A FINANCIAL ANALYSIS OF A MONEY YIELD TABLE

BY

W. E. HILEY, M. A.,

LECTURER IN FORESTRY ECONOMICS,
IMPERIAL FORESTRY INSTITUTE, OXFORD

The prologed controversy between the Soil Rental and Forest Rental Theories has been fought on the plane of opinion rather than ascertainable fact and clearness of thinking can only come by a careful analysis of the financial positions which may be expected to be produced by the adoption of various systems of management. The controversy between the two schools has been waged chiefly round the issues of length of rotation and grade of thinning, and, though the latter question is receiving more attention at the present time, the present analysis is entirely confined to the question of rotation. It is necessary to select some money yield table for the basis of these calculations and I have consciously chosen a table in which the prices are out of date in order that the discussion may be maintained on an academic plane. It is a discussion in pure finance and the manner in which the results should be applied in forest policy is a matter that will be left to the reader.

The money yield table chosen for this analysis is SCHWAPPACH's table for Scots pine, Quality II (of 5 qualities) published in 1908. This table is based on a large number of measurements and price observations in Prussia and, as SCHWAPPACH's deductions are very different from the present writer's, it cannot be suggested that the table is unfairly constructed to show any particular conclusions. In this table allowance has been made for the difference between the actual volumes obtained in forest practice and the ideal yields of the yield table but, as no deduction has been made for loss by rot, insects or fire it has been thought advisable to reduce the money valuet given in this table for »remaining stands» by 25 per cent., so tha there may be no question of over-valuing the yields. Certain irre-

[1] Dr. A. SCHWAPPACH, Die Kiefer. Neudamm, 1908, p. 146.

gularities have been smoothed and the yields, both of remaining stands and thinnings, have been rounded off to the nearest 50 marks in order to simplify calculations.

TABLE I. SCHWAPPACH's money yield table for Scots Pine, Quality II, with remaining Yields reduced by 25 per cent. and values rounded off

Age	Remaining stand. marks per ha. Zu	Thinnings. mks. per ha. Du	Final Yield. mks. per ha. Au
10	40	—	40
20	200	—	200
30	700	100	800
40	1150	250	1400
50	1550	300	1850
60	1950	350	2300
70	2300	350	2650
80	2700	400	3100
90	3150	450	3600
100	3600	450	4050
110	4000	500	4500
120	4450	500	4950
130	4800	500	5300
140	5000	500	5500

It is assumed in the calculations that plantations are clear-felled when mature and planted the same winter at a cost of 80 mks. per ha., and that the annual cost of management is 6 marks per ha.; both these figures are SCHWAPPACH's estimates. It is further assumed that the sale value of the land is 100 mks. per ha. Thus C = 80, v = 6, B = 100.

On the basis of this money yield table the following amounts have been calculated.

1) The financial yield, or mean annual forest per cent. for various rotations.
2) The net annual income from a normal forest worked on various rotations and reduced to the average value per hectare.

3) The capital value of a normal forest worked on various rotations and reduced to the average value per hectare.
4) The net income expressed as a rate of interest on the capital value.

1) The Financial Yield. This has been worked out from the equation

$$B = \frac{Au + \Sigma\, Da \cdot 1.0p^{u-a} - C \cdot 1.0p^{u}}{1.0p^{u} - 1} - \frac{v}{.0p}$$

In this equation all the quantities are known for any rotation except p, and the financial yield is the rate of interest (p) which satisfies the equation. The solution is found for each rotation by trial and error and subsequent interpolation and the values are given to the nearest ,05 per cent. in Table IV Col. 2.

This value is the actual rate of compound interest earned on all the capital invested in the plantation when the final yield has been sold. It assumes that the land also is sold, but this may be reserved for another rotation and its value credited to the first rotation as a book entry. It will be seen that the highest rate of interest (4.45 %) is earned on a 40 year rotation and in this sense 40 years is the financial rotation.

2) The net annual income from a normal forest. The income from a single plantation is discontinuous and culminates in a very large amount when the plantation is felled. Thus no figure can be estimated as the annual income from a single plantation. It is easy, however, to estimate the annual income from an ideally normal forest in which an equal area is cut each year yielding an equal net profit and in which the costs remain constant from year to year. The income from a given area will be greatly influenced by the length of the rotation adopted on the area and in order to compare these incomes it is necessary to reduce the income to the average income per hectare.

The net income from u hectares is

$$A_u + D_a + D_b + \dots\dots - C - uv$$

The net income per hectare is, therefore,

$$(A_u + D_a + D_b + \dots\dots - C - uv)\frac{1}{u}$$

This amount has been computed for each rotation from 30 to 140 years (Table IV, Column 5). The income becomes greater as the rotation becomes longer and it appears to culminate at about 140 years. This is therefore the rotation of highest income.

3) The capital value of a normal forest. The next step is to estimate the various capitals on which these incomes are earned, i.e. the average capital value per hectare of ideally normal forests worked on various rotations. Various methods are set out in the textbooks of forest valuation for computing the capital value of a forest and none of these is entirely satisfactory. Most of them involve compound interest at some arbitrary rate and the calculated value can be made anything the writer likes by taking a suitable rate of interest. What is required is the price which the forest would fetch in an open competitive market and this will be determined, in the first place, by the realisable value of the growing stock and the value of the land. When the timber is ripe for felling then its standing value is the basis for valuation, and as, in the present case, the plantations are financially mature at 40 years all stands of 40 years or over can be valued at the price they would fetch for felling. When they are less than 40 years old, however, it is financially profitable to let them grow on, i.e. their expectation value is greater than their utilisation value. Thus the utilisation value of a 5 year old plantation is probably nil; but money has been invested in producing it and it clearly has a value to the forester.

The textbooks of forest valuation present two methods of calculating the value of an immature plantation; one gives the cost value, the other the expectation value. The cost value is obtained by bringing forward all costs at some rate of compound interest and deducting from the total the value of all income brought forward in the same manner. The expectation value is obtained by discounting all future returns at some rate of interest and deducting the discounted value of all future costs.

Applying these two methods to the problem of valuing a hectare of pine plantation 20 years old and adopting the money yield table costs and returns we obtain values in the following way:

a) Cost value. The costs of producing such a plantation are the prices of the land and planting, 20 years previously, and the annual expenditure each year. This amounts to

$$(B + C)\,1.0p^{20} + v(1 + 1.0p + \cdots + 1.0p^{19})$$

or

$$(100 + 80)\,1.0p^{20} + 6\left(\frac{1.0p^{20} - 1}{.0p}\right)$$

b) Expectation value. The highest return from the plantation will be obtained by felling it when it is 40 years old. The land will also then be available for sale or replanting. In addition a thinning worth 100 mks. will be obtained in the year 30, and 6 mks. must be spent per annum on maintenance. The value is therefore:

$$(A_{40} + B)\frac{1}{1.0p^{20}} + D_{30}\frac{1}{1.0p^{10}} - v\left(\frac{1}{1.0p} + \frac{1}{1.0p^2} + \cdots + \frac{1}{1.0p^{20}}\right)$$
$$= (1400 + 100)\frac{1}{1.0p^{20}} + 100\frac{1}{1.0p^{10}} - 6\,\frac{1.0p^{20} - 1}{.0p \times 1.0p^{20}}$$

Both these values will be greatly affected by the value chosen for p, the rate of interest, and a high value of p will increase the cost value but diminish the expectation value. Table II. shews the cost value and the expectation value if p is put at 3, 4, 4 1/2 and 5 per cent.

It will be seen from this table that the two values are nearest to each other when the calculation is made with 4 1/2 per cent. interest, and they will be equal if the rate of interest is just below 4 1/2 per cent. Now it has already been shewn that the financial yield on a 40 year rotation is 4.45 per cent. and it is quite easy to prove from the formulae that if the cost value and expectation value are worked out for

TABLE II. Cost and Expectation values of a plantation at 20 years.

Rate of interest %	Cost value marks	Expectation value marks
3	486	816
4	574	670
4 1/2	623	609
5	676	551

any age less than the financial rotation with a rate of interest equal to the financial yield, then the two values will be equal. From this it appears legitimate to assess the value of immature plantations at the cost value or expectation value (since they are equal) worked out with a rate of interest equal to the financial yield at the financial rotation (in this case 4.45 per cent.)

The cost value of a complete series of 1 hectare each of all ages from 0 to 39 years is the summation of the series

$$(B + C)\, 1.0p^q + \frac{v}{.0p}(1.0p^q - 1)$$

Where q has all values from 0 to 39. But for the ages 30 to 39 we must deduct $D_{30} \times 1{,}0p^{q-30}$ since a thinning D_{30} (= 100 mks) is taken at 30 years which has helped to pay for some of the cost of the older plantations. This summation is

$$(B + C)\frac{1.0p^{40} - 1}{.0p} + \frac{v}{.0p}\left(\frac{1.0p^{40} - 1}{.0p} - r\right) - D_{30}\frac{1.0p^{10} - 1}{.0p}$$

putting $p = 4.45$, this becomes

$$(100 + 80)\frac{5.707 - 1}{.0445} + \frac{6}{.0445}\left(\frac{5.707 - 1}{.0445} - 40\right) - 100\frac{1.546 - 1}{.0445}$$

$\doteqdot$ 25 650 marks.

The value may also be calculated as follows. If this forest of 40 hectares is worked as a normal forest the net income will be $A_{40} + D_{30} - C - 40\,v$

$= 1400 + 100 - 80 - 240 = 1180$ marks.

If this income represents the interest on a capital which yields 4.45 per cent., then the value of the capital is $\frac{1\,180}{4.45} \times 100$ or 26 500 marks. This value is slightly different from the cost value because the true financial yield on a 40 year rotation is a little less than 4.45 per cent. It is actually about 4.44 per cent. and the value of the complete series of age gradations (1 to 39 years) may be taken as 26 600 marks. We can now proceed to calculate the value of a complete series of age gradations from 0 to 49 years.

The series from 0 to 39 years is valued at 26 600.

The sale value of the timber on 1 hectare of age 40 after thinning is 1150, and of age 50 before thinning is 1850. If growth in value is constant between these ages the sum of the values of the timber on 1 ha. each of ages 40 to 49 is $\frac{1}{2}(11 \times 1150 + 9 \times 1850)$. To this we must add the value of 10 ha. of land which is 1000 marks. Therefore the value of 50 ha. covering the ages 0 to 49 is

$$26\,600 + \frac{1}{2}(11 \times 1150 + 9 \times 1850) + 1000 = 42\,250$$

Dividing by 40 and 50 respectively we see that the average value per ha. of a normal forest worked on a 40 year rotation is $\frac{26\,600}{40} =$ 665 marks, and of a normal forest worked on a 50 year rotation is $\frac{42\,250}{50} = 845$ marks. By continuing these processes to each successive decade the series of figures in Table IV, Column 3, is obtained.

This is not the only way of valuing a forest but for accountancy purposes it is probably the most satisfactory. It assumes that a prospective purchaser is at liberty to cut all the timber that is marketable without delay which can only be done if the forest is free from restrictions and not too large for immediate cutting. If the forest took 20 years to cut it would be worth rather less to a timber company, but for the use which is made of the figures in this chapter these reservations are not significant.

4) The net income expressed as a rate of interest on the capital value. This rate of interest is obtained by the formula $p = \frac{I}{C} \times 100$, where I is the income in Table IV, Column 5 and C the capital in Column 3. The result is given in Column 7.

If Column 7 is compared with Column 2, which is the financial yield, also an expression of income as a percentage on capital, it will be seen that the two are equal for a 40 year rotation but at other rotations they are unequal, and column 7 is invariably lower than column 2. The rate of interest as computed in column 7 will here be called the rate on realisable capital and the reason why the rate on realisable capital is not the same as the financial yield is as follows. The financial yield on a 100 year rotation is 3.80 per cent; this means that if all the costs of a single plantation are debited to a forest account at 3.8 per cent compound interest and all income is credited at the same rate then the two amounts exactly balance when the final yield at 100 years and the land have been sold. Translating this into terms of a normal forest it means that if the capital value of the normal forest is the cost value computed with a rate of interest of 3.8 per cent. then the income from the normal forest is 3.8 per cent. of this capital. Now the realisation value of a 40 year old plantation is equal to the cost value computed with 4.45 per cent. compound interest since if the plantation is felled then the investment yields 4.45 per cent. The realisation value of a 40 year old plantation is therefore greater than the cost value computed at 3.8 per cent. compound interest, and is thus greater than the value allowed to it in estimating the financial yield on a 100 year rotation. The same is true of all other ages, so that the realisation value of the normal forest as a whole is greater than the value allowed to it when the financial yield of 3.8 per cent. is calculated. The rate on realisable capital, on the other hand, is the rate of interest which the net income represents on the realisable capital of the forest and is thus lower than the financial yield [1]. The distinction between these two rates of interest has its parallel in ordinary modern investement. If an investor buys bonds at 20s. which yield 9 per cent. and these bonds subsequently appreciate in value to 30s. then the investor receives a rate of 9 per cent. on his original investment but only 6 per cent. on the market value of the bonds. In the same manner, if a normal forest managed on a 100 year rotation has been created and costs and returns follow the premises of this article, then the owner is obtaining a yield of 3.8 per cent. on the money invested in the forest but only 3.07 per cent. on the market value of the forest. This distinction only arises when the rotation in the forest is different from the financial rotation.

5) The principle of diminishing returns applied to the length of rotation. The longer the rotation in a normal forest of given type the greater is the capital invested in it per hectare or per acre. Also the longer the rotation, up to a point (viz. the rotation of highest income), the greater is the net income per hectare or per acre. Now let it be supposed, for the use of abstract reasoning, that the length of rotation can be changed at will. It will then be seen (Table IV: columns 4, 6 and 8) that for each increase of 10 years in rotation from 40 to 140 years there is an increase of from 180 to 210 marks per hectare in the capital invested and an increase of from 6.9 to 0.2 marks per hectare in net income. Each additional dose of capital yields an additional dose of income and each dose of income can be expressed as a rate of interest on the corresponding dose of capital.

[1] It should be noted that what is here called the realisable capital of the forest is slightly higher than the amount that could be obtained from felling it since a value has been given to immature plantations, i.e., those under 40 years. If, however, no value were attached to plantations under 40 years the average capital value per hectare of a normal forest conducted on a 100 year rotation would still be 1554 marks on which capital the net income represents a return of 3.59 per cent. This is also below the financial yield.

To make this clear the following figures are extracted from Table IV.

TABLE III. Application of the law of diminishing returns to the length of rotation.

Change in Rotation	Capital increase per hectare mks.	Income increase per hectare mks.	Rate of interest earned on increase of capital %
30—40	162	11.5	7.10
40—50	180	6.9	3.83
50—60	190	5.4	2.84
60—70	190	3.2	1.68
70—80	192	3.6	1.87
80—90	198	4.0	2.02
90—100	205	3.1	1.51
100—110	210	2.6	1.24
110—120	206	2.5	1.21
120—130	207	1.4	0.68
130—140	202	0.2	0.10

The figures are not quite continuous and no attempt has been made to »smooth» them. But it remains clear that in general each additional dose of capital yields a smaller return than the last so that the table demonstrates in a striking manner the law of diminishing returns. Up to 40 years, the financial rotation, the yield of each capital dose is greater than the highest financial yield obtainable on the whole rotation, i.e. 4.44 per cent., but beyond 40 years each dose yields a lower return than 4.44 per cent. and the yield of successive doses becomes progressively smaller. In agriculture and other industries it is customary to apply additional doses of capital up to a point when they cease to yield a satisfactory return. In this case 4.44 per cent. is the highest financial yield that can be obtained and, since, under present conditions, this is lower than the rate of interest that can be earned in other investments, purely financial considerations would lead us to forego the application of further financial doses of capital when they ceased to yield this rate of interest.

This analysis provides a new definition for the rotation of highest income. It is the rotation at which any money invested in lengthening the rotation still further would yield no return. In the case of Scots pine, quality II, with the data used in this chapter the rotation of highest income is approximately 140 years and it is probable that, if the rotation were further lengthened, the income per hectare would be reduced though the capital invested would be increased.

The rotation of highest income has frequently been advocated as having a financial raison d'être. This is entirely fallacious and, though there may be good reasons for employing a rotation in excess of the financial rotation, the rotation of highest income, as such, has no financial attractiveness and should therefore not be made a basis for fixing the rotation.[1]

The graph in Fig. 1 shews the relation between the capital invested per hectare in a normal forest and the net income derived from this capital. The highest rate of interest is received when the proportion of income to capital is at its maximum, i.e. where the tangent through the origin strikes the curve (*P*). This point on the curve represents the relation of income to capital at the financial rotation. The income is greatest at the point *Q*, and this point represents the relation of income to capital at the rotation of highest income.

A great deal of care would have to be exercised in applying the results of these calculations to a forest of quality II pine; nevertheless they shew that the practice of working pine forests on very long rotations, which is found in Germany and elsewhere, has little in its favour from an economic standpoint and is likely to bring forestry into financial disrepute. Thus, if a forest of Quality II pine which is worked on a rotation of 120 years could be converted into a forest with a rotation of 80 years, then the capital value (pre-war figures) would be reduced from 2236 mks. to 1417 mks. per ha.,

[1] As is done today in many forests in Central Europe and is advocated for America.

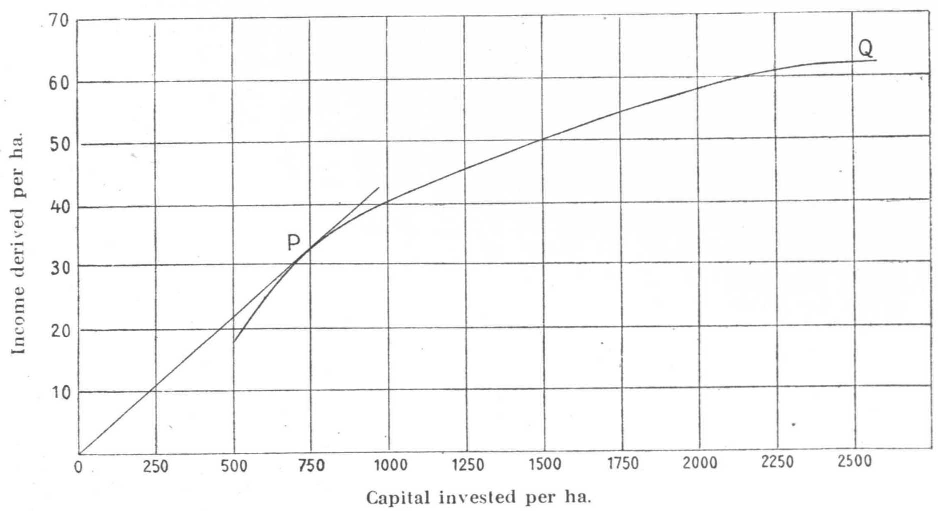

Fig. 1. Relation between the capital invested per ha. on various rotations and the net income derived from it.

a reduction of 819 mks. At the same time the net income would be reduced from 60.8 mks. to 48.7 mks. a loss of 12.2 mks. This capital, which might be taken out of the forest, is only earning 1.5 per cent. and the remaining capital in the forest would earn over 3.4 per cent.

If such a change were made in the rotation the relative supply of various timber sizes would be materially altered and this might be expected to reduce the market price of small sizes and increase that of large sizes. A new money yield table would then have to be constructed and it is probable that the financial rotation would then be more than 40 years. For this reason any reduction in the working rotation of a forest should be gradual

SUMMARY.

SCHWAPPACH's money yield table for *Pinus silvestris*, Quality II has been modified to allow for loss by fungi, insects and fire and has then been subjected to a detailed analysis with the following results.

1) The »financial yield» or mean annual forest per cent. culminates at a rotation of 40 years when its value is about 4.45 per cent.

progressively lower rate of interest; the dose which is required to raise the rotation from 130 to 140 years yields only 0.1 per cent. The income from the successive capital doses required to lengthen rotations follows the law of diminishing returns and the rotation of the highest net income is the rotation at which an increased dose of capital yields no income whatever. The rotation of highest income has, therefore, no financial attractiveness.

2) The rotation of highest net income is about 140 years when this income is 62.4 mks.

3) The capital invested per hectare is much greater for a long rotation than a short one and each increase of 10 years in the rotation involves an increase of about 200 marks in the capital invested per hectare.

4) Each successive dose of capital, which is required if the rotation is increased by jumps of 10 years from 30 to 140 years, yields a

NOTE. When I wrote this article I had not read VON SPIEGEL's Praktische Waldwertrechnung auf wirtschafttheoretischer Grundlage (Hannover, 1926). VON SPIEGEL has calculated the rate of interest on realisable capital earned by each quality class of pine, spruce, beech and oak, and has obtained very valuable results. His estimated rates of interest are lower than mine because he worked with higher cost prices of land, planting and annual maintenance.

TABLE IV. Deductions from money yield table for Scots Pine, Quality II.

Rotation	Financial Yield	Capital value per ha.	Capital increase per ha.	Annual income per ha.	Income increase per ha.	Ann. income as percentage of capital	Income increase as percentage of cap. increase
1	2	3	4	5	6	7	8
years	%	marks	marks	marks	marks	%	%
30	3.90	503		18.0		3.58	
			162		11.5		7.10
40	4.45	665		29.5		4.44	
			180		6.9		3.83
50	4.40	845		36.4		4.31	
			190		5.4		2.84
60	4.30	1035		41.8		4.04	
			190		3.2		1.68
70	4.10	1225		45.0		3.67	
			192		3.6		1.87
80	3.95	1417		48.6		3.43	
			198		4.0		2.02
90	3.95	1615		52.6		3.25	
			205		3.1		1.51
100	3.80	1820		55.7		3.06	
			210		2.6		1.24
110	3.70	2030		58.3		2.87	
			206		2.5		1.21
120	3.65	2236		60.8		2.72	
			207		1.4		0.68
130	3.60	2443		62.2		2.54	
			202		0.2		0.10
140	3.55	2645		62.4		2.36	

W. E. Hiley