuracy. As the covariance in small distances has an important effect, additional sampling units with distances less than usual may be needed in estimation. Of course, the serial variation function may be used instead of the covariance function, if this provides any advantage.

It is to be expected that the formulae so derived will be very complicated, and possibly feasible only for electronic computers. The work required to measure additional sampling units needs also be remembered. Moreover, it must be noted that this method, even at best, can give an estimate of $E\sigma_{\bar{x}}^2$ only, and not of $\sigma_{\bar{x}}^2$, as would be the case in simple and stratified random sampling.

In literature, several methods belonging to the present category have been suggested. Most of these can be divided roughly into two groups (cf. e.g. MATÉRN 1960, pp. 110—120; YATES 1960, pp. 229—233; COCHRAN 1963, pp. 224—227).

1. The sample is analysed as if it were of some random type (e.g. a stratified random sample). It is generally known that this method results in over-estimation of the variance; it is used only by virtue of its convenience.

2. A quadratic form is set up of the sampling unit values, so as to eliminate »the systematic component of variation» (as stated by YATES 1960, p. 213). The estimate is obtained by summing the squares of a linear form of sampling unit values as this is moved in the sample, and finally the sum is divided by a constant which would make the estimation unbiased for a random sample. For a more complete description see YATES (1960, p. 231); MATÉRN (1960, pp. 115—116).

The behaviour of sample-based variance estimation formulae for systematic sampling has hardly been discussed; this in spite of the fact that for instance ÖSTLIND (1932), FINNEY (1948) and ZINGER (1964) studied variance of the sample mean in strip sampling on the basis of data from different forests with either partial or complete enumerations. The material of Area 1, Evo, measured in detail, makes possible a comprehensive study of plot sampling.

We denote the values of sample plots in a systematic sample by $x_1, x_2 \ldots, x_n$. Consideration is given to the following five methods for the estimation of $\sigma_{\bar{x}}^2$:

1. The sample is analysed as a random sample with replacement, i.e. we use the estimator

$$s_{\bar{x}}^2 = \frac{1}{n(n-1)} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

2. The estimator

$$s_{\bar{x}}^2 = \frac{1}{2kn} \sum_{i,j} (x_i - x_j)^2$$

is used, the sum being taken over all neighbouring sample plots (i.e. which are side by side in x- or y-direction), and k the number of terms in the sum. This method was presented by LINDEBERG (1924). It can be regarded as a special case of analysis of the sample as a stratified random sample with two units in each stratum, and overlapping strata.

In the remaining methods, mentioned by MATÉRN (1960, pp. 115—116), linear forms with the following coefficients are employed to set up the quadratic form:

3.	−0.5	0.5		
	0.5	−0.5		
4.	−0.05	0.15	−0.15	0.05
	0.15	−0.45	0.45	−0.15
	−0.15	0.45	−0.45	0.15
	0.05	−0.15	0.15	−0.05
5.	−0.1	0.2	−0.2	0.1
	0.2	−0.4	0.4	−0.2
	−0.2	0.4	−0.4	0.2
	0.1	−0.2	0.2	−0.1

The coefficients have here been so normed that the estimate for $n\sigma_{\bar{x}}^2$ is the mean of the squared linear forms. For example, Method 3 could be expressed in another form as

$$1/4\,(x_1 - x_2 - x_3 + x_4)^2$$

the plots being located in the following arrangement:

$$\begin{matrix} .\,x_3 & .\,x_4 \\ \\ .\,x_1 & .\,x_2 \end{matrix}$$

To perform the computations, there was devised a computer program in which the information concerning the sampling is given by the same parameter cards as in the main program. From every systematic sample, the program computes the values[1] for $s_{\bar{x}}^2$ by Methods 1—5 and gives for every set of systematic samples of the same type the means and the standard deviations of the estimators. Note especially that we study the distribution of $s_{\bar{x}}^2$, and not that of $s_{\bar{x}}$; this is because the formulae are derived for estimation of the variance, and not the standard deviation without bias.

Table 8 presents the true values of the variances in simple uniform systematic plot sampling, here in the form of variance per sample point ($= n\sigma_{\bar{x}}^2$) for elimination of the effect of sample size. For comparison, the corresponding values in simple random sampling are included in the last row; the ratios of the results of the two methods were in fact given in another form in Fig. 12 (p. 31).

To facilitate their examination, the results proper for five methods are given in Tables 9.1 to 9.5 in the form of the ratios of the mean and standard deviation of the estimator to the true value. All the formulae examined overestimate in general the variance of sample mean; overestimation rises with increasing plot size and decreasing plot interval. Method 1 is the worst as regards overestimation. On the average, Method 3 seems to give slightly better estimates than

[1] In methods 3, 4 and 5, the set of coefficients has been moved step by step in all possible locations in the sample, e.g. if we have 25 plots in a 5 × 5 pattern, we have 16 locations in Method 3, and 4 locations in Methods 4 and 5. In Methods 4 and 5, there is thus marked overlapping.

Table 8. Variance per sample point in simple uniform systematic plot sampling in Area 1, Evo.

Plot distance, metres	Plot type (cf. p. 13)				
	1	2	3	4	5
40	5 616.00[1)] 576 16	3 542.40 576 8	1 359.36 576 8		
60	8 555.52 256 36	5 580.80 256 18	5 923.84 256 18	4 628.48 256 9	1 008.64 256 4
90	9 247.00 100 81	3 562.00 100 36	6 389.00 100 36	2 230.00 100 16	1 458.00 100 9
120	9 964.16 64 144	6 020.48 64 72	6 017.28 64 72	4 143.36 64 36	2 773.12 64 16
150	9 820.80 36 225	5 839.56 36 105	5 509.44 36 105	3 577.68 36 49	2 387.16 36 25
180	9 682.75 25 324	4 973.25 25 162	5 288.75 25 162	2 884.25 25 81	2 431.25 25 36
240	11 952.64 16 576	8 358.08 16 288	7 994.40 16 288	6 259.04 16 144	4 806.24 16 64
Random sampling	13 215.80 1 10 000	9 387.67 1 5 000	9 729.47 1 5 000	7 361.64 1 2 500	5 829.32 1 1 089

1) 5 616.00 = $n\sigma_{\bar{x}}^2$
576 = number of plots in the sample
16 = number of samples of this type

Method 2, but the standard deviation of the estimates is also higher. On the average Methods 4 and 5 do not appear to afford any advantage over Method 3, and their standard deviations are considerably higher. However, it is quite natural that the standard deviation of estimate is large if the estimate has been calculated as a mean of no more than a few terms. Thus Methods 4 and 5 might give better results in larger areas.

It is to be concluded that none of the methods discussed has given fully reliable results, but that those achieved by Methods 3 and 2, for example, may be sufficiently precise in the cases which appear most frequently in practice, although they seem to overestimate the error, and are consequently on the safe side.

Table 9.1. Estimation of variance by Method 1 (cf. p. 42).

Plot distance, metres	Plot type (cf. p. 13)				
	1	2	3	4	5
40	2.409[1] .163[2]	2.699 .133	6.983 .411		
60	1.582 .149	1.714 .149	1.602 .143	1.620 .135	2.786 .208
90	1.502 .280	2.761 .379	1.501 .267	3.399 .404	4.111 .441
120	1.362 .246	1.596 .240	1.585 .239	1.820 .258	2.116 .214
150	1.421 .377	1.691 .398	1.765 .430	2.144 .492	2.535 .544
180	1.447 .483	1.980 .532	1.823 .524	2.615 .572	2.507 .570
240	1.139 .455	1.152 .398	1.198 .442	1.209 .422	1.225 .418

1 The mean of the estimates $s_{\bar{x}}^2$, divided by $\sigma_{\bar{x}}^2$
2 The standard deviation of the estimates $s_{\bar{x}}^2$, divided by $\sigma_{\bar{x}}^2$

Table 9.2. Estimation of variance by Method 2 (cf.p. 42).

Plot distance, metres	Plot type (cf. p. 13)				
	1	2	3	4	5
40	1.801 .156	1.738 .138	4.440 .381		
60	1.290 .133	1.245 .124	1.163 .112	1.047 .099	3.190 .126
90	1.293 .261	2.187 .330	1.184 .245	2.440 .279	2.725 .366
120	1.230 .275	1.364 .267	1.348 .282	1.470 .274	1.589 .229
150	1.300 .391	1.475 .429	1.521 .406	1.760 .476	1.967 .436
180	1.332 .488	1.737 .552	1.599 .500	2.185 .580	2.028 .482
240	1.091 .517	1.069 .440	1.112 .489	1.084 .439	1.049 .435

Explanations in Table 9.1.

Table 9.3. Estimation of variance by Method 3 (cf. pp. 42-43).

Plot distance, metres	Plot type (cf. p. 13)				
	1	2	3	4	5
40	1.722	1.608	4.046		
	.175	.182	.411		
60	1.254	1.176	1.089	.951	2.720
	.171	.125	.144	.113	.223
90	1.267	2.076	1.137	2.234	2.391
	.328	.434	.319	.424	.557
120	1.195	1.292	1.274	1.356	1.395
	.368	.362	.399	.332	.298
150	1.258	1.400	1.434	1.609	1.742
	.522	.554	.581	.607	.541
180	1.273	1.595	1.473	1.929	1.755
	.617	.739	.645	.823	.631
240	1.085	1.046	1.086	1.034	.965
	.755	.665	.703	.645	.603

Explanations in Table 9.1.

Table 9.4. Estimation of variance by Method 4 (cf. pp. 42-43).

Plot distance, metres	Plot type (cf. p. 13)				
	1	2	3	4	5
40	1.764	1.647	3.978		
	.266	.282	.550		
60	1.281	1.202	1.074	.922	2.512
	.293	.232	.261	.238	.634
90	1.284	2.019	1.157	2.115	2.032
	.549	.739	.544	.851	1.039
120	1.244	1.303	1.270	1.321	1.313
	.677	.667	.639	.556	.754
150	1.269	1.368	1.445	1.571	1.693
	1.113	1.230	1.395	1.454	1.485
180	1.169	1.376	1.248	1.500	1.393
	1.148	1.288	1.251	1.262	1.187
240	1.287	1.233	1.278	1.218	1.162
	1.930	1.741	1.822	1.732	1.606

Explanations in Table 9.1.

Table 9.5. Estimation of variance by Method 5 (cf. pp. 42-43).

Plot distance, metres	Plot type (cf. p. 13)				
	1	2	3	4	5
40	1.752	1.637	3.927		
	.288	.312	.567		
60	1.274	1.199	1.063	.911	2.479
	.313	.249	.292	.264	.725
90	1.251	1.931	1.118	1.991	1.838
	.598	.805	.572	.860	1.032
120	1.224	1.280	1.240	1.292	1.298
	.740	.729	.686	.611	.858
150	1.258	1.370	1.444	1.577	1.719
	1.194	1.301	1.520	1.563	1.576
180	1.150	1.370	1.226	1.480	1.333
	1.226	1.390	1.362	1.427	1.326
240	1.199	1.135	1.181	1.101	1.012
	1.801	1.595	1.763	1.639	1.474

Explanations in Table 9.1.

62. Regression equations for standard error

621. Plot sampling

As there is no general equation for the error calculation in systematic sampling, an endeavour was made to develop an equation on the basis of the material from Area 1, Evo. The dependence of standard error on the different characteristics of samples and forest areas is described by equations derived by regression analysis. Dependent variables were both the absolute and the relative standard error of the mean. Independent variables were the plot size, distance between plots, size of area, and mean volume. In addition, the variation in plot volumes belonging to the sample could be an interesting characteristic, although it could not be used at this phase. The distance between plots, which refers to the distance between the edges of plots, was regarded as a characteristic superior to the number of plots, since it is not correlated with the size of the forest area.

As the preliminary tests indicated that no common model could be used for full expression of the effect exercised by the distance between plots for various plot sizes, regression coefficients were calculated separately for each size of plot. Nevertheless, these additional coefficients did not appreciably improve the reliability of equations, and were for this reason omitted from the final treatment.

In the regression analysis, the material consisted of the results of samples taken. Since the number of observations underlying the analysis varied substantially in accordance with the plot size the observation values were weighted, by taking into account the observations carried out on sample plots of different sizes as many times as is indicated by the following set-up:

1)	100	sq.m.	plots	8 times
2)	200	»	»	2 »
3)	200	»	»	2 »
4)	400	»	»	2 »
5)	900	»	»	1 time

This weighting was intended to safeguard an approximately correct representation of small plot sizes, and to improve the fit of regression equations in the extreme zones of observation values. However, the weighting described entails that the characteristics of reliability calculated for the equations lose a part of their meaning.

Since no significant difference was discovered between the absolute and the relative standard error as a dependent variable, only the equations of relative standard error are given below.

The different variables were transformed into a logarithmic form before the regression analysis was carried out; prior to transformation, graphic inspection of the results was made.

The final regression equation, which still includes the logarithms of the variables, is as follows:

$$\log y = 1.189 - 0.2552 \log x_1 + 1.070 \log x_2 - 0.5390 \log x_3 - 0.6537 \log x_4 \quad (7)$$

where y = standard error as a percentage
x_1 = size of sample plot in 100 sq. metres
x_2 = distance of sample plots, m. — $\sqrt{\text{plot size in sq.m.}}$
x_3 = area of survey unit, hectares
x_4 = mean volume, cu.m./ha.

The variation range of different variables in the material was as follows:

y:	from	0.5	to	24.3
x_1:	»	1	»	9
x_2:	»	30	»	310
x_3:	»	20.2	»	96.0
x_4:	»	89.2	»	204.4

The multiple correlation coefficient in Equation (7) is 0.963, and the t-values of independent variables in the order listed above, are as follows: 39, 115, 54 and 21.

When the anti-logarithms are taken from both sides of the equation, and the correction due to the logarithmic transformation is made (cf. JEFFERS 1960), there is obtained the following equation for standard error of entire volume in simple uniform systematic plot sampling, based on data from Area 1, Evo:

$$y = 15.7\, x_1^{-0.255}\, x_2^{1.07}\, x_3^{-0.539}\, x_4^{-0.654} \quad (8)$$

In subsequent calculations concerned with the numbers of sample plots, the equation took the following form:

$$x_5 = 100\, x_3/(0.00761\, x_1^{0.238}\, x_3^{0.504}\, x_4^{0.611}\, y^{0.935} + \sqrt{x_1})^2 \quad (9)$$

where x_5 = plot number. It is to be noticed that x_2 is set automatically as soon as x_3 is given and x_5 is calculated.

For a given plot size, plot distance and area of survey unit, the standard error of the volume of trees exceeding 20 cm. D.B.H. was generally 1/10 to 1/3 higher than that of the entire volume. Large-sized trees have not been studied by quarter-areas, and thus the effect of neither mean volume nor area is ascertainable for this portion of the growing stock. The following equation illustrates the effect of plot size and distance between plots on the standard error in this particular portion of growing stock:

$$\log y = -1.203 - 0.2620 \log x_1 + 1.081 \log x_2 \quad (10)$$

R is equal to 0.971, and the t-values of the variables x_1 and x_2 are 35 and 105, respectively.

To test the reliability of Equation (8), the averages were calculated by groups of sample means, and also the results obtained from the corresponding equations. Fig. 20 presents the standard errors calculated as actual standard deviation of sample means, and those calculated by means of Equation (8). Only the plot sizes 100 and 900 sq.m. are involved. For these extreme variants, the fit seems to be satisfactory, with the possible exception that for large plots with wide spacing the equation appears to underestimate the error to some extent, by reason of the more pronounced actual curvature of standard error compared with that given by the equation.

Table 10 further illustrates the results given by Equation (8) in respect of sample plots of varying size within the total area and quarter areas in Evo. Taken as a whole for the area, the systematic deviations can be regarded as relatively small in general. Instead, on quarter areas the deviations are greater, particularly as regards the extreme values of mean volume.

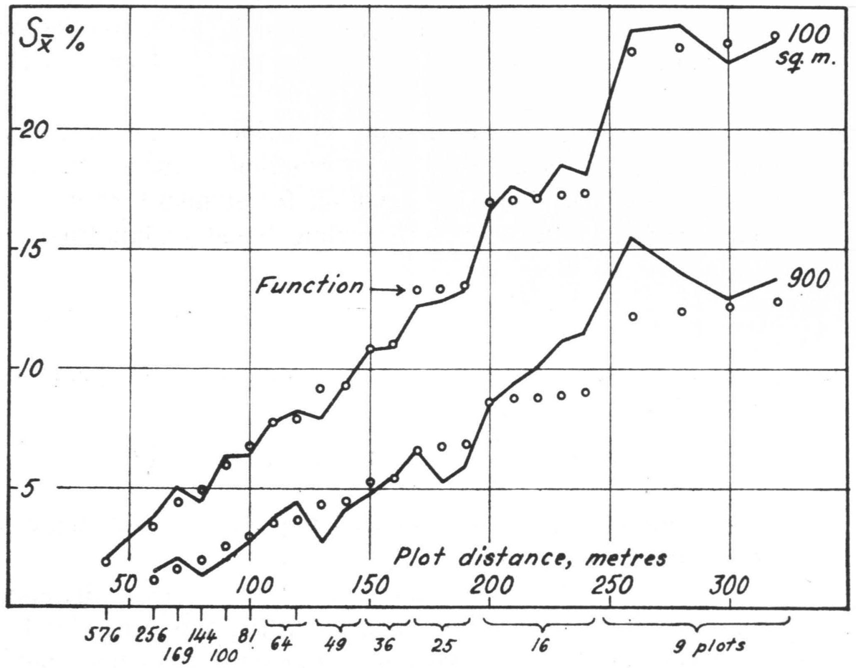

Fig. 20. Comparison of standard errors in Area 1, Evo; the entire volume. The zig-zag lines indicate the results calculated as actual standard deviation of sample means, the circles (»Function») those given by Equation (8).

Fig. 21, which presents the results of systematic survey on sub-areas in Area 2, Toivala, also indicates the effect of mean volume on standard error. This Figure also presents the results obtained on the application of Equation (8). The effect of mean volume is even more marked in Toivala than in Evo; the equation has somewhat overestimated the standard error in Toivala. The outcome of comparison is partly affected by the area of Toivala not having been fully measured, which is likely to result in smaller standard deviations of sample means. However, it does not seem that there are essential differences between the two areas.

In this context, comparison can be made of the results of non-uniform systematic plot sampling. This implies a method in which the distance between plots is greater in one direction than in another. Such a kind of plot location is interesting, particularly in surveys of large areas, as in this way it may be possible to reduce the time required to measure line; this is because the plots are placed closer to each other along the lines than the distance between the lines.

Following the case-by-case comparison of non-uniform sampling, a comparison was made on Area 1, Evo, with the mean results given by Equation (8). The standard error of uniform survey corresponding to each non-uniform sampl-

Table 10. Comparison of standard errors in Area 1, Evo; the entire volume.

Plot distance, metres	Plot size, sq.m. 100	200	400	900
	Difference, per cent[1]			
Whole area				
40—90	+ 4.6	— 2.7	— 2.4	— 4.4
100—140	— 2.6	— 8.9	— 9.0	— 5.1
150—190	— 2.5	— 9.6	— 11.3	— 9.4
200—240	+ 2.7	+ 0.2	+ 4.7	+ 13.6
260—320	+ 0.9	+ 1.2	+ 2.6	+ 12.0
Quarter areas				
40	+ 9.3	+ 9.2	...	...
60	— 2.9	— 4.5	— 4.1	+ 10.2
90	+ 1.1	— 7.2	— 11.0	— 1.1
120	— 4.8	— 5.5	+ 0.2	+ 13.1
Mean volume, cu.m./ha.				
92	— 17.8	— 12.5	— 2.3	+ 16.8
150	+ 3.2	— 1.1	— 1.7	+ 12.1
160	+ 14.1	+ 6.8	+ 2.8	+ 10.7
200	— 2.9	— 10.3	— 21.8	— 20.2

[1] The figures indicate the percentage by which the actual standard errors are less (—) or more (+) than those given by Equation (8).

ing was calculated, using the same plot size, the distance between plots based on the same number of plots, the same size of area, and the same mean volume of area.

The results have been compiled in Table 11. If the sign of difference is minus, non-uniform sampling has given more precise results than uniform sampling with the same number of plots. The results do not follow any consistent pattern, but considerable variations are discernible, particularly as regards the periodicity of area. However, the results indicate that in »lucky» cases there is a possibility of time-savings. In particular, the distance between lines of 180 m. in each direction seems to be such an advantageous case in the present material. Thus the combination of 180 m. in distance between lines and 40 m. between plots applied within a forest area of 100 ha. requires 5.5 km. of survey line measurement, whereas in a uniform survey of an equivalent degree of precision the amount of travel is doubled. An extremely unfavourable case is provided by the use of 240-m. plot distances in the E-W direction, i.e. the running of lines in the N-S direction at 240-m. intervals.

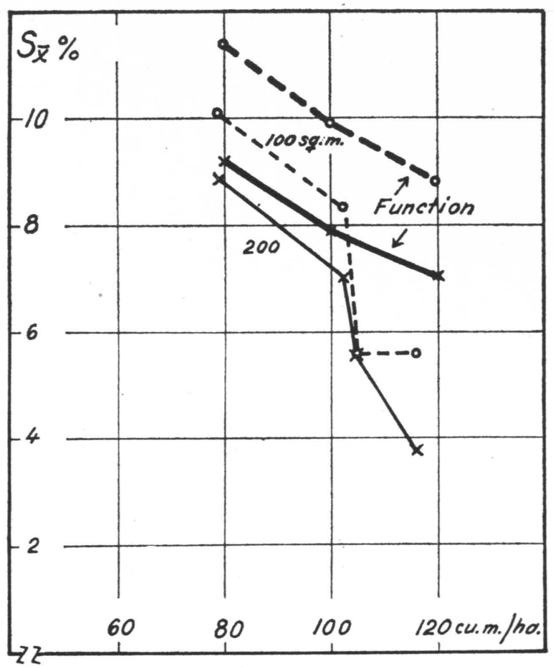

Fig. 21. The effect of mean volume on standard error in Area 2, Toivala, and according to Equation (8) based on Evo data; the latter indicated by word »Function». Broken lines: 100-sq.m. plots; solid lines: 200-sq.m. plots.

Table 11. Comparison of standard errors in Area 1, Evo. Entire volume, all plot sizes.

Direction E-W	Direction N-S	Number of plots per sample	Difference, per cent[1]
Plot distance, metres			
60	120	128	+ 26.5
120	60		+ 13.6
60	180	80	— 14.7
180	60		— 3.8
60	240	64	+ 7.3
240	60		+ 74.0
40	180	120	— 0.1
180	40		+ 0.2
40	240	96	+ 30.6
240	40		+ 113.6

[1] The figures indicate the percentage by which the errors of nonuniform systematic plot sampling are less (—) or more (+) than those given by Equation (8) for the same area, mean volume and plot number.

622. Strip sampling

The dependence of standard error on different characteristics in strip surveys was also submitted to regression analysis based on the entire volume of Area 1, Evo, and by combining all the strips, regardless of their compass direction, for treatment. Analysis resulted in the following equation, of which the variables proved significant:

$$\log y = 1.102 + 0.8947 \log x_2 — 0.5803 \log x_3 — 0.6201 \log x_4 \qquad (11)$$

where y = standard error as percentage
x_2 = strip interval — strip width (strip width = 10 m.)
x_3 = area, hectares
x_4 = mean volume cu.m./ha. (x_1 was used in earlier equations for plot size; here it is unnecessary).

The multiple correlation coefficient R is equal to 0.900, and the t-values were 17, 11 and 3 respectively. In calculating the equation, the strips used were placed at 40- to 250-m. intervals.

If anti-logarithms are taken of both sides of Equation (11), and the correction for logarithmic transformation is made, the following equation is obtained for standard error of the entire volume in systematic strip sampling based on data from Area 1, Evo:

$$y = 13.3 \; x_2^{0.895} \; x_3^{—0.580} \; x_4^{—0.620} \qquad (12)$$

The coefficients pertaining to area and mean volume are quite close to the corresponding coefficients of plot surveys in Equation (8). The effect of the size of area is very much the same in Equation (12) as that reported earlier by Östlind (1932) and Langsaeter (1932) for strip sampling, but the importance of mean volume is here somewhat greater.

With respect to a given level of precision, the total length of strip needed was calculated by the equation:

$$x_5 = 10000 \; x_3 / (0.0555 \; x_3^{0.648} \; x_4^{0.693} \; y^{1.117} + 10) \qquad (13)$$

where x_5 = strip length, m. The strip interval (x_2) is set automatically as soon as x_3 is given and x_5 determined.

63. Plot numbers and strip lengths for given degrees of precision

Equations (9) and (13), calculated on the basis of data from Area 1, Evo, and to be found in section 62, can be utilized to present the number of plots and the length of strips for a given degree of precision. Six forest areas were taken as examples in the calculations. These were square in shape, with superficial areas of 25, 100 and 400 ha., and mean volumes of 100 and 150 cu.m./ha.

Sample sizes were determined for standard errors of ± 2.5, ± 5 and ± 10 per cent. The numbers of plots were calculated for plots of 100 to 1000 sq.m. in simple uniform systematic plot surveys, and the lengths of 10 m. strips in systematic strip surveys. For the area of 400 ha., appreciable extrapolation was made, as regards the Evo area alone, but in Toivala the material available extended to the area exceeding 400 ha., and no contradictions have been found on comparison of these areas.

Table 12 presents the sample sizes needed in different cases. Furthermore, Table 13 illustrates the number of 300 sq.m. plots in different cases.

Table 12. Plot numbers in simple uniform systematic plot sampling and strip lengths in systematic strip sampling, calculated by Equations (9) and (13).

Mean volume, cu.m./ha.	Area, hectares	Plot size, sq. m. 100	200	300	400	500	600	700	800	900	1 000	10-meter strips, m.
		Plot number										
Standard error ± 2.5 per cent												
100	25	396	244	183	148	126	110	98	89	81	75	6 200
	100	613	400	310	258	223	198	179	164	151	141	11 800
	400	791	538	427	362	318	286	262	242	225	210	20 700
150	25	290	183	138	113	97	85	76	69	64	59	5 000
	100	417	277	217	182	158	141	128	118	109	102	9 200
	400	513	353	282	240	212	191	175	163	152	143	15 900
Standard error ± 5 per cent												
100	25	165	107	83	69	60	53	48	44	40	37	3 300
	100	216	146	116	98	86	78	71	65	61	57	5 800
	400	249	173	140	120	106	97	89	83	77	73	9 800
150	25	113	75	58	49	42	38	34	32	29	27	2 600
	100	140	96	77	65	58	52	48	44	41	39	4 500
	400	157	110	89	77	68	62	57	53	50	47	7 500
Standard error ± 10 per cent												
100	25	59	40	31	27	23	21	19	18	16	15	1 600
	100	68	47	38	33	29	26	24	23	21	20	2 800
	400	74	52	42	37	33	30	27	26	24	23	4 600
150	25	38	26	21	18	16	14	13	12	11	11	1 300
	100	43	30	24	21	19	17	16	15	14	13	2 100
	400	46	32	26	23	20	19	17	16	15	14	3 500

Table 13. Number of 300-sq.m. plots, calculated by Equation (9).

Area, hectares	Mean volume, cu.m./ha. 100			150		
	Standard error, per cent ± 2.5	± 5	± 10	± 2.5	± 5	± 10
	Number of plots					
25	183	83	31	138	58	21
100	310	116	38	217	77	24
400	427	140	42	282	89	26

7. Summary and conclusions

This paper reports on tests made for the study of alternative methods in forest survey. Data were acquired by measurements in five areas, varying from 20 to 900 hectares in size. Four of the test areas are in Finland between 60 and 67 degrees N.lat., and one in Mexico at about 24° N.lat. The main area is 100 hectares in size; this was measured in 10 000 squares, each of 100 sq.m. On the basis of tree tally, for each plot there was calculated the basal area and volume of the entire stock and the stock in trees exceeding 20 cm. D.B.H. The principal characteristic used in the analysis was the entire volume. By the combination of neighbouring-plots, the variation could be studied for different plot sizes and survey strips. Variable (relascope) plots could also be compared; on these plots, it was found necessary to make careful checks by means of caliper and measuring tape for the exclusion or inclusion of boundary trees, to avoid systematic deviations.

As a starting point for the comparison of different sampling methods, calculations were made of the coefficients of variation for each plot type; total and within the strata. The amount of decrease of variation with an increasing plot size could be established. The effect of stratification was rather similar, irrespective of the basis for differentiating strata: treatment class, entire volume, or dominant height and volume. Most of the calculations for stratified sampling methods in this study were based on the groups of treatment classes.

Comparisons have been made of the following sampling methods: simple random, stratified random, simple systematic, and stratified systematic sampling. For random surveys, an estimate of the variance of sample mean could be calculated by the methods generally known. To permit of comparing systematic sampling, thousands of surveys were made with varying plot types and plot intervals. Comparisons between the different methods were effected by means of an equal plot number in each case.

On comparisons of the standard error of sample mean it was found that in both stratified random sampling and different types of systematic sampling there is, with increasing size and diminishing interval of sample plots, an increase in the relative improvement of the result as against simple random sampling. Only in exceptional cases did systematic surveys give results which were less precise than those derived by other methods. As in addition the application of systematic sampling is relatively easy, it was decided to take the results of simple uniform systematic sampling as a basis for the calculation of relative numbers of sample plots of different types.

In discussion of some methods for determination of the precision of systematic sampling, brief consideration was first given to the possibilities of theoretical determination of the degree of precision, by means of a mathematical model of the forest and sampling, and not by some equations applied to sample values. Secondly, an empirical study was made of the behaviour of some equations based on the sample itself. The larger the plot size and the shorter the plot interval, the more the equations overestimated in general the variance of sample mean.

As none of the equations studied gave reliable results, regression equations were calculated for the relative standard error on the basis of the data measured. The independent variables were plot size, plot or strip interval, area of survey unit, and mean volume. From these equations, there was calculated the number of 100 to 1000 sq.m. plots and the length of survey strip 10 metres in width for the following combinations: forest area 25, 100, or 400 hectares, standard error $\pm$ 2.5, $\pm$ 5, or $\pm$ 10 per cent, and mean volume 100 or 150 cu.m./ha.

The results arrived at are based mainly on the complete measurement of one area only. To enable extension of the scope of application, there is needed more material with a complete enumeration of trees; the acquisition of this material is already under way.

In continued studies, more work is also necessary on the variation to be found on variable plots and on combinations of fixed-area plots in which large trees are tallied on a larger circle than small ones. As regards systematic plot sampling, more attention should be paid to systems in which the plot interval along survey lines is less than the distance between lines.

Since the final aim of the investigations discussed here is to evaluate the relative efficiency of different alternatives, time studies need to be made and combined with variation studies. It is intended that the report on these studies will be presented in a further paper.

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SELOSTE:

ERÄIDEN METSÄNARVIOIMISMENETELMIEN TARKKUUDESTA

Tutkimuksessa tarkastellaan vaihtoehtoisia metsänarvioimismenetelmiä. Aineistoa on kerätty viideltä alueelta, joista pienin on 20 ja suurin noin 900 hehtaaria. Koealueista neljä on Suomessa 60. ja 67. leveysasteen välillä ja viides sijaitsee Meksikon mäntyalueella n. 24 leveysasteella. Tärkein on Evolta mitattu 100 ha:n suuruinen neliönmuotoinen alue, joka mitattiin 10 000 ruutuna, kukin 100 m². Suoritetun puidenluvun nojalla voitiin joka ruudulle laskea koko puuston ja rinnantasalta yli 20 cm:n puuston pohjapinta-ala ja kuutiomäärä, joista päätunnuksena tutkimuksessa käytettiin koko puuston kuutiomäärää. Naapuriruutuja yhdistämällä voitiin tutkia erisuuruisilla koealoilla esiintyvää vaihtelua. Myös relaskooppikoealoja koskevia vertailuja voitiin tehdä. Osoittautui, että relaskooppia käytettäessä on välttämätöntä suorittaa systemaattisia rajapuiden tarkistuksia kaulaimen ja mittanauhan avulla.

Eri otantamenetelmien vertailun lähtökohdaksi laskettiin kunkin koealatyypin edustama variaatiokerroin sekä kokonaisuutena että metsikköluokittain. Vaihtelun pieneneminen oli jokseenkin samanlainen riippumatta siitä, oliko luokitusperusteena kehitysluokka, kuutiomäärä vai valtapituus ja kuutiomäärä. Useimmissa tapauksissa käytettiin tässä tutkimuksessa luokitusperusteena kehitysluokkien muodostamia ryhmiä.

Seuraavia otantamenetelmiä verrattiin keskenään: yksinkertainen ja luokiteltu satunnaisvalinta sekä yksinkertainen ja luokiteltu systemaattinen valinta. Satunnaisvalintaa käytettäessä tulosten luotettavuus voitiin laskea yleisesti tunnetuilla kaavoilla. Käsityksen saamiseksi systemaattisten arviointien tarkkuudesta suoritettiin tietokoneen avulla tuhansia arviointeja eri koealatyyppejä ja -välejä käyttäen. Vertailut eri menetelmien kesken tehtiin käyttämällä kussakin tapauksessa samaa koealalukua.

Vertailujen tulokset osoittivat ensiksikin sen, että luokiteltua satunnaisvalintaa ja systemaattisia arviointeja käytettäessä koealojen koon suuretessa näiden lukumäärä vähenee voimakkaammin kuin yksinkertaista satunnaisvalintaa sovellettaessa. Systemaattiset arvioinnit antoivat vain poikkeustapauksissa heikompia tuloksia kuin muut menetelmät. Kun systemaattisten arviointien käyttö on lisäksi suhteellisen helppoa, päätettiin yksinkertaisen systemaattisen arvioinnin tulokset ottaa pohjaksi erityyppisten koealojen lukumäärän laskennalle.

Tarkasteltaessa menetelmiä systemaattisten arviointien luotettavuuden määrittämiseksi kiinnitettiin ensiksi huomiota teoreettisiin menetelmiin, jotka nojaavat matemaattisiin malleihin. Toiseksi tutkittiin empiirisesti eräiden näytteeseen perustuvien kaavojen ominaisuuksia. Mitä suurempi koealakoko ja mitä pienempi koealaväli, sitä enemmän ko. kaavat yleensä yliarvioivat keskivirheen.

Kun mikään tutkituista kaavoista ei antanut luotettavia tuloksia, laskettiin koottujen aineistojen perusteella regressioyhtälöitä kuvaamaan suhteellista keskivirhettä. Selittävät muuttujat olivat koealakoko, koeala- ja linjaväli sekä metsän pinta-ala ja keskikuutiomäärä. Näiden yhtälöiden avulla laskettiin 100—1 000 m^2:n koealojen lukumäärä ja 10 m leveän arviointikaistan pituus seuraaville yhdistelmille: metsäala 25, 100 tai 400 ha, keskivirhe $\pm$ 2.5, $\pm$ 5 tai $\pm$ 10 % sekä keskikuutiomäärä 100 tai 150 m^3/ha.

Tulokset perustuvat pääosin vain yhden alueen täydelliseen puidenlukuun. Soveltamismahdollisuuksien laajentamiseksi tarvitaan lisää aineistoa. Jatkotutkimuksia tarvitaan myös relaskooppikoealojen ja sellaisten kiinteäalaisten koealojen osalta, joilla suuret puut luetaan isommalta alalta kuin pienet. Enemmän huomiota olisi kiinnitettävä vielä menetelmiin, joissa koealaväli on pienempi kuin linjaväli.

Lopullisena tarkoituksena on eri vaihtoehtojen tehokkuuden arviointi, mikä edellyttää aikatutkimusten suorittamista ja näiden tulosten yhdistämistä vaihtelututkimuksiin. Tekijät toivovat voivansa selostaa näiden jatkotutkimusten tuloksia eri julkaisussa.