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Reijo Mykkänen

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Editorial Office Unioninkatu 40 A, FIN-00170 Helsinki, Finland
Phone +358 0 857 051, Fax +358 0 625 308, E-mail silva.fennica@metla.fi

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Reijo Mykkänen

Aspiration-Based Utility Functions in a Planning Model for Timber Flow Management

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The study presents a theory of utility models based on aspiration levels, as well as the application of this theory to the planning of timber flow economics. The first part of the study comprises a derivation of the utility-theoretic basis for the application of aspiration levels. Two basic models are dealt with: the additive and the multiplicative. Applied here solely for partial utility functions, aspiration and reservation levels are interpreted as defining piecewisely linear functions. The standpoint of the choices of the decision-maker is emphasized by the use of indifference curves. The second part of the study introduces a model for the management of timber flows. The model is based on the assumption that the decision-maker is willing to specify a shape of income flow which is different from that of the capital-theoretic optimum. The utility model comprises four aspiration-based compound utility functions.

The theory and the flow model are tested numerically by computations covering three forest holdings. The results show that the additive model is sensitive even to slight changes in relative importances and aspiration levels. This applies particularly to nearly linear production possibility boundaries of monetary variables. The multiplicative model, on the other hand, is stable because it generates strictly convex indifference curves. Due to a higher marginal rate of substitution, the multiplicative model implies a stronger dependence on forest management than the additive function. For income trajectory optimization, a method utilizing an income trajectory index is more efficient than one based on the use of aspiration levels per management period. Smooth trajectories can be attained by squaring the deviations of the feasible trajectories from the desired one.

Keywords: bounded rationality, income trajectories, capital theory, intertemporal choice, resource economics.

Author's address: Pellervo Economic Research Institute, Revontulentie 8 A, FIN-02100 Espoo, Finland. Fax +358 0 463 041.

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Contents

PREFACE	5
LIST OF SYMBOLS	6
1 INTRODUCTION	9
1.1 Aspiration-Based Planning	9
1.2 Core of Timber Management Planning	11
1.3 Overview of Previous Studies	12
1.4 Need for Analytical and Numerical Methods	13
1.5 Purpose of the Study	14
2 DERIVATION OF THE UTILITY-THEORETIC BASIS	15
2.1 Scope of the Analysis	15
2.2 Compound Utility Functions	15
2.2.1 Function Forms	15
2.2.2 Relative Importances	16
2.3 Partial Utility Functions	16
2.4 Indifference Curves	17
2.4.1 Relationships Between Utility Functions and Indifference Curves ..	17
2.4.2 Restrictions of the Analysis	18
2.4.3 Aspiration-Based Indifference Curves	18
2.4.4 Relationships Between the Additive and the Multiplicative Form ...	20
2.4.5 Characterization of the Optimal Choice	21
2.4.5.1 Location of the Aspiration Point	21
2.4.5.2 Additive Form	22
2.4.5.3 Multiplicative Form	24
2.5 Utility Maximization	25
2.5.1 Analytical Method	25
2.5.2 Outline of an Iterative Solution Procedure	27
3 A MODEL FOR TIMBER FLOW MANAGEMENT	29
3.1 Overview	29
3.2 Planning Horizon	29
3.3 Ingredients of the Model	30
3.3.1 Concepts of Exploitation and Conservation	30
3.3.2 Intertemporal Choices	31
3.3.3 Formulation and Computation	32
3.3.4 Consideration of Income Trajectories	34
3.3.5 Production Possibility Boundaries	35
3.3.5.1 Theoretic Assumptions	35
3.3.5.2 Final Net Income and Final Capital	36
3.3.5.3 Final Net Income, Final Capital and Return on Capital	36
3.3.5.4 Return on Capital and Temporal Distribution of Income	37
3.3.5.5 Periodical Net Income	37

3.4	Utility Model	38
3.4.1	Compound Utility Functions	38
3.4.2	Partial Utility Functions	39
3.4.3	Maximization of Utility	39
4	NUMERICAL TESTING OF THE MODEL	41
4.1	Purpose and Basic Data	41
4.2	Choices of the Decision-Maker	41
4.3	Optimizing Income Trajectories	46
4.3.1	Methods to Control Income Flow	46
4.3.2	Comparison of Methods Using the Income Trajectory Index	47
4.3.3	Methods Using Periodical Net Income	47
4.4	Endogenous vs. Exogenous Determination of Income Trajectories	48
4.4.1	Profitability Maximization	48
4.4.2	A Sustained Yield Formulation	50
4.4.3	Profitability and Evenness	51
5	DISCUSSION	52
5.1	Main Results	52
5.2	Evaluation	53
5.2.1	Resource Model for Timber Flow Economics	53
5.2.2	Utility Model	54
5.2.2.1	Compound Utility Functions	54
5.2.2.2	Partial Utility Functions	55
5.2.2.3	The Search for the Solution	56
5.2.3	Aspects of Income Flow Determination	57
5.3	Conclusions and Future Work	58
	REFERENCES	59
	Appendix. Generating Alternatives with Simulation	64

Preface

The present study is a document on an individual project with only loose connections to any official research programs, and the contents of the study consequently reflect my own research ideas. The subject of the dissertation has its roots in my professional background. Several years as a systems expert in private forestry proved to me that conceptual simplicity is one of the most important aspects of planning models. Moreover, aware of the importance of theoretic consistency I have made a serious effort to form a theoretically well-based model framework.

I would like express my thanks to Dr. Heikki Vesikallio, who was one of the first contributors to this study. He aroused my interest in the capital-theoretic elements of forest management. I am very grateful for the contribution of Prof. Pekka Kilkki, who repeatedly encouraged me to participate in research work. Many of his ideas and initiatives can be recognized in this study. I am indebted to Prof. Timo Pukkala, whose guidance and suggestions were a great help particularly during the first phases of the study.

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I have prepared this dissertation on my own time in addition to my full-time job at Oy Silvadata Ab. I wish to express my warmest thanks to the personnel of Silvadata for supporting me during the hardest times of my research. Acknowledgement is also due to the board of Silvadata, which offered financial support for the language check.

Finally, my deepest gratitude goes to my wife Marjatta and my daughter Inka for their patience, support and understanding during the years I spent with this study.

Espoo, December 1994

Reijo Mykkänen

List of Symbols

NOTE: In scalar notations, the superscripts used in vectors and matrices are replaced by subscripts, e.g., FNI_A and FNI_A and vice versa.

Abbreviations

CONI 1–3 computation method of ZNI, FNI and CNI, respectively

FH1–FH3 forest holdings 1–3

Determinants

$|H|$ Hessian determinant

Functions

B_c budget constraint

B_p production possibility boundary

f unspecified function (to be defined later)

F_g function of the natural growth rate of a resource stock

f_w probability density function of Weibull

F_w cumulative distribution function of Weibull

f_v value growth

g constraint

h rate of removal (harvesting)

I indifference curve

I_a indifference curve of the additive model

I_m indifference curve of the multiplicative model

L Lagrangian function

RM_1 Resource Model 1

RM_2 Resource Model 2

U utility function

V stumpage value of the growing stock, $V=V(F_g, P, t)$

I,II,

III,IV compound utility functions

Matrices

a second-order relative importances

C silvicultural costs (logging costs excluded)

D diameters

I felling income

n^s number of schedules per period and compartment

N number of stems

P^B prices of timber assortments at the beginning of the period

P^E prices of timber assortments at the end of the period

Q contribution of schedules to each objective

R removal percentages

S contribution of current schedule to each objective

u second-order partial utility functions

V volumes of timber assortments

V^B volumes of timber assortments at the beginning of the period

V^E volumes of timber assortments at the end of the period

Operators and denotations

\max maximum element of a set, $\max \{x_1, \dots, x_n\}$

Max maximization

x_Δ value of x after a change

Δx change in the value of x

x^* optimal value of x

x^*, x^*_{II} optimal values of x yielded by functions I and II

x_0 initial value of x

Ordered pairs

A aspiration point

E_p production optimum

E_p^I production optimum produced by function I

E_p^{II} production optimum produced by function II

$P_{I/U=x}$ point of an indifference curve with utility equal to x

P_{LC} left corner of an indifference curve

P_{RC} right corner of an indifference curve

P_T tangency point

Scalars

a, b, c parameters of the Weibull function

A age of the growing stock, years

A_h asymptote of the height function

CNI final net income, sum of **FNI**

d diameter of a tree, cm

d_5^{EP} growth forecast for 5 years, peatlands

d_5^{EU} growth forecast for 5 years, uplands

d_{ref}^E basic growth

dz_2/dz_1^{\max} maximum gradient of indifference curves of the multiplicative model

dz_2/dz_1^{\min} minimum gradient of indifference curves of the multiplicative model

D basal area median diameter, cm

DD number of degree days

$|D|_{\max}$ maximal single deviation from a point of an objective trajectory

FC final capital

FNI final net income, compounded sum of **NI**

g basal area of a tree, m^2

g_{\max} basal area of the largest tree, m^2

G basal area, m^2/ha

G_u basal area of trees larger than the subject tree, m^2/ha

h height of a tree, m

i interest rate, discrete time

i_B interest rate, borrowing

i_L interest rate, lending

IC initial capital

ITI income trajectory index

ITI_c income trajectory index, even temporal distribution

k undefined constant

n_c number of compartments

n_M number of management plans

n_P number of management periods

n_T number of stems per hectare

N length of the planning horizon

NI net income, general expression

\bar{NI} average net income

p number of objectives

p_d mortality forecast for a tree for the next five years

P management period

P_{np} length of the last management period

r_F rate of value growth of forest

\bar{r}_F average rate of value growth of forest

RNI relative net income

ROC return on capital

SAD sum of absolute deviations

SAD_{\max} maximum sum of absolute deviations

SD sum of deviations expressed by SSD, SUD or SAD

SD_{\max} maximum of SSD, SUD or SAD

SSD sum of squared relative deviations

SSD_{\max} maximum sum of squared relative deviations

SUD sum of unsigned relative deviations

SUD_{\max} maximum sum of unsigned relative deviations

t time

T management period

T_B beginning of the planning horizon

T_E end of the planning horizon

TNI periodical net income

U utility

V stumpage value of the growing stock

x decision variable

x_S stock size

z objective variable

ZNI net income, intertemporal sum with 0 % interest rate

λ Lagrangian multiplier

Sets

N^+ positive integers

Vectors

a relative importances

c slopes of partial utility functions

C temporal consumption

d^L lower limits of diameter classes

d^U upper limits of diameter classes

E extreme values in trajectory comparisons

FC final capital

FNI compounded net income per period

FNI^A aspiration levels of **FNI**

G^{FC/FNI} gradients of indifference curves between **FC** and **FNI** (functions I,II)

G^G	goal level of the basal area	V^E	stumpage values at the end of the planning period
i	interest rates	V^T	stumpage values in the treatment year
IC	initial capital	VG	value growth
I^T	felling income from treatment years	W	maximum temporal consumption
n	number of stems	x	decision variables
n^s	number of schedules per compartment	Y	temporal income
NI	net income per period	z	objective variables
NI^A	aspiration levels of net income per period	z^A	aspiration levels
NI^0	net income per period, reference values	z^C	chosen levels of objective variables
NI^1	net income per period, values after a change	z^{max}	maximum values of objective variables
P	prices	z^{min}	minimum values of objective variables
r^F	rates of value growth	z^{umax}	utility-maximizing levels of objective variables
RNI	relative net income (desired trajectory)	z^{umin}	lowest acceptable values of objective variables (reservation levels)
RNI^a	relative net income (alternative trajectory)	β	estimated parameters of basal area growth models
ROC	return on capital	γ	estimated parameters of basal area growth models
T^T	treatment years	ω	shape parameters of the height function
TV^E	stumpage value of the thinned growing stock at the end of the period	κ	parameters of the asymptote of the height function
u	partial utility functions	ζ	parameters of the mortality function
u^{min}	minimum values of partial utilities		
u^{max}	maximum values of partial utilities		
U	utility values of functions I–IV		
U^*	utility values (currently nonoptimal schedules for one compartment)		

1 Introduction

1.1 Aspiration-Based Planning

A plan is a predetermined course of action. Various plans are the outcomes of a procedure which involves essentially the same steps as any rational choice or decision. According to Banfield (1973), a rational decision has three underlying elements that are also characteristic of planning processes:

- (1) All the alternative courses of action are listed.
- (2) All the consequences of all the possible courses of action are identified.
- (3) The course of action with the preferred set of consequences is chosen.

Maximization of utility is assumed to be the ultimate goal of rational decision-makers. This assumption is taken as the starting-point in evaluating consequences and alternative plans. Such evaluation, however, calls for the formation of a utility function. A utility function is a mathematical transformation that associates a utility with each alternative so that all alternatives may be ranked (Cohon 1978). Utility functions, thus, capture the preferences of the decision-maker. The estimation of the utility functions presupposes that these preferences are known, i.e., they have to be inquired. On the basis of this information, compound (multiattribute) utility functions can be used to provide the overall utility produced by the examined objective variables (e.g. Keeney and Raiffa 1976, Starr and Zeleny 1977, von Winterfeldt and Edwards 1988).

For computational convenience, the utility functions of the decision-maker are usually expressed in a mathematical form. However, it is often difficult, if not impossible, to find a mathematical presentation for these functions (Halme 1992). Estimation procedures tend to be laborious at the least (Tell 1976) and, in addition, these methods have to cope with the possible inconsistencies of the decision-maker (e.g. von Winterfeldt and Edwards 1988).

The principle of satisficing decision-making (e.g. Simon 1979, Wierzbicki 1980a) – also called “bounded rationality” – states that the decision-maker does not strictly optimize when making decisions (e.g. Lilly 1994). This is due to difficulties in the optimization operations, uncertainty of the decision environment, and complexity of the decision situation. Instead, so-called aspiration levels are used to evaluate the various outcomes of decisions. Aspiration levels represent those values of the objectives that can be accepted as reasonable or satisfactory by the decision-maker. (Lewandowski and Wierzbicki 1989)

The theory behind satisficing decision-making is based on the observation that every decision is actually a compromise (Simon 1979). If the decision-maker recognizes this, it is possible to apply an approach involving the determination of desirable levels of the objectives to find a compromise solution (e.g. Davis and Olson 1985). This process seems to be justifiable because of the purposive behavior of the decision-maker originating from his/her learning ability (Simon 1979, Honkapohja 1993). In this sense, a distinction can be made between an “economic man” who maximizes, and an “administrative man” who satisfices (Davis and Olson 1985), i.e., tries to meet a certain aspiration level (e.g. Keltikangas 1971).

In its full extent, the conceptualization of bounded rationality comprises the following items (Wall 1993):

- (1) Decision-making is dominated by the effects of complexity on the limited abilities of humans to process large amounts of information.
- (2) New solutions are synthesized by modifying the current solution (local search).
- (3) Alternatives are considered one at a time, not simultaneously (sequential search).
- (4) The search for a new and better solution is undertaken only when deemed necessary, i.e., when goals are not being met.
- (5) A satisficing mode is utilized: the first solution

that is good enough is implemented.

- (6) Goals are stated in terms of aspirations, which are formed by adaptation and learning from experience.
- (7) Search strategies are developed on the basis of learning and adaptation through experience.
- (8) The attention paid by the decision-maker to the environment is the result of learning and adaptation driven by experience.

The concept of utility maximization can be interpreted using the mathematical framework of satisficing decision-making (Wierzbicki 1986). By virtue of this interpretation, it is assumed that the decision-maker has a nonstationary utility function that changes in time due to learning. Intuitive and tentative utility maximization determines the aspiration level (Lewandowski and Wierzbicki 1989).

Microeconomic theory assumes that utility functions are continuous, increasing, strictly quasiconcave and twice differentiable (Malinvaud 1985). A strictly quasiconcave utility function has a bell-shaped form (Fig. 1a) and it implies a diminishing marginal rate of substitution between objectives (e.g. Henderson and Quandt 1985). Kallio et al. (1985a) have presented an outline of utility functions based on aspiration levels (Fig. 1b). This kind of function form is typical, e.g., in business management where the implementation of a project requires a certain amount of liquid assets.

Aspiration levels as suggestions of tentative utility maximization result in an iterative procedure to find the final form of a utility function. This approach is consistent with the category of the interactive estimation process mentioned by Tell (1976). The iterative method can also be applied to the determination of scaling factors in multicriteria utility models (Malakooti et al. 1994).

In its simplest form, the aspiration-based method presupposes knowledge about the aspiration (desired) level and the reservation (minimum) level based on an inquiry (Halme 1992). The iterative estimation process, based on successive formulations and presentations of solutions to the decision-maker, is particularly promising, since there is nothing to guarantee that a once-and-for-all estimation of a utility function yields a satisfactory result. Besides, aspiration levels are often seen as more intuitive than the weights of objectives thereby providing a natural representation of preferences in multicriteria decision problems (Vetschera 1994).

Due to the consecutive formulations and examination of solutions, the aspiration-based, iterative estimation method immediately yields information about the sensitivity of the solutions. In addition, the decision-maker has direct contact to the computation model for observing the effect of any changes in the aspiration levels. This method, which comprises formulating, solving and learning phases, is of substantial importance in business management (Jääskeläinen 1971). The itera-

tive method also contributes to a general understanding of the characteristics of an efficient solution (Vetschera 1994). This aspect is especially important if the decision-maker is unaware of whether the found solution is the most preferred (Halme 1992).

1.2 Core of Timber Management Planning

Timber production still seems to be one of the most important elements in forest management. According to Ihalainen (1992), about 54 percent of forest owners (71 percent of farmers) regard timber production as the primary objective of forest ownership. This percentage clearly exceeds the importance of the other objectives dealt with in that study. The result seems to imply that even though environmental factors are given increasing emphasis (Pukkala 1992, Kangas et al. 1992, Ovaskainen and Kuuluvainen 1994), forest owners are still dependent on the income flow from timber production.

Timber management has solid points of contact to profitability in forestry. As this topic was a frequent subject of debate in Finland in the 1980s, a specific project was established to find out the consequences of forest treatment methods under different economic circumstances (Mykkänen 1988a, Mykkänen 1988b, Simula 1988). The issue of profitability has also been studied by several authors in the past (e.g. Kilkki 1968a, Kilkki 1968b, Hämäläinen 1973, Hämäläinen 1982, Ringbom 1985), and new projects with different leading ideas have been launched (Harvennushakkuiden... 1992, Kinnunen et al. 1993, Simula 1994). The continuous interest in the profitability of forestry indicates the importance of this subject.

Sustained yield, on the other hand, is commonly considered as the main thread in forest management (e.g. Kilkki 1989, Hänninen and Karppinen 1991). Actually this concept has two primary objectives: conservation of the forest resource stock and determination of timber flow. Both of these are closely related to security (Jacobsson 1986, Ovaskainen and Kuuluvainen 1994). The aspiration to regulate timber flows usually originates from money needs related to, e.g., real investments and consumption (e.g. Archer et al.

1983, Karppinen and Hänninen 1986, Tikkanen and Vehkamäki 1987, Järveläinen 1988).

Economic theory presumes that consumption decisions and timber production decisions are independent (separable) in circumstances of a perfect capital market (Hirshleifer 1970). This being the case, the decision-maker can borrow and lend unlimited amounts of money with a known interest rate. In practice, the capital market is imperfect for at least the following reasons. First, only limited amounts can be borrowed. Second, the marginal cost of borrowing increases. Third, the interest rate for borrowing differs from that of lending. (Nautiyal 1988)

Capital-theoretic studies of forest management, in fact, show evidence of market imperfections. According to Kilkki (1968a), forest owners fail to follow the cutting policy based on present value maximization because of liquidity constraints (for theoretic analysis, see Ollonqvist and Kajanus 1992). This is why forest owners tend to be "income-oriented" instead of "investment-oriented" (Ollikainen 1984). It follows that the decision-maker has to set a certain minimum level for the felling income. In reality, liquidity constraints along with nontimber benefits result in consumption and felling decisions being made simultaneously (Ovaskainen and Kuuluvainen 1994). Unlike what is stated in the separation theorem (Dixit 1976), the preferences of the decision-maker do affect the short-term timber supply (Kuuluvainen 1989).

Timber management planning deals primarily with the regulation of timber flows over time. For most forest owners, timber flows as such are meaningless; monetary elements are incorporated through the timber and the capital market to yield the desired economic outcome. The inclusion of forest dynamics in the economics of the decision-maker calls for the use of planning models. Economic planning models combine the prediction of future events and the estimation of economic characteristics to trace the consequences of different management strategies. A utility model is one which takes into account the preferences of the decision-maker.

Empirical studies have shown that the planning model referred to in this context implies the following assumptions:

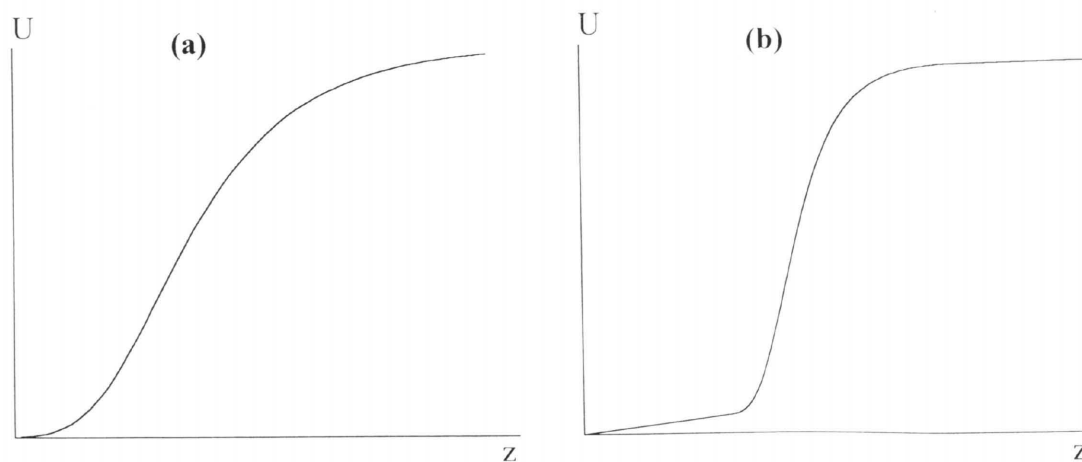


Figure 1. Strictly quasiconcave (a) and aspiration-based (b) utility functions (Kallio et al. 1985a).

- (1) The decision-maker wants to guarantee a certain income level.
- (2) The decision-maker wants to control exogenously the size of the forest resource stock.
- (3) The decision-maker wants to allow for a predetermined intertemporal consumption pattern when making cutting decisions.
- (4) Cutting order is determined by profitability criteria.

The underlying purpose of a model that fulfills these conditions is to achieve the appropriate management of a forest resource stock and timber flows, representing the consumption of the stock.

The first two points pertain to the realm of the economics of renewable resources (e.g. Clark 1976, Neher 1990): a resource stock can either be consumed or conserved. This approach can be depicted by resource models with a differential equation form (Conrad and Clark 1987)

$$\frac{dx_S}{dt} = F_g(x_S) - h(t) \quad (1)$$

which implies, e.g.,

$$h(t) > F_g(x_S) \Rightarrow \frac{dx_S}{dt} < 0 \quad (2)$$

resulting in reduction of the stock size. The implications of this general model will be used as a guideline throughout the timber management part of this study.

The last two points are related to the timing of treatments as well as to income flows. The third point assumes that the decision-maker has an exogenously determined consumption schedule based on preferences, whereas the fourth one searches for an endogenous treatment order, i.e., one based on the optimal use of resources.

1.3 Overview of Previous Studies

Modern forest management planning in Finland is largely based on the ideas of Kilkki (1968a). The essence of the approach is to use numerical simulation with mathematical optimization to find the most feasible management plan. This fundamental work was further developed by Pökälä (1973), Kilkki and Pökälä (1975) and Kilkki et

al. (1975). Stem-level operations were combined to the framework of simulation and optimization in the MELA cutting budget (Kilkki 1985). The basic methodology has been complemented by, e.g., profitability comparisons (Mykkänen 1988b) and multiobjective optimization (Pukkala 1988, Kangas and Pukkala 1992, Lappi 1992, Pukkala and Kangas 1993). The standpoint of the entire farm enterprise as a simultaneously optimizing unit is dealt with in the work of Hyttinen (1992) and of Hämäläinen and Kuula (1992).

Traditionally, an even flow of net income has been one of the most decisive criteria in evaluating utility produced by forest management (e.g. Jacobsson 1986, Kilkki 1989, Hänninen and Karpinen 1991). In addition to temporal distribution of income, net income level and interest rate have been the most important grounds for decision-making (Kangas 1992). This implies that production and consumption optima have not been considered as separable as the theory of the perfect capital market assumes. There is a need to consider consumption and production decisions simultaneously.

Profitability and sustained yield have often been considered complementary goals in forest management (Kilkki 1968a, Jacobsson 1986). A management alternative consistent with these goals is characterized by a high present value and an even and sustained yield of net income over time (Jacobsson 1986). Jonsson et al. (1993) state that a treatment option which generates a high degree of utility is a compromise between high net present value and a reasonable temporal distribution of net income.

If sustained yield is regarded as a constraint of the original model, methods of traditional mathematical optimization can be applied to find a feasible solution. Examples of establishing flow constraints have been presented in linear programming (Dykstra 1984) and goal programming (Buongiorno and Gilles 1987). Hof et al. (1986) applied a MAXMIN approach to maximize the minimum harvest during any time period in the planning horizon.

Theoretically, the management of intertemporal timber flows pertains to the category of trajectory optimization (Wierzbicki 1982). The task of trajectory optimization is to find a multiperiod path that most closely matches a goal trajectory

(Steuer 1986). Trajectory optimization has applications, e.g., in forest sector modeling (Kallio and Soismaa 1982, Kallio et al. 1985b), generation of energy supply strategies (Grauer et al. 1982), and macroeconomic modeling (Wierzbicki 1982).

Until recently, little attention has been paid to the utility theoretic formulation of forest management problems. In the model of Lappi and Siitonen (1985), the utility function was based on the maximization of even consumption. Kilkki et al. (1986) presented an application that involved the use of shadow prices in the derivation of the utility model of the decision-maker. Jacobsson (1986) chose a multiplicative, nonlinear utility model, because the use of a linear additive model results in an uneven temporal distribution of income. The utility model comprises parameters for interest rate and importance of sustained yield. By choosing the most appropriate net income trajectory, the decision-maker implicitly determines the values of these parameters. The values of the sustained-yield parameter may vary between 0 and 1: small values stress the evenness requirement, while values close to 1 refer to present values.

Contemporary utility-oriented studies of forest management incorporate preference estimation into numeric simulation and optimization. Kangas et al. (1992) applied the Analytic Hierarchy Process (Saaty 1980) to the formulation of a utility model for multiple-use planning. Pukkala and Kangas (1993), in turn, introduced a procedure in which utility was maximized by means of heuristic optimization. The additive utility function consists of partial utility functions estimated by means of the Analytic Hierarchy Process. In general utility-theoretic studies, interactive methods seem to be arousing increasing interest: examples are given by Corner (1994) and Malakooti et al. (1994).

Applications of satisficing decision-making have been implemented by reference point optimization (Wierzbicki 1980b) in several studies (e.g. Kallio et al. 1980, Kindler et al. 1980, Grauer et al. 1982, Kallio and Soismaa 1982, Lewandowski and Grauer 1982). Although utility functions are usually mentioned in the context of aspiration levels (e.g. Korhonen and Laakso 1986), there are no studies which combine the aspiration-level approach and the microeconomic utility theory. The main reason for this lack of research may be attributable to the different roles

assigned to preference representations. In utility-based examinations, the decision-maker is assumed to have a consistent system of preferences, whereas interactive procedures aim at forming preferences by means of learning (Vetschera 1994). The concept of bounded rationality, on the other hand, is a basis of the current research attempting to explain adaptive economic behavior (Lant 1992, Evans and Honkapohja 1993, Honkapohja 1993, Marinon 1993, Board 1994) to form decision-making models (Wall 1993, Norman and Shimer 1994).

1.4 Need for Analytical and Numerical Methods

The standpoint of microeconomic utility theory has attracted only limited attention in recent studies on forest management planning. This is unfortunate because an examination of the analytical background is a key element in understanding numerically produced results. On the other hand, aspiration levels have traditionally had only loose connections to utility theory. Therefore, the first part of the present study is devoted to developing a general framework of aspiration-based utility functions.

Because the aspiration-level method of forming utility functions differs from traditional economic theory, the derivation of theoretic fundamentals will be given particular stress. This approach results in a somewhat mathematical presentation. The chosen standpoint has definite advantages, however, since the derivation of an analytical framework contributes to a general understanding of the choices of the decision-maker.

Models for regulating timber or income flows are customary in theoretic examples of mathematical programming (e.g. Dykstra 1984, Buongiorno and Gilles 1987). Contemporary forest planning models are characterized by the lack of a flexible incorporation of intertemporal decision-making. The aspiration-level approach, however, seems to be particularly appropriate for this purpose. This is because the method has an inherent feature which allows for certain minimum and desired levels of objectives. Consideration of the desired income trajectories is, therefore, given special emphasis in the present study.

The focus in the model presented in this study is on income, since monetary variables are assumed to be of primary importance to the decision-maker. Despite this aspect, the outcome is termed "a model for timber flow management": the amount of timber to be cut is an essential decision variable, with income generation being one of the objectives.

1.5 Purpose of the Study

The study has three main aims:

- (1) To apply the aspiration-level approach to utility functions and to derive mathematical rules characterizing the choices of the decision-maker.
- (2) Based on the derived theoretic framework, to formulate a deterministic planning model for the economics of timber flow management.
- (3) To test the planning model on nonindustrial forest holdings.

The derivation of the theoretic basis will consist of the conceptualization and development of necessary inquiry and computation routines. The mathematical rules will be derived mainly by means of differential calculus to facilitate analytical examination.

The approach to model formulation will be general and oriented to natural resource economics. The forest resource stock is described by means of management units called compartments. The utility model will be composed on the basis of a few alternative formulations. Since the utility model consists of multiple objectives, a multiobjective optimization algorithm has to be included in the model.

Formulation of the model will be based on the theoretic considerations. The inquiry about the preferences of the decision-maker and other practical tests fall beyond the scope of this study. The use of numerical computations is, therefore, restricted to testing the consistency of the theoretic framework and the model.

2 Derivation of the Utility-Theoretic Basis

2.1 Scope of the Analysis

Before aspiration levels can be applied to utility models, a general theoretic framework has to be worked out. The following steps can be distinguished in the process:

- (1) Specification of function forms for compound utility functions.
- (2) Determination of the shape of partial utility functions.
- (3) Development of the inquiry and computation technique for determining the partial utility functions.
- (4) Indifference curve analysis to characterize the choices of the decision-maker.
- (5) Outline of solution methods.

Partial utilities are combined via compound utility functions to produce total utility. For achieving this, there are several function forms available (e.g. Tell 1976, von Winterfeldt and Edwards 1988). A decision must, therefore, be made as to what function forms to include in the examination.

The second point deals with defining an exact mathematical formulation of partial utility functions. This is closely related to the inquiry of the decision-maker's preferences and to the computation of partial utilities (3).

Points (4) and (5) attempt to outline the choices of the decision-maker from the standpoint of indifference curves and utility surfaces. This section is of particular interest because it combines production possibilities with aspiration-based utility functions. The final item involves the introduction of two tentative solution procedures.

2.2 Compound Utility Functions

2.2.1 Function Forms

A utility-theoretic basis will be derived here for two forms of compound utility functions: the additive and the multiplicative. These two forms seem to be the most widely used (e.g. Tell 1976, Jacobsson 1986, Kilkki et al. 1986, von Winterfeldt and Edwards 1988, Kangas 1992). In addition, these models are mathematically manageable, and although their utilization has sometimes been compared in the context of forest management economics (Jacobsson 1986), it has not involved any fundamental microeconomic analysis.

The additive function can be written

$$U = \sum_{i=1}^p a_i u_i(z_i) \quad (3)$$

and the multiplicative one

$$U = \prod_{i=1}^p u_i(z_i)^{a_i} \quad (4)$$

If

$$\sum_{i=1}^p a_i = 1 \quad (5)$$

the multiplicative function is linearly homogeneous. This can be seen by multiplication of j

$$U = \prod_{i=1}^p u_i(jz_i)^{a_i} \quad (6)$$

$$\Leftrightarrow U = j^{\sum_{i=1}^p a_i} \prod_{i=1}^p u_i(z_i)^{a_i}$$

This is

$$U = j \prod_{i=1}^p u_i(z_i)^{a_i} \quad \text{if } \sum_{i=1}^p a_i = 1 \quad (7)$$

Function (4) can be made separable by taking logarithms, that is

$$\ln(U) = \sum_{i=1}^p a_i \ln(u_i(z_i)) \quad (8)$$

In (3) and (4), the objective variables z are assumed to be utility-independent (Keeney and Raiffa 1976).

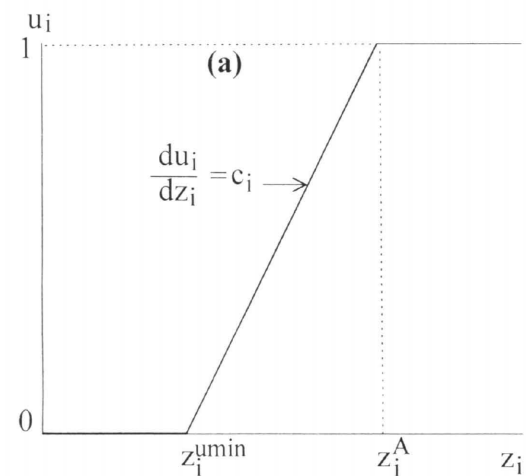
2.2.2 Relative Importances

The weight coefficients a determine the relative importances of the objectives. Equal weights indicate that the objectives are equally important. If some coefficient equals 0, the corresponding partial utility has no effect on total utility. By utilizing this feature, it is possible to eliminate objective variables from the compound utility model.

Before the utility function can be applied, the relative importances of the objectives have to be defined. Several different methods exist to determine the weights (Tell 1976, von Winterfeldt and Edwards 1988). They can also be evaluated directly (Render and Stair 1992). Direct evaluation is relevant if the only choices are 1 (included) and 0 (not included).

Problems related to the estimation of relative importances fall beyond the scope of this study. It is, therefore, assumed that the decision-maker is able to define the relative importances without difficulty. It is assumed that

$$\sum_{i=1}^p a_i = 1 \quad (9)$$



2.3 Partial Utility Functions

Functions (3) and (4) only determine techniques for computing total utility. This is because the content of partial utility functions remains undefined. To obtain computational capabilities, partial utility functions have to be determined for each objective variable.

The first aim of this study was to apply the aspiration-level approach to utility models. Aspiration levels – originating from satisficing decision-making (Simon 1979, Wierzbicki 1980a) – will, therefore, be used in defining the partial utility functions of single objectives.

Aspiration levels have been defined as reflecting those values of objectives that the decision-maker accepts as reasonable or satisfactory. On the other hand, aspiration levels also indicate when to stop optimizing (Lewandowski and Wierzbicki 1989). By this definition, it is assumed that

$$\begin{aligned} 0 < u_i(z_i) < 1, & z_i < z_i^A \\ u_i(z_i) = 1, & z_i \geq z_i^A \end{aligned} \quad (10)$$

if $u_i^{min} = 0$ and $u_i^{max} = 1 \quad \forall i = 1, \dots, p$

In other words, the aspiration level is interpreted as showing the point where maximum partial utility is reached. It is further assumed that the segment between u_i^{min} and u_i^{max} is linear. This definition constitutes a piecewisely linear partial utility curve (Fig. 2a; for mathematical implications, see Kannai 1992).

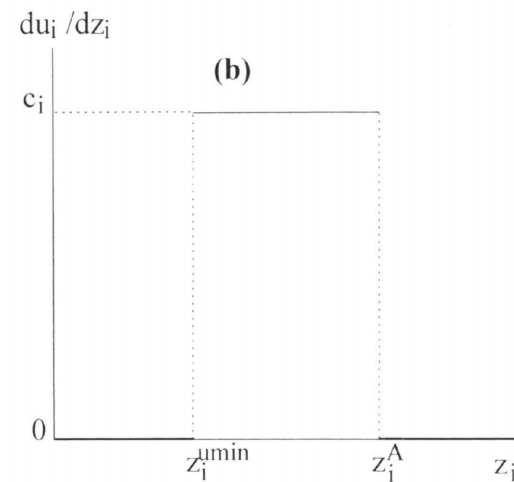


Figure 2. Piecewisely linear utility function (a) and corresponding marginal utility function (b).

By virtue of (10), the range of partial utility is restricted to between 0 and 1. This solution eliminates the differences of the measuring scale, i.e., partial utilities are made commensurate. In addition, scaling guarantees that the utility function is not weighted due to differing measurement units of the variables. Scaling of utility values is a normal practice, although it does not render the values produced by distinct utility functions comparable (Kangas 1992).

The aspiration-based method of composing partial utility functions may be interpreted from the standpoint of marginal utility (Fig. 2b). The increasing segment of the partial utility curve yields

$$\frac{du_i}{dz_i} = c_i, \quad z_i^{u,min} < z_i < z_i^A \quad (11)$$

where c_i is a positive constant. The horizontal plane segments of the partial utility curve produce

$$\frac{du_i}{dz_i} = 0, \quad z_i^A \leq z_i \leq z_i^{max} \vee z_i^{min} \leq z_i \leq z_i^{u,min} \quad (12)$$

i.e., additional units of the objective variable do not increase utility. Hence, marginal utility is a piecewisely decreasing function of total utility.

The determination of partial utility functions consists of three steps.

- (1) The decision-maker is asked to specify an aspiration level between the minimum and maximum of each objective.
- (2) The decision-maker is asked to define the lowest acceptable value of each objective (reservation level).
- (3) Partial utility curves are computed according to formulas (13)–(16).

$$u_i^{min} = u_i(z_i^{u,min}) \quad (13)$$

$$c_i = \frac{u_i^{max} - u_i^{min}}{z_i^A - z_i^{u,min}}, \quad z_i^{min} < z_i^A \leq z_i^{max} \quad (14)$$

$$\begin{aligned} u_i &= u_i^{min} + c_i(z_i - z_i^{u,min}), & z_i^{min} \leq z_i < z_i^A \\ &= 0, & z_i < z_i^{u,min} \\ &= 1, & z_i^A \leq z_i \leq z_i^{max} \text{ or } z_i^{min} = z_i^A \end{aligned} \quad (15)$$

$$u_i^{min} = 0, \quad u_i^{max} = 1 \quad \forall i = 1, \dots, p \quad (16)$$

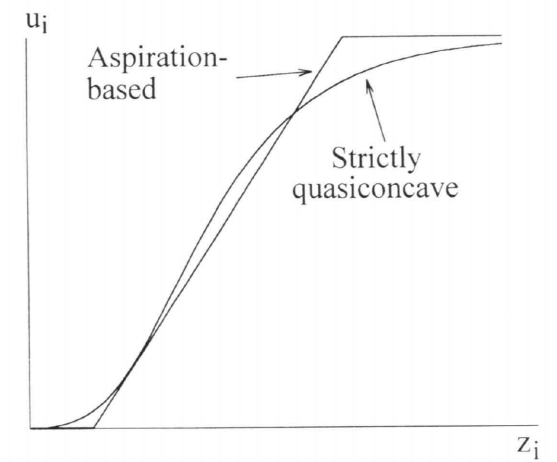


Figure 3. Aspiration-based utility function seen as a piecewisely linear approximation of a strictly quasiconcave function.

The purpose of this three-step procedure is to form a piecewisely linear approximation of a strictly quasiconcave utility function (Fig. 3).

The range of partial utility functions is closely related to the scaling of functions. By virtue of (14), the steepness of the partial utility functions is affected both by the lowest acceptable value and the aspiration level of an objective. If the lowest acceptable value is raised or the aspiration level reduced, the partial utility function becomes steeper. In the opposite case, the partial utility function becomes slanting.

Compared to the basic assumptions, the aspiration-based partial utility functions are continuous and increasing. The differentiability condition is not met because the first derivative is undefined at the points determined by the aspiration and reservation levels. Partial utility functions are (non-strictly) quasiconcave.

2.4 Indifference Curves

2.4.1 Relationships Between Utility Functions and Indifference Curves

An indifference curve is the locus of the combinations of two objectives that yields a constant utility level. For more than two variables, indifference surfaces and hypersurfaces are analogous

concepts. The optimal choice of the decision-maker is at the point where the indifference curve is tangent to the linear budget constraint. In forest management, a concave production possibility boundary constitutes the resource use constraint.

If utility functions are smooth, increasing, and strictly quasiconcave, the corresponding indifference curves are strictly convex (Chiang 1984). Strictly convex indifference curves are a prerequisite for the existence of a unique solution if the budget constraint is linear. This is because, if an indifference curve contains linear segments, the curve coincides with a portion of the budget line. Indifference curves of this type imply that the corresponding utility functions are explicitly quasiconcave. The definition of explicit quasiconcavity rules out any horizontal plane segments on the surface; nonhorizontal plane segments are allowed.

The same reasoning applies to the production possibility boundary. If the boundary contains linear portions, unique solutions are guaranteed only by strictly quasiconcave partial utility functions.

2.4.2 Restrictions of the Analysis

The utility functions used in this study contain both horizontal and nonhorizontal plane segments. To outline the shape of corresponding indifference curves, formulas defining the features of indifference curves are indispensable.

The derivation of indifference curves will be based on an examination of two objective variables denoted by z_1 and z_2 , respectively. The reasons for this restriction are solely illustrative. It follows that the reduced forms of functions (3) and (4), i.e.

$$U = a_1u_1(z_1) + a_2u_2(z_2) \tag{17}$$

$$U = u_1(z_1)^{a_1} u_2(z_2)^{a_2} \tag{18}$$

are used in the analysis.

If more than two variables are incorporated into the examination, isoutility levels appear as surfaces or hypersurfaces. To depict the production possibilities, the concept of the production possibility boundary is analogous even in multidimensional cases.

One notational aspect has to be taken into consideration. The positively sloped segment of a partial utility function u_i on z_i is characterized by

$$u_i = \left(\frac{1}{z_i^A - z_i^{u\min}} \right) z_i \tag{19}$$

For notational conciseness it is assumed that

$$z_i^{u\min} = 0 \Rightarrow u_i = \frac{1}{z_i^A} z_i \tag{20}$$

It follows that in formulas containing z_i^A

$$U = f(z_i^A, \dots) \neq f(z_i^A - z_i^{u\min}, \dots) \tag{21}$$

in cases of nonzero values of the lowest acceptable value of the objective.

2.4.3 Aspiration-Based Indifference Curves

If aspiration levels are used in partial utility functions (Fig. 4a,b), the indifference curves are composed of a vertical, a horizontal and a negatively sloped portion (Fig. 4c,d). The corners correspond to the aspiration levels. As utility increases, i.e., the indifference curves are located at higher positions, the negatively sloped segments shorten. At the point determined by the aspiration levels (referred to as the aspiration point), an indifference curve has a rectangular form.

The negatively sloped part of an indifference curve is composed of the nonhorizontal segments of the partial utility functions. Let us first examine the additive model. In this area, total utility is

$$U = a_1c_1z_1 + a_2c_2z_2 \tag{22}$$

Alterations in objectives affect total utility

$$\Delta U = a_1c_1\Delta z_1 + a_2c_2\Delta z_2 \tag{23}$$

To have a constant utility level,

$$\Delta U = 0 \tag{24}$$

This means that

$$a_2c_2\Delta z_2 = -a_1c_1\Delta z_1 \tag{25}$$

Hence, the slope of an indifference curve is

$$\frac{\Delta z_2}{\Delta z_1} = -\frac{a_1c_1}{a_2c_2} \tag{26}$$

For infinitesimal changes

$$\lim_{\Delta z_1 \rightarrow 0} \frac{\Delta z_2}{\Delta z_1} = \frac{dz_2}{dz_1} = -\frac{a_1c_1}{a_2c_2} = -\frac{a_1 \partial U / \partial z_1}{a_2 \partial U / \partial z_2} \tag{27}$$

which is a weighted form of the general case

$$\frac{dz_2}{dz_1} = -\frac{\partial U / \partial z_1}{\partial U / \partial z_2} \tag{28}$$

The corner points can be determined as follows. First consider changes in z_2 (right corner). Hence, let us write

$$\Delta U = a_1c_1\Delta z_1 + a_2c_2\Delta z_2 \tag{29}$$

Assume that

$$\Delta z_1 = 0 \tag{30}$$

It follows that

$$\Delta z_2 = \frac{\Delta U}{a_2c_2} \tag{31}$$

For changes in z_1 (left corner),

$$\Delta z_1 = \frac{\Delta U}{a_1c_1} \tag{32}$$

In a difference form, the corner points are thus

$$P_{RC} = \left(z_1^A, z_2^A - \frac{\Delta U}{a_2c_2} \right) \tag{33}$$

$$P_{LC} = \left(z_1^A - \frac{\Delta U}{a_1c_1}, z_2^A \right) \tag{34}$$

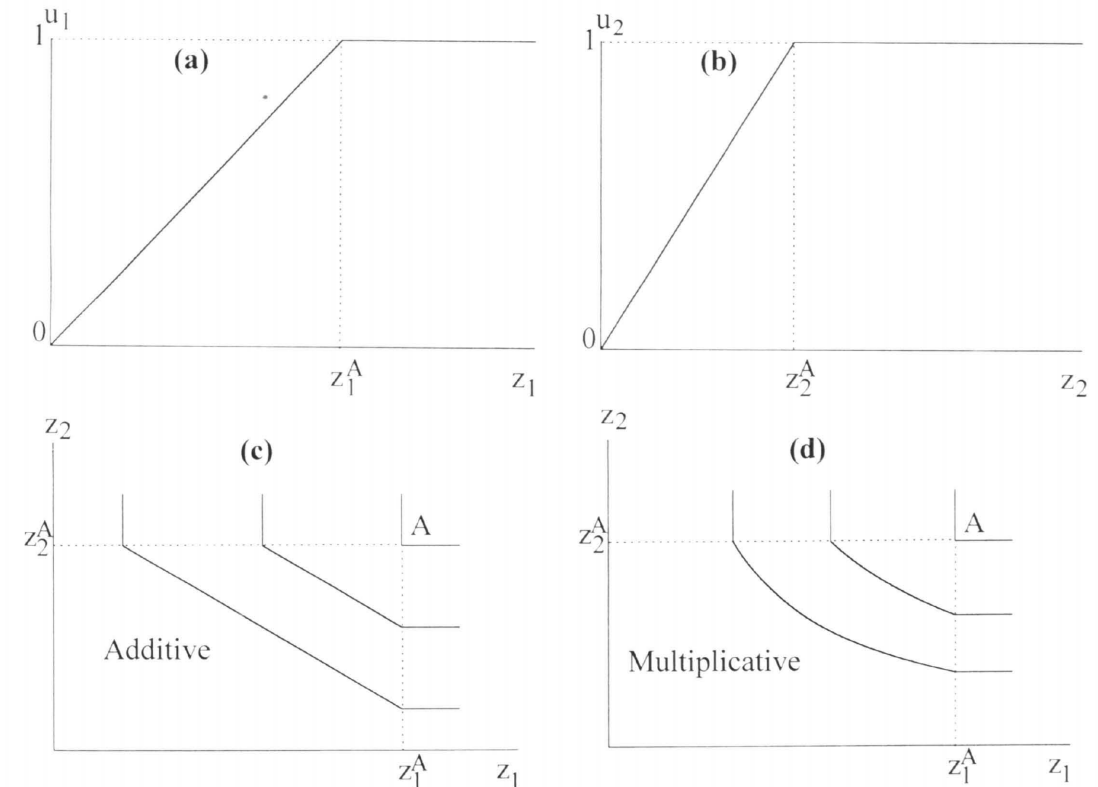


Figure 4. Partial utility functions of z_1 and z_2 (a,b) and three indifference curves (c,d).

A nondifference form can be derived by solving z_1 and z_2 from (17). Because in the right corner $z_1 = z_1^A$

$$P_{RC} = \left(z_1^A, \frac{U - a_1 c_1 z_1^A}{a_2 c_2} \right) \quad (35)$$

and in the left corner $z_2 = z_2^A$

$$P_{LC} = \left(\frac{U - a_2 c_2 z_2^A}{a_1 c_1}, z_2^A \right) \quad (36)$$

where $U (<1)$ denotes the utility of the indifference curve. In a function form, the linear indifference curve can be written

$$z_2 = z_2^A - \frac{a_1 c_1}{a_2 c_2} \left(z_1 - \frac{U - a_2 c_2 z_2^A}{a_1 c_1} \right) \quad (37)$$

To derive the slope of the multiplicative model, let us first write the logarithmic transformation

$$\ln U = a_1 \ln(u_1(z_1)) + a_2 \ln(u_2(z_2)) \quad (38)$$

in the form

$$\ln U = a_1 \ln(c_1) + a_1 \ln(z_1) + a_2 \ln(c_2) + a_2 \ln(z_2) \quad (39)$$

By differentiating both sides with respect to z_1

$$\frac{1}{U} \frac{\partial U}{\partial z_1} = \frac{a_1}{z_1} \quad (40)$$

$$\Leftrightarrow \frac{\partial U}{\partial z_1} = \frac{a_1}{z_1} U$$

and z_2

$$\frac{1}{U} \frac{\partial U}{\partial z_2} = \frac{a_2}{z_2} \quad (41)$$

$$\Leftrightarrow \frac{\partial U}{\partial z_2} = \frac{a_2}{z_2} U$$

Thus, the slope of an indifference curve is

$$\frac{dz_2}{dz_1} = - \frac{a_1 z_2}{a_2 z_1} \quad (42)$$

which is independent of the slopes of the partial utility functions.

For the multiplicative form, the corner points can be solved from (39) to give

$$z_2 = \exp\left(\frac{\ln(U) - a_1 \ln(c_1) - a_1 \ln(z_1) - a_2 \ln(c_2)}{a_2}\right) \quad (43)$$

$$z_1 = \exp\left(\frac{\ln(U) - a_1 \ln(c_1) - a_2 \ln(z_2) - a_2 \ln(c_2)}{a_1}\right) \quad (44)$$

The corner points are

$$P_{RC} = \left(z_1^A, \exp\left(\frac{\ln(U) - a_1 \ln(c_1) - a_1 \ln(z_1^A) - a_2 \ln(c_2)}{a_2}\right) \right) \quad (45)$$

$$P_{LC} = \left(\exp\left(\frac{\ln(U) - a_1 \ln(c_1) - a_2 \ln(z_2^A) - a_2 \ln(c_2)}{a_1}\right), z_2^A \right) \quad (46)$$

2.4.4 Relationships Between the Additive and the Multiplicative Form

The indifference curves span the entire objective space. Therefore, an indifference curve originating from the multiplicative model has to be tangent to an indifference curve of the additive form. This point is characterized by

$$- \frac{a_1 c_1}{a_2 c_2} = - \frac{a_1 z_2}{a_2 z_1} \quad (47)$$

It follows that

$$z_2 = \frac{c_1}{c_2} z_1 \quad (48)$$

By setting

$$a_1 c_1 z_1 + a_2 c_2 z_2 = c_1 z_1^{a_1} c_2 z_2^{a_2} \quad (49)$$

and replacing z_2 by (48) on both sides

$$(a_1 + a_2) c_1 z_1 = c_1 z_1^{a_1 + a_2} \quad (50)$$

Because of (9), $a_1 + a_2 = 1$, which means

$$c_1 z_1 = c_1 z_1 \quad (51)$$

which is identically true. It follows that the tangent has the same utility as the linear indifference curve (Fig. 5). As U , a_1 , a_2 , c_1 and c_2 are known, the tangency point can be solved from

$$\begin{cases} z_1 = \frac{U - a_2 c_2 z_2}{a_1 c_1} \\ z_2 = \frac{c_1}{c_2} z_1 \end{cases} \quad (52)$$

Substitution of z_2 by the lower equation in the upper one yields

$$z_1 = \frac{U - a_2 c_2 \left(\frac{c_1}{c_2} z_1\right)}{a_1 c_1} \quad (53)$$

By arranging the terms, the tangency point P_T can be written

$$P_T(z_1, z_2) = \left(\frac{U}{(a_1 + a_2)c_1}, \frac{c_1}{c_2} z_1 \right) \quad (54)$$

Because $a_1 + a_2 = 1$

$$P_T(z_1, z_2) = \left(\frac{U}{c_1}, \frac{c_1}{c_2} z_1 \right) \quad (55)$$

The indifference curves of the multiplicative model are strictly convex below the aspiration levels. This is because

$$\frac{d^2 z_2}{dz_1^2} = \frac{a_1 z_2}{a_2 z_1^2} > 0 \quad \forall a_1 > 0 \wedge a_2 > 0 \wedge z_2 > 0 \wedge z_1 > 0 \quad (56)$$

In summary, the following statements characterize the relationships of the indifference curves:

- (1) Piecewisely linear partial utility curves give rise to strictly convex indifference curves, if the multi-attribute model is multiplicative.
- (2) The indifference curves of the multiplicative model are tangent to the indifference curves of the additive model at the same utility level.

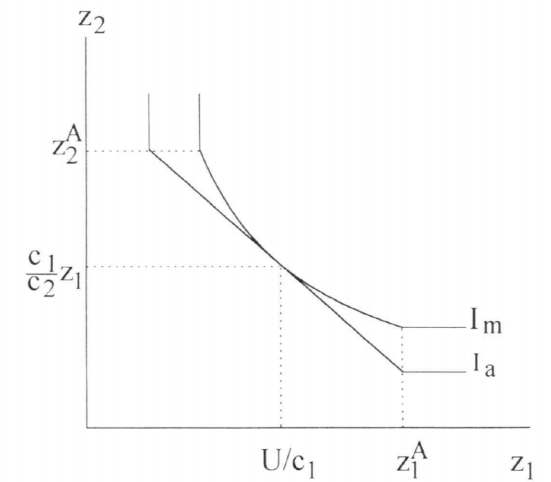


Figure 5. Indifference curves of additive and multiplicative models corresponding to the same utility level.

2.4.5 Characterization of the Optimal Choice

2.4.5.1 Location of the Aspiration Point

An aspiration point may lie inside, outside or exactly on the production possibility boundary. The uniqueness of the solution of utility maximization is closely related to the location of the aspiration point. Accordingly, three different situations can be distinguished (Fig. 6).

- (1) The aspiration point lies exactly on the production possibility boundary. The solution is unique.
- (2) The aspiration point lies inside the production possibility boundary. The solution is nonunique. The number of solutions is determined by the possible combinations found on the boundary.
- (3) The aspiration point lies outside the production possibility boundary. If the production possibility boundary is strictly concave, the solution is unique. Otherwise, for the additive form, the solution is nonunique in case of a tangency solution and unique in case of a corner solution, i.e., when a corner of an indifference curve touches the boundary. For the multiplicative model, the concavity of the boundary is a sufficient condition to guarantee the uniqueness of the solution.

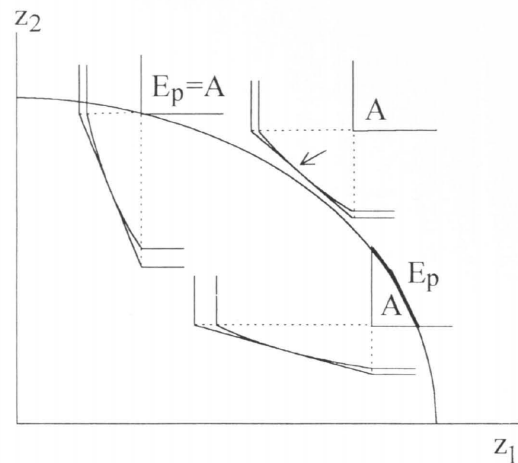


Figure 6. Indifference curves with different locations of aspiration points.

In the first situation, the aspiration levels isolate a variation range that is actually a single point on the production possibility boundary. In other words, alterations in the levels of objective variables are not needed to attain maximal utility, i.e.

$$z_1^{u_{max}} = z_1^A \text{ and } z_2^{u_{max}} = z_2^A \quad (57)$$

In the second situation, the aspiration levels are too low. Consequently, each point on the boundary fulfills the initial conditions and produces maximal utility. For these points,

$$z_1^{u_{max}} \geq z_1^A \text{ and } z_2^{u_{max}} \geq z_2^A \quad (58)$$

In the third situation, the aspiration levels are too high. It follows that maximal attainable utility is reached at a point for which

$$z_1^{u_{max}} < z_1^A \text{ and } z_2^{u_{max}} < z_2^A \quad (59)$$

Alterations in relative importances a_1 and a_2 affect the slope of the indifference curves. For example, if a_1 increases, the negatively sloped part becomes steeper. In the opposite case, i.e., if a_2 increases, the slope decreases.

In the third situation and from the viewpoint of the additive form, production possibility boundaries that fail to meet the condition of strict con-

cavity are disadvantageous. This is a serious problem if the boundary is linear. Then, in principle, the slope of the indifference curves being suitable, the entire boundary may represent a set of optimal solutions.

2.4.5.2 Additive Form

Let us examine more closely the effects of aspiration levels and relative importances. For the additive model, the effects can be observed by studying the slope and its elements

$$\frac{dz_2}{dz_1} = -\frac{a_1 c_1}{a_2 c_2} \quad (60)$$

If an aspiration level increases (Fig. 7a), the slope of the corresponding partial utility function decreases. Consequently, the indifference curves become slanting (Fig. 7b). For example, for z_1 this means

$$z_1^{A\Delta} > z_1^A \Rightarrow c_1^A < c_1 \Rightarrow \left(\frac{dz_2}{dz_1}\right)^A < \frac{dz_2}{dz_1} \quad (61)$$

A linear partial utility function is a special case in which the aspiration level equals the maximum value of the objective.

The effects of a decrease in an aspiration level can be observed by reverse reasoning. For z_1 ,

$$z_1^{A\Delta} < z_1^A \Rightarrow c_1^A > c_1 \Rightarrow \left(\frac{dz_2}{dz_1}\right)^A > \frac{dz_2}{dz_1} \quad (62)$$

If the aspiration level is equal to the minimum of the objective (Fig. 7c), the corresponding indifference curve forms a rectangle at point (z_1^{min}, z_2^A) . The indifference curves are horizontal to this point (Fig. 7d). As a result, utility is completely determined by the other variable.

The effects of relative importances a_1 and a_2 can be summarized as follows:

$$\text{if } a_1 = 0, a_2 \neq 0 \Rightarrow \frac{dz_2}{dz_1} = 0 \quad (63)$$

$$\text{if } a_1 \neq 0, a_2 = 0 \Rightarrow \frac{dz_2}{dz_1} = \infty \quad (64)$$

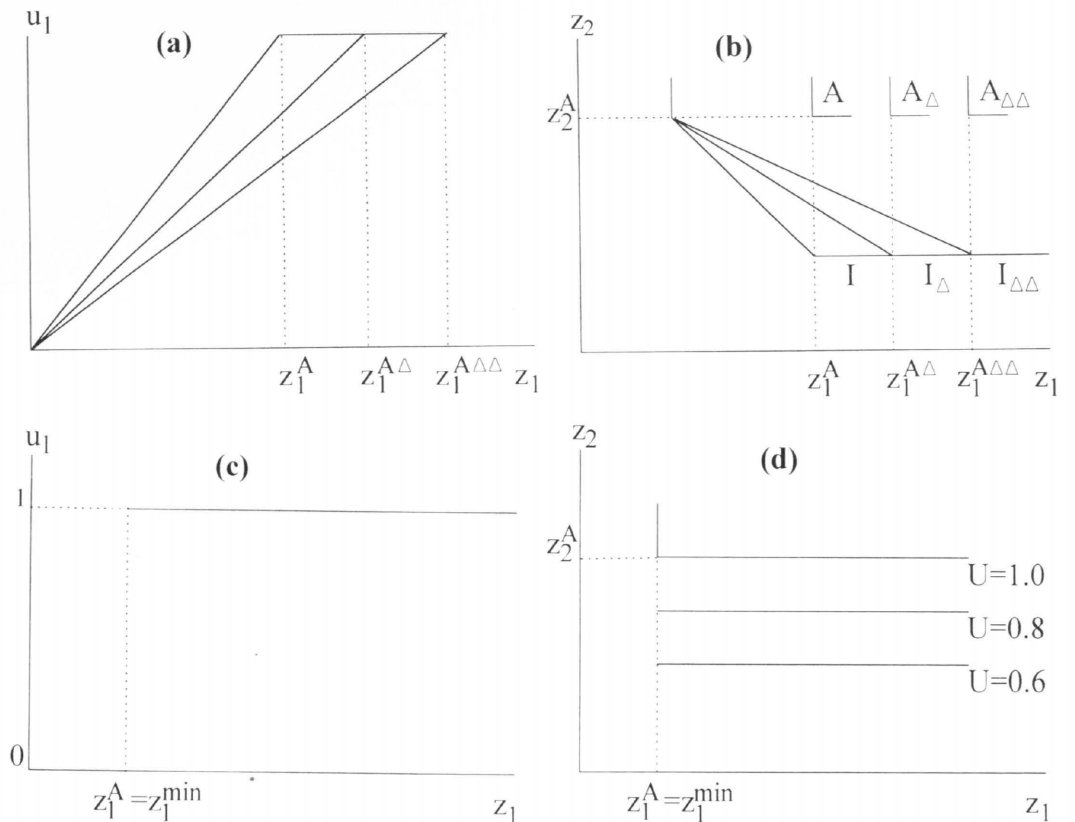


Figure 7. Relationship between aspiration levels and corresponding indifference curves.

$$\text{if } a_1 \neq 0, a_2 \neq 0 \Rightarrow \frac{dz_2}{dz_1} < 0 \quad (65)$$

In (63), the indifference curves are horizontal, whereas in (64) they are vertical. Formula (65) describes a situation in which the indifference curves are negatively sloped.

Because the slopes of indifference curves are affected both by relative importances and aspirations levels, the combined effects of these factors need to be studied. For this examination, it is useful to make a distinction between two cases.

First, if the aspiration levels determine a point inside the production possibility boundary or exactly on the boundary, the solution is dependent only on the location of the aspiration point. In other words, alterations in relative importances do not affect the solution (Fig. 8a).

Second, if the aspiration point lies outside the production possibility boundary, both relative importances and aspiration levels affect the solution. For z_1 , the effects can be summarized

$$a_1^A < a_1 \Rightarrow \left(\frac{dz_2}{dz_1}\right)^A < \frac{dz_2}{dz_1} \Rightarrow z_1^A < z_1 \quad (66)$$

$$z_1^{A\Delta} < z_1^A \Rightarrow \left(\frac{dz_2}{dz_1}\right)^A > \frac{dz_2}{dz_1} \Rightarrow z_1^A > z_1 \quad (67)$$

The effects caused by alterations in relative importances are, in fact, logical. For example, if the level of either objective is too low, a better result can be attained by assigning a greater weight to that objective or, as in this case, by decreasing the weight of the other objective. The effects of

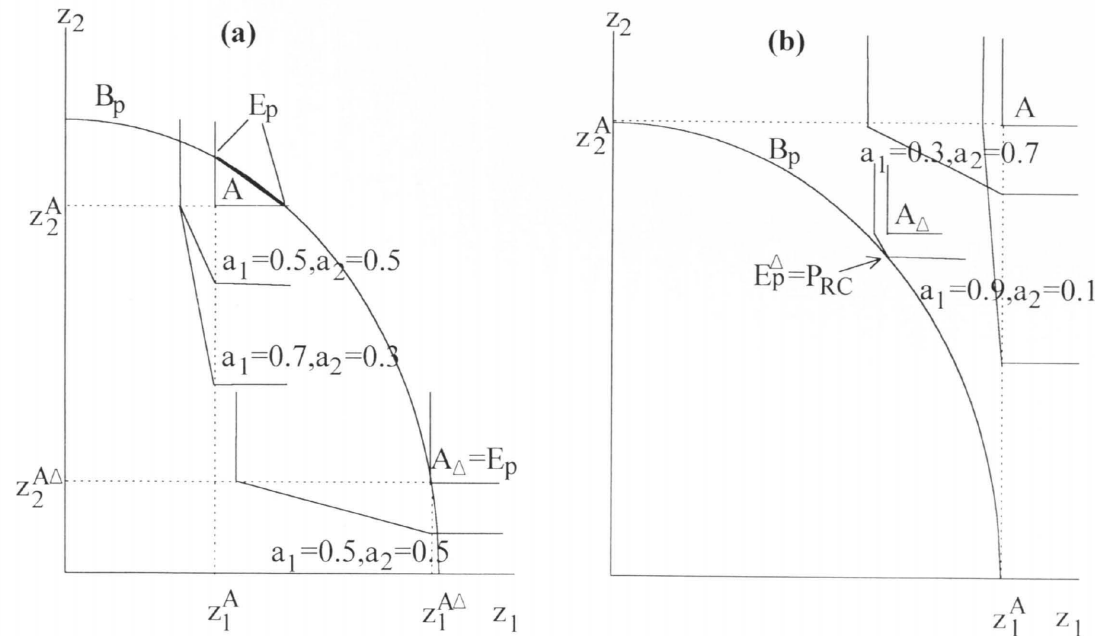


Figure 8. Aspiration points and effects of changes in aspiration levels and relative importances.

changes in aspiration levels, on the other hand, are unexpected: a decrease in the aspiration level results in an increase in the value of that objective.

These results seem to imply that if unrealistically high aspiration levels are used, the solutions cannot be fully predicted by means of aspiration levels. This is because in such a situation the solution can be determined not only by the tangency of an indifference curve, but also by the touch of a corner of the curve (Fig. 8b).

2.4.5.3 Multiplicative Form

The rules of the multiplicative model are similar to those of the additive model, except that the slope of an indifference curve is not affected by the slopes of partial utility curves. The slope of an indifference curve

$$\frac{dz_2}{dz_1} = -\frac{a_1 z_2}{a_2 z_1} \tag{68}$$

changes continuously along the curve, being affected by relative importances. As z_2 increases

(or z_1 decreases), the slope increases, and vice versa.

Changes in relative importances turn the indifference curves so that, for example,

$$a_1^A > a_1 \Rightarrow \left(\frac{dz_2}{dz_1}\right)^A > \frac{dz_2}{dz_1} \tag{69}$$

The bound values 0 and 1 of relative importances give rise to either vertical or horizontal indifference curves.

In the additive model, aspiration levels affect both the slope of the indifference curves and their valid domain. In the multiplicative model, the slope of the curves is not directly affected by the slopes of partial utility functions. Instead, aspiration levels determine only the domain of the negatively-sloped (strictly convex) part of the indifference curves (Fig. 9). This can be seen from the definitions of the minimum and maximum of the slope of an indifference curve corresponding to total utility U

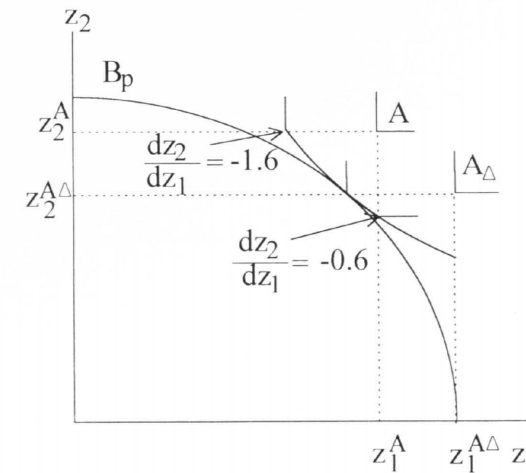


Figure 9. Indifference curves of the multiplicative model in the case of two different aspiration points.

$$\begin{aligned} \text{Max } U &= U(z_1, \dots, z_p) \\ z_1, \dots, z_p &\in B_p \end{aligned} \tag{72}$$

Hence, the optimization problem is to locate the point on the production possibility boundary with the highest total utility.

In three-dimensional space, total utility can be depicted by a surface. In the additive model, this surface contains plane segments that are either nonhorizontal or horizontal (Fig. 10a-c). Analogously, in n -dimensional space, utility forms a hypersurface with similar features. Nonhorizontal plane segments are a direct consequence of the linear, positively sloped parts of the partial utility functions. Horizontal plane segments, in turn, are caused by the existence of aspiration levels.

For the multiplicative model, the segments bend. Only the area restricted by aspiration levels is a plane segment (Fig. 10d).

In the case of the additive model, the surface constituted by partial utility functions is (nonstrictly) concave. For the multiplicative model, with aspiration levels equal to single-objective maxima, the utility surface is strictly concave. Formally, this can be proved as follows. The partial derivatives of (18) are

$$\frac{dz_2}{dz_1}^{\min} = -\frac{a_1}{a_2} \frac{\exp\left(\frac{\ln(U) - a_1 \ln(c_1) - a_1 \ln(z_1^A) - a_2 \ln(c_2)}{a_2}\right)}{z_1^A} \tag{70}$$

$$\frac{dz_2}{dz_1}^{\max} = -\frac{a_1}{a_2} \frac{z_2^A}{\exp\left(\frac{\ln(U) - a_1 \ln(c_1) - a_2 \ln(z_2^A) - a_2 \ln(c_2)}{a_1}\right)} \tag{71}$$

$$\begin{aligned} \frac{\partial U}{\partial z_1} &= \frac{a_1 U}{z_1} \\ \frac{\partial U}{\partial z_2} &= \frac{a_2 U}{z_2} \end{aligned} \tag{73}$$

and the second partial derivatives and cross partials are

$$\begin{aligned} \frac{\partial^2 U}{\partial z_1^2} &= -\frac{a_1 U}{z_1^2} \\ \frac{\partial^2 U}{\partial z_2^2} &= -\frac{a_2 U}{z_2^2} \\ \frac{\partial^2 U}{\partial z_1 \partial z_2} &= \frac{\partial^2 U}{\partial z_2 \partial z_1} = 0 \end{aligned} \tag{74}$$

In the multiplicative model, the probability of corner solutions is small compared to the additive model. This is because alterations in aspiration levels have no effect on the slope of the indifference curves. On the other hand, the shape of the curves favors tangency solutions.

2.5 Utility Maximization

2.5.1 Analytical Method

The problem of utility maximization is that of typical constrained optimization. In a general form, this problem can be expressed as

Because

$$\frac{\partial^2 U}{\partial z_1^2} = -\frac{a_1 U}{z_1^2} < 0 \quad (z_1 \neq 0) \tag{75}$$

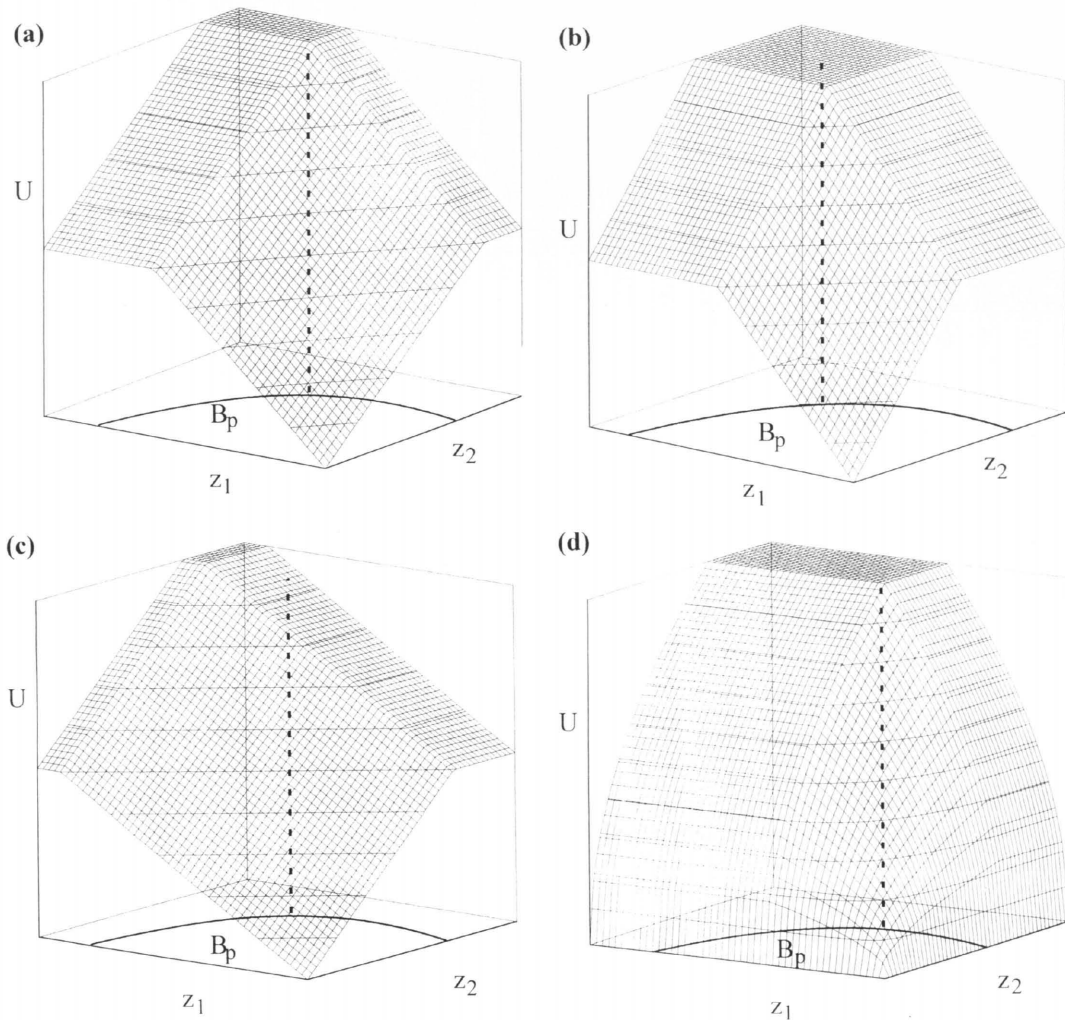


Figure 10. Utility surfaces of the additive model (a-c) and the multiplicative model (d) with respect to a production possibility boundary.

and the Hessian determinant

$$|H| = \begin{vmatrix} \frac{\partial^2 U}{\partial z_1^2} & \frac{\partial^2 U}{\partial z_1 \partial z_2} \\ \frac{\partial^2 U}{\partial z_2 \partial z_1} & \frac{\partial^2 U}{\partial z_2^2} \end{vmatrix} = \begin{vmatrix} -a_1 U & 0 \\ 0 & -a_2 U \end{vmatrix} \quad (76)$$

$$= \frac{a_1 a_2 U^2}{z_1^2 z_2^2} > 0 \quad (z_1 \neq 0, z_2 \neq 0)$$

the function is strictly concave.

The problem (72) can be solved by the ordinary Lagrange multiplier method. For the additive

model, the increasing part of the objective (utility) function is in the form

$$U = a_1 c_1 z_1 + \dots + a_p c_p z_p \quad (77)$$

subject to the constraint

$$g(z_1, \dots, z_p) = B_p \quad (78)$$

It follows that the Lagrangian function is

$$L = a_1 c_1 z_1 + \dots + a_p c_p z_p + \lambda (B_p - g(z_1, \dots, z_p)) \quad (79)$$

for which the first-order condition consists of the following $p+1$ simultaneous equations:

$$\begin{aligned} \frac{\partial L}{\partial \lambda} &= B_p - g(z_1, \dots, z_p) = 0 \\ \frac{\partial L}{\partial z_1} &= a_1 c_1 - \lambda \frac{\partial g}{\partial z_1} = 0 \\ &\dots \\ \frac{\partial L}{\partial z_p} &= a_p c_p - \lambda \frac{\partial g}{\partial z_p} = 0 \end{aligned} \quad (80)$$

The same method can be applied to the multiplicative form.

The uniqueness of the solutions corresponds directly to the aspects revealed by the indifference curves. In the first situation, the aspiration point lies exactly on the production possibility boundary. The solution is unique, because utility increases up to that point, and the production possibility boundary contains no other points with equal utility. In other words, the optimal point is situated exactly at the corner of the horizontal plane segment of the utility surface (Fig. 10a,d).

If the aspiration point lies inside the production possibility boundary, multiple solutions are possible. This is because the total differential is zero for each point on the horizontal plane segment (Fig. 10b), i.e.

$$dU = \frac{\partial U}{\partial z_1} dz_1 + \frac{\partial U}{\partial z_2} dz_2 = \frac{\partial(1)}{\partial z_1} dz_1 + \frac{\partial(1)}{\partial z_2} dz_2 = 0 \quad (81)$$

Hence, each of these points are stationary. The number of solutions is determined by the features of the production possibility boundary.

If the aspiration point lies outside the production possibility boundary, the horizontal plane segment is never reached (Fig. 10c). For the solution point

$$dU = \frac{\partial U}{\partial z_1} dz_1 + \frac{\partial U}{\partial z_2} dz_2 \neq 0 \quad (82)$$

because the value of the total differential changes as a result of infinitesimal changes in z_1 or z_2 . The uniqueness of the solution is guaranteed only if the production possibility boundary bends

so that equal utility levels are not attainable at different combinations of z_1 and z_2 .

2.5.2 Outline of an Iterative Solution Procedure

If the decision-maker is not satisfied with the solution found by, e.g., the analytical method presented above, an iterative procedure has to be applied. The term is used here to refer to successive specification of aspiration levels and relative importances as well as successive examination of the solutions.

The outline for an iterative procedure comprises the following steps. A similar procedure for reference point optimization has been presented by Wierzbicki (1980b).

- (1) The minima and maxima of the objective variables are presented to the decision-maker.
- (2) The decision-maker is asked to specify a vector of aspiration levels and a vector of reservation levels.
- (3) The computed solution is presented to the decision-maker along with the location of the aspiration point.
- (4) If the decision-maker is satisfied with the solution, stop. Otherwise, define a new set of aspiration and/or reservation levels and compute a new solution (Fig. 11).

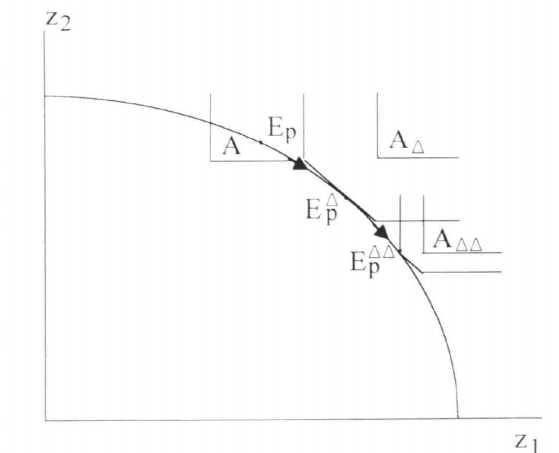


Figure 11. Examples of aspiration points and solutions obtained by the iterative procedure.

If the decision-maker is willing to scan the production possibility boundary, this can be implemented either by changing the relative importances (weighting method) or aspiration levels (aspiration-level method). The scanning procedure can be used for sensitivity analysis or to give an overview of the problem to be solved.

The chain of formulations and solutions is continued until a satisfactory solution is obtained.

In practice, the iterative procedure requires the use of a numerical optimization method. This is because the utility function in many cases is multidimensional and the production possibility boundary may not be depictable in a function form.

3 A Model for Timber Flow Management

3.1 Overview

The planning model is based on an examination of a forest economic unit divided into subareas called compartments. The idea of the model is to find a treatment schedule for each stand compartment that is optimal in terms of the objectives defined for the whole forest holding (Fig. 12). Compartments are assumed to be nondivisible management units.

The starting-point of the model is an alternative set describing the production possibilities of the resource stock. In numeric computation, this information is generated through simulation (see Appendix for details). The aim of the simulation model is to create a finite number of feasible management schedules for each compartment over a predetermined planning horizon.

The preferences of the decision-maker are expressed through the utility model. The model includes only such objective variables as are related to the regulation of timber flow to reach a satisfactory economic result. The fundamental idea in the formulation of the partial utility functions is the use of aspiration levels determined by the decision-maker.

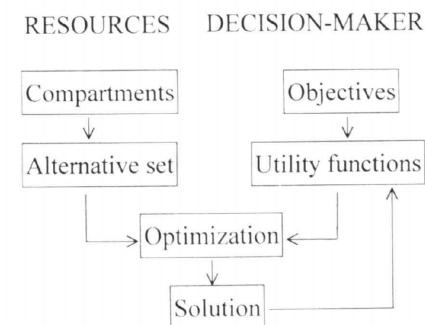


Figure 12. Structure of the planning model.

Problems defined by the utility model and restricted by the production possibilities will be solved by optimization. This procedure is used to find the solution which maximizes the utility to the decision-maker.

3.2 Planning Horizon

In terms of dynamic optimization, the planning model presented in this study is based on the concept referred to as a fixed-time-horizon problem or a vertical-terminal-line problem (Chiang 1992). The finite planning horizon is divided into parts called management periods. It is assumed that

$$N \in N^+ , P \in N^+ \quad (83)$$

implying that the shortest management period is one year. From an operational standpoint, the management period shows the time gap between successive forest treatments.

Formally, the following rules characterize the planning horizon.

$$T_E = T_B + N - 1 , N > 0 \quad (84)$$

$$n_p = \frac{N}{P} + \theta , P \leq \frac{N}{2} \text{ or } P = N \quad (85)$$

$$T_i^T = T_B + (i - 1) \times P , i = 1, \dots, n_p \quad (86)$$

$$P_{n_p} = N - (n_p - 1) \times P \quad (87)$$

$$\theta = 0, \text{ if } \text{remainder}(N/P) = 0 \\ 1, \text{ if } \text{remainder}(N/P) > 0 \quad (88)$$

This equation system has three exogenous variables: T_B , N and P . Formula (84) simply states that the planning horizon is a closed interval

Table 1. Number of management plans as a function of number of compartments and treatment schedules.

Number of compartments	Treatment schedules/compartments		
	3	5	10
5	243	3125	1×10^5
10	59049	9.8×10^6	1×10^{10}
30	2.1×10^{14}	9.3×10^{20}	1×10^{30}

$[T_B, T_E]$ from the beginning of T_B to the end of T_E . Because there are several possibilities to combine variables N and P , the correction term (88) in formula (85) is necessary when the length of the planning horizon is not evenly divisible by the length of the management period. In such cases, the last period is always shorter than the others (87).

The determination of the planning horizon and management period originates from the need for discretization. It is difficult to apply infinite time horizons as well as continuous time examinations to forest management planning. It follows that finiteness and some level of discretization are necessary. Accuracy in this respect has an effect, e.g., on profitability because of discrepancies between the optimal and possible timing of treatments. On the other hand, as the number of possible management plans is determined by

$$n_M = \prod_{i=1}^{n_c} \sum_{j=1}^{n_p} n_{ij}^s \quad (89)$$

the alternative set grows rapidly as management periods shorten, giving rise to schedules with different timing (Table 1).

Calculations here are based on a planning horizon of ten years with management periods of one year. The examination of different time horizons and discretization degrees falls beyond the scope of this study. Despite this limitation, formulas will be presented in a general form allowing for flexible time horizons.

3.3 Ingredients of the Model

3.3.1 Concepts of Exploitation and Conservation

The utility model to be presented here is an extension of the resource model introduced in Section 1.2. By definition, a resource stock can either be conserved or consumed. In the case of renewable resources, the stock possesses capacity for growth. If consumption exceeds the natural growth rate, the size of the stock will decrease.

With concepts of resource economics as a guideline, felling income was chosen to represent consumption, i.e., exploitation of the resource stock. This characteristic can be measured against the rate of value growth of forest. Thus, the following relationships are analogous to the general resource model (1):

$$\sum_{i=1}^{n_p} Y_i = \sum_{i=1}^{n_p} VG_i \Rightarrow \frac{\Delta V}{\Delta t} = 0 \quad (90)$$

$$\sum_{i=1}^{n_p} Y_i > \sum_{i=1}^{n_p} VG_i \Rightarrow \frac{\Delta V}{\Delta t} < 0 \quad (91)$$

$$\sum_{i=1}^{n_p} Y_i < \sum_{i=1}^{n_p} VG_i \Rightarrow \frac{\Delta V}{\Delta t} > 0 \quad (92)$$

These identities correspond to a stable, a decreasing and an increasing value of the resource stock, respectively. Net income, i.e., felling income subtracted by silvicultural costs, is used in the computations to incorporate related costs.

From the standpoint of business management, net income represents profit: the difference between revenues and costs. The classical objective of a firm is to maximize this difference (e.g. Neher 1990).

The resource model is extended by introducing an additional variable to describe the size of the resource stock. A variable of this kind can be justified by the need to stabilize resource use. This is in line with the principles of sustained yield management. The approach differs slightly from that of general resource economics: instead of differential comparisons, it enables static examinations between distinct points in time.

In this sense, the stumpage value of the growing stock at the end of the planning horizon was chosen to describe the size of the resource stock. The computation of this variable – referred to as “final capital” – has the general form

$$FC = P^E V^E \quad (93)$$

which can be compared with “initial capital”

$$IC = P^B V^B \quad (94)$$

This computation method omits the time after the end of the planning horizon, i.e., the future production potential of the land. This approach was chosen for the following reasons. First, the reliability of long forecasts is weak because of uncertain growth conditions and economic circumstances. Second, human time horizons tend to be short, emphasizing current and near-future events (e.g. Keltikangas 1971). Third, the land is assumed to be occupied by timber production pursued by a single owner, meaning that there is no need to include the opportunity cost of the land in the computations.

3.3.2 Intertemporal Choices

According to the capital-theoretic approach (e.g. Solow 1963, Hirshleifer 1970), the optimal size of the capital stock is determined by the equivalence of the interest rate and the marginal productivity of the capital stock. The production schedule is dependent on the applied interest rate. Incorporation of the intertemporal preferences of the decision-maker to this solution indicates what consumption level to follow. With aspiration levels, however, things are different. The aspiration level approach calls for an interpretation with respect to traditional economic theory.

If consumption level and stock size are decided exogenously, the above marginal rule is no longer applicable. On the contrary, it is consumption and stock size which determine productivity and return on capital. As the possibility to choose the aspiration levels of objectives is a fundamental precondition in this model, a variable controlling the return on capital (ROC) is included. The purpose of this variable is to enable the inclusion of

the return requirement for net income and the whole capital stock as well. On the other hand, ROC measures the profitability of forest management activities and is directly dependent on the timing of treatments per compartment.

In this context, the return on capital is determined by

$$ROC = \left(\sqrt[n]{\frac{FNI + FC}{IC}} - 1 \right) \times 100 \quad (95)$$

to measure the overall profitability derived from timber growing and treatments depicted by compounded net income. The return on capital equals the internal rate of return, which is a frequently used variable in profitability evaluations.

If interest rate or return on capital alone are used to constitute an income distribution, the temporal flow of income tends to be uneven. This is a harmless feature in circumstances of a perfect capital market. An additional requirement is that the decision-maker accepts the state of the forest resulting from capital-theoretic decision-making.

Exogenous determination of desired income trajectories is based on the assumption that production and consumption optima are not strictly separable (e.g. Ollikainen 1984, Ovaskainen and Kuuluvainen 1994). This results in a need to control cutting schedules, e.g., on the basis of consumption decisions. Such behavior may be due to a difference in the interest rates for borrowing and lending, on one hand, or to an unwillingness to borrow, on the other. In any case, the nonseparability assumption implies that the decision-maker is subject to liquidity constraints affecting decisions in forest management.

If the borrowing (i_B) and lending (i_L) rates differ, the budget line contains a kink. It is usually assumed that $i_B > i_L$ (Maddala and Miller 1989). In the case of liquidity constraints, it may be more profitable to use forest resources as a financing source for consumption (Fig. 13). The advantage depends on the marginal productivity of the stock.

If the decision-maker is unwilling or unable to borrow, the financing needs exceeding the income determined endogenously have to be covered by additional cuttings. In the case of diminishing returns on production – giving rise to a strictly concave production possibility boundary – this

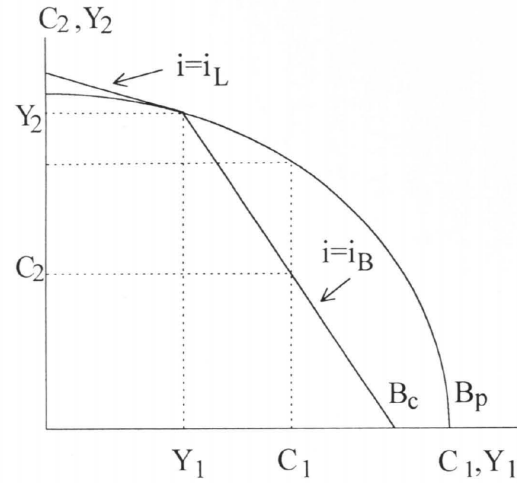


Figure 13. Effect of different interest rates on a production decision, provided that liquidity constraints prevail.

measure results in decreased maximum consumption possibilities for each time period (Fig. 14). This is because a lower budget line has to be accepted due to the exogenously determined production schedule.

Assuming that the decision-maker can choose freely between lending and consumption, the lending rate represents the corresponding opportunity cost. This restriction – which excludes borrowing – is definitely a simplifying assumption. A real situation may involve a mixture of consumption, lending and borrowing. Aspiration levels of falling income, however, have their clearest interpretation in the consumption-saving framework.

3.3.3 Formulation and Computation

The utility model related to timber management economics has two variations referred to as RM₁ and RM₂, respectively. The formulation of the models is drawn from the assumptions introduced in Section 1.2.

Resource Model 1 (RM₁):

$$U = U(NI, FC, ROC, ITI) \tag{96}$$

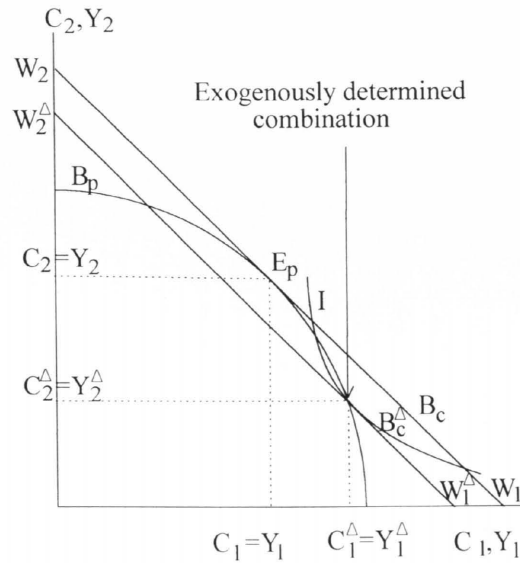


Figure 14. Effects of an exogenously determined production schedule in the case of no lending or borrowing possibilities.

Temporal income distribution can be presented either as a trajectory index (ITI) or as absolute values of income per period. The latter method is described by Resource Model 2 (RM₂):

$$U = U((NI_1, \dots, NI_{n_p}), FC, ROC) \tag{97}$$

The values of the objective variables have to be computed for each management plan. There are several computation methods especially for determining net income levels and income trajectories. These are referred to as CONI (computation of net income), and three of them are presented below.

CONI 1: No time discounting or compounding of income in each period. The total net income is a sum

$$ZNI = \sum_{j=1}^{n_c} \sum_{k=1}^{n_j} (I_{jk} - C_{jk}) \tag{98}$$

corresponding to the zero value of the interest rate.

CONI 2: No time discounting or compounding of income in each period. The total net income is a compounded sum

$$FNI = \sum_{j=1}^{n_c} \sum_{k=1}^{n_j} (I_{jk} - C_{jk})(1+i)^{T_E - T_k} \tag{99}$$

corresponding to the “Stalinist decision-maker” (Intriligator 1971): all income is saved to increase future consumption possibilities.

CONI 3: Time compounding for income per period. The total net income is a sum

$$CNI = \sum_{j=1}^{n_c} FNI_j \tag{100}$$

The first computation method is easiest to understand. It has the weakness of involving an implicitly determined interest rate that may conflict with the time-preference of the decision-maker and the actual discount rate. Moreover, the lack of a time-discounting element hampers the application of the capital-theoretic approach to the timing of treatments.

The second method takes economic timing into account. In addition, it has a clear interpretation of income per period. A drawback is that specifying an even compounded income trajectory, e.g., actually means a declining income distribution in the case of $i > 0$. The third method involves interest compounding at the level of each year.

Corresponding computation procedures can be presented for time discounting. Though not included here, discounting is not regarded as less appropriate. On the contrary, exogenous determination of income trajectories can be assumed to originate from needs related to consumption rather than to investment. This aspect implies that discounting would be even more appropriate than compounding. The presented methods stress the lending-oriented standpoint of the model: saving is seen as the best alternative use of realizable capital. Besides, both discounting and compounding convey the time-preference through financial losses in case of a deviation from the optimum.

The calculation of ROC has the formulation

$$ROC = \left(\frac{\sum_{j=1}^{n_c} (FC_j + \sum_{k=1}^{n_j} (I_{jk} - C_{jk})(1+i)^{T_E - T_k})}{\sum_{j=1}^{n_c} IC_j} - 1 \right) \times 100 \tag{101}$$

If the interest rate is not constant over time, interest is compounded by using the recursive function

$$FNI_{T_j} = FNI_{T_{j-1}}(1+i_{T_j})^P \tag{102}$$

to replace the compounded net income in (101).

If time discounting or compounding is included in NI, the terms NI and ROC contain a capital-theoretic timing element. A comparison of the essential elements of FNI and ROC

$$FNI = NI(1+i)^N \quad ROC = \frac{NI(1+i)^N + FC}{IC} \tag{103}$$

gives a reason for the following statements:

- (1) The maximization of FNI as the only objective corresponds to the maximization of net present value.
- (2) If ROC is maximized, treatments with a valuable final state are emphasized.
- (3) If the final state is not valued, the treatment sequences obtained through the maximization of FNI or ROC are equal.

The economic timing of treatments based on FNI or ROC determines the production optimum. If the consumption bundle has to equal the production bundle and the latter deviates from the optimum, it is necessary to specify the desired income trajectories. In RM₁, an additional variable ITI takes this into account. This variable allows a direct definition of the curvature of income flow. For RM₂, the shape variable is unnecessary because of the implicit determination of income flow. On the other hand, ROC is indispensable for RM₂ to enable the economic timing of the treatments.

3.3.4 Consideration of Income Trajectories

Model RM_1 has a variable for controlling the temporal distribution of income. This objective was included because of difficulties to regulate the dynamics of income flow only by means of income level or interest rate. For RM_2 this problem does not exist, because the net income trajectory is specified by the income levels of successive time periods.

In RM_1 , the desired income trajectory and income distributions produced by different management plans are compared by means of a specified algorithm. The idea of this algorithm is to calculate the values of the variables describing the deviations between preferred and feasible income trajectories.

Phase 1: Characterization of the desirable net income trajectory

- (1) The decision-maker is asked to outline a desirable net income trajectory (Fig. 15a).
- (2) The values corresponding to the points of the objective trajectory are scaled between 0 and 1. As a result, a vector of relative net income is formed

$$\mathbf{RNI} = [RNI_1, \dots, RNI_{n_p}] \quad (104)$$

- (3) The maximum deviation from the objective trajectory is calculated from

$$SSD_{\max} = \sum_{i=1}^{n_p} (E_i - RNI_i)^2 \quad (105)$$

in which \mathbf{E} is determined by

$$E_i = 0, \text{ if } RNI_i \geq 0.5 \\ = 1, \text{ if } RNI_i < 0.5, \quad i = 1, \dots, n_p \quad (106)$$

Phase 2: Comparison of the objective trajectory and a feasible trajectory

- (1) A feasible trajectory of net income is calculated by summing up the compartmental values of net income occurring in successive management periods.
- (2) Values determining the feasible trajectory are scaled between 0 and 1. A vector corresponding to that of (104) is formed, i.e., $\mathbf{RNI}^a = [RNI_1^a, \dots, RNI_{n_p}^a]$.

- (3) The discrepancy between the objective trajectory and a feasible trajectory is described by the sum of the squared deviations

$$SSD = \sum_{i=1}^{n_p} (RNI_i^a - RNI_i)^2 \quad (107)$$

From a set of feasible trajectories (Fig. 15b–d), the most appropriate temporal distribution of net income can be found by choosing the vector of relative net income \mathbf{RNI}^a such that (107) is minimized.

An alternative formulation makes use of unsigned deviations instead of squared ones. The maximum deviation then is

$$SUD_{\max} = \sum_{i=1}^{n_p} |E_i - RNI_i| \quad (108)$$

Since unsigned values of the deviations are used, the discrepancy between the objective trajectory and a feasible trajectory is described by the sum of the deviations

$$SUD = \sum_{i=1}^{n_p} |RNI_i^a - RNI_i| \quad (109)$$

The squaring of the deviations should result in smaller single deviations than those obtained from the minimization of the unsigned deviations. Examination of other differences is left to the numerical analysis to be presented later in the study.

The goal of an even income distribution requires a linear objective trajectory with a constant relative net income level. Consequently, the maximum sum of squared deviations is always $n_p(0-1)^2 = n_p$ (by virtue of (105)).

A comparison of absolute deviations can be made in the case of an even objective trajectory. In this procedure, the average net income is computed from

$$\bar{NI} = \frac{\sum_{i=1}^{n_p} NI_i}{n_p} \quad (110)$$

Intertemporal deviations from the average net income are expressed as

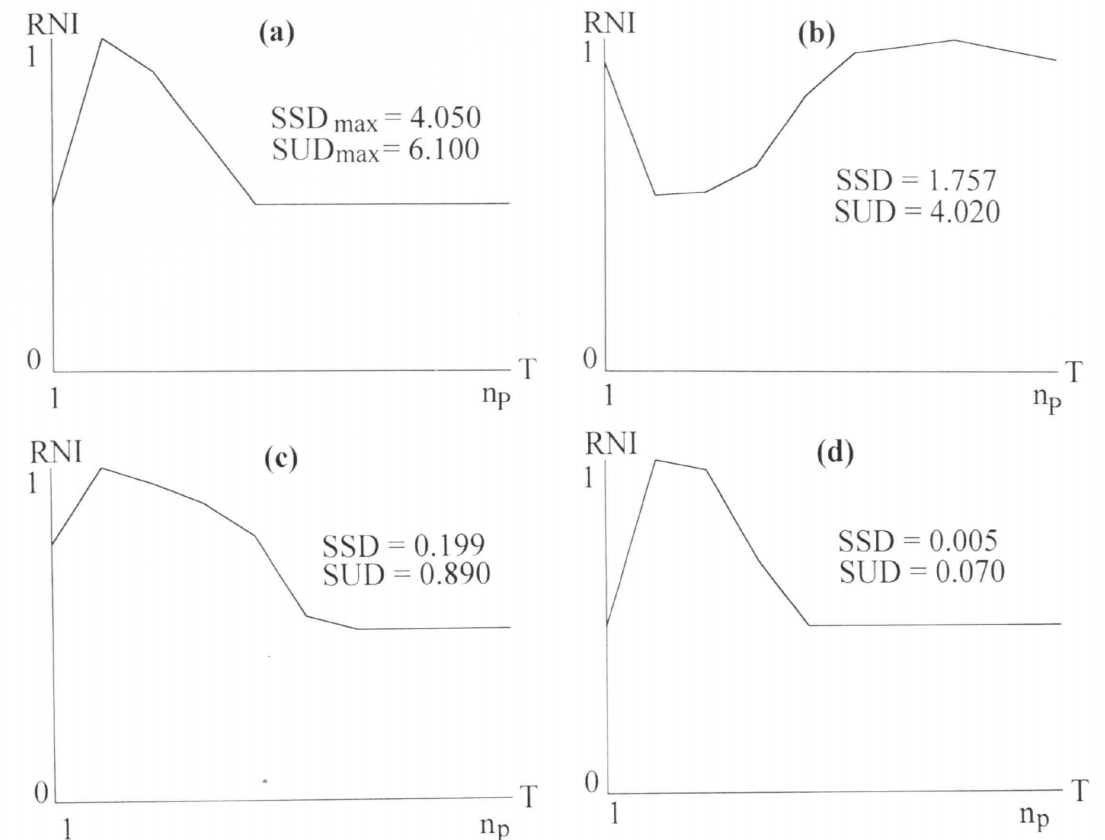


Figure 15. Objective trajectory of relative net income (a) and three feasible trajectories (b–d). Assumption: $n_p = 10$.

$$SAD = \sum_{i=1}^{n_p} (\bar{NI} - NI_i) \quad (111)$$

The vector of absolute net income is sorted in an ascending order. The maximum deviation is then

$$SAD_{\max} = (NI_{n_p} - \bar{NI})^2 + \sum_{i=1}^{n_p-1} (0 - \bar{NI})^2 \quad (112)$$

The use of nonlinear objective trajectories makes it possible to exclude management periods from the planning horizon. This is because this kind of definition results in a solution involving no cuttings in the periods with a relative income equal to zero.

Deviations as such are not used as a basis for the ITI objective. Instead, to obtain an increasing utility function, ITI is expressed as a percentage

$$ITI = \left(\frac{SD_{\max} - SD}{SD_{\max}} \right) \times 100 \quad (113)$$

This transformation makes ITI a decreasing function of the summed deviations.

3.3.5 Production Possibility Boundaries

3.3.5.1 Theoretic Assumptions

The concept of the production possibility boundary is an important analytic tool in forest management and economics. The notion is especially useful in examining the characteristics of a forest resource stock. The theory underlying production possibility boundaries is closely related to the concepts of joint production (Gregory 1987)

and rival products (Nautiyal 1988), and more fundamentally to the general theory of production functions (e.g. Baumol 1965). In multiobjective programming, the theory of noninferior sets (Cohon 1978) has an analogous content.

Theoretically, production possibility boundaries are often assumed to be concave (Nautiyal 1988) and graphic illustrations frequently contain the implication of strict concavity (e.g. Kilkki 1985). Strict concavity originates from the law of diminishing returns: an intensified use of inputs results in a marginally decreasing yield.

The concept of the "production possibility boundary" will be used here to depict the noninferior sets of the multiobjective optimization problem. This procedure differs somewhat from the convention of forest economics: the boundary is usually derived directly from production functions. Despite this difference, production functions are always the ultimate basis of the noninferior sets of this study.

3.3.5.2 Final Net Income and Final Capital

In RM_1 , the objectives NI and FC are strong competitors and, therefore, suitable to be presented by a production possibility boundary. The requirement of an increase in final capital leads to an inclined net income, and vice versa, that is

$$NI_{\Delta} > NI \Rightarrow FC_{\Delta} < FC \text{ if } NI, FC \in B_p \quad (114)$$

The magnitudes of these alterations depend on growth, applied treatments and timber prices, as well as on the opportunity cost, if NI includes compounding.

Let us examine the production possibility boundary of FNI and FC. Because FNI involves compounding, an increased interest rate results in a more slanting production possibility boundary. If the interest rate is multiplied by a constant k (>1), the difference in compounded net income is

$$\Delta FNI = FNI(1+ki)^N - FNI(1+i)^N \quad (115)$$

To simplify notation, let us write

$$1+i = \theta \quad (116)$$

and introduce an additional constant α .

Now

$$\begin{aligned} \Delta FNI &= FNI(\alpha\theta)^N - FNI\theta^N \\ \Leftrightarrow \Delta FNI &= \alpha^N FNI\theta^N - FNI\theta^N \\ \Leftrightarrow \Delta FNI &= (\alpha^N - 1)FNI\theta^N \end{aligned} \quad (117)$$

Because

$$\begin{aligned} \theta &= 1+i \wedge \alpha\theta = 1+ki \\ \Rightarrow \alpha &= \frac{1+ki}{1+i} \end{aligned} \quad (118)$$

the difference can be written in the form

$$\Delta FNI = \left(\left(\frac{1+ki}{1+i} \right)^N - 1 \right) FNI(1+i)^N \quad (119)$$

In summary, if FC is held at a constant level FC_0 , the change in opportunity cost i to ki shifts the production possibility boundary from FNI_0 to

$$FNI_0 + \left(\left(\frac{1+ki}{1+i} \right)^N - 1 \right) FNI_0(1+i)^N \quad (120)$$

The above has two interpretations. First, an increased interest rate results in a higher compounded income with a constant final capital (income effect of interest rate). On the other hand, if a certain net income level is aspired, an increased interest rate leads to a higher final capital.

An alternative way to express the above analysis is to separate the compound rate from the production possibility boundary. This being the case, the budget line has a slope equal to the average rate of return on realized capital.

3.3.5.3 Final Net Income, Final Capital and Return on Capital

The interactions between the objectives FNI, FC and ROC can be studied by outlining the boundary in three-dimensional space. To avoid problems related to illustration, the following analysis is based solely on analytic reasoning without graphical examination.

To begin with, it is useful to distinguish between three situations:

$$i > \bar{r}_F \quad (121)$$

$$i \cong \bar{r}_F \quad (122)$$

$$i < \bar{r}_F \quad (123)$$

In (121), the interest rate exceeds the average value growth rate of forest. Thus, an increasing net income results in an increasing return on capital, because every monetary unit yields a higher return on average when invested instead of when preserved as forest capital. Formally stated this means that

$$NI(1+i) > V(1+r_F) \quad (124)$$

Consequently,

$$i > \bar{r}_F \text{ and } FNI_{\Delta} > FNI \Rightarrow ROC_{\Delta} > ROC \quad (125)$$

If the interest rate is approximately equal to the value growth rate (122), alterations in net income have no major effects on return on capital, that is

$$i \cong \bar{r}_F \text{ and } FNI_{\Delta} > FNI \Rightarrow ROC_{\Delta} \cong ROC \quad (126)$$

If the interest rate falls below the value growth rate (123), an increasing net income leads to a decreased return on capital, because it would be more profitable to preserve the forest capital instead of consuming it.

$$i < \bar{r}_F \text{ and } FNI_{\Delta} > FNI \Rightarrow ROC_{\Delta} < ROC \quad (127)$$

Two additional aspects have to be taken into account. First, because cuttings affect the rate of value growth of forest, the above rules are only generalizations. Alterations in value growth may make extremely intensive cutting regimes disadvantageous. The exact effects at different net income levels can be figured out only by numerical computation. Second, according to (114), increased income usually results in a decreased value of the growing stock when movements along the boundary are examined. Again, the effects on ROC depend on the interest rate and the shape of the value growth functions.

3.3.5.4 Return on Capital and Temporal Distribution of Income

When used in forest management planning, return on capital measures the combined profitability of forest growing and further investments. This objective is useful in determining economical cutting schedules, because both timing and final capital are considered simultaneously.

Temporal income distribution produced by the application of ROC depends on the opportunity cost and the growth potential of forest. If the interest rate is high, immediate cuttings become profitable. A low interest rate leads to delayed cuttings. If the decision-maker is willing to control the income flow by other means, losses depend on the desired flow shape.

In RM_1 , the objective ITI allows the direct control of income flow. In general, any attempt to change the income flow dictated by ROC means losses in profitability. This applies even when the desired income flow corresponds perfectly to that determined by ROC. This is because the latter includes information about the treatment sequence of the compartments – an aspect not included in ITI.

3.3.5.5 Periodical Net Income

To study the periodical net income in RM_2 , production possibility boundaries can be examined by a pairwise analysis. Let us consider a case of two successive management periods.

The slope and location of the boundaries are affected by growth, treatments, and timber prices, as well as by interest rate in the case of CONI 3. These are exactly the same variables that affect the boundaries of NI and FC. From this viewpoint, FC can be interpreted as being a hypothetical felling income.

Changes in timber prices shift the boundary if the changes are equal for each management period. This is because it is possible to earn a higher felling income with equal fellings regardless of the net income level. In the case of intertemporal changes, i.e., price fluctuations over time, the slope and location of the boundary change. Alterations in interest rate cause changes of the latter type if compounding is included (CONI 3); slope and

location are different after the changes.

Movements along the production possibility boundary mean changes in felling times. If NI^0 denotes income at the original point and NI^1 at a point after the movement, these choices can be characterized by

$$NI_{i+1}^1 = NI_{i+1}^0 + NI_i^0 + f_V \quad (128)$$

In other words, delayed cuttings cause an income increase in the second period equal to the reduction in the first period added by the value increase (Fig. 16). Value growth is affected by prices: a constant change over time has no effects, whereas price fluctuations may result in notable changes in value growth percentages.

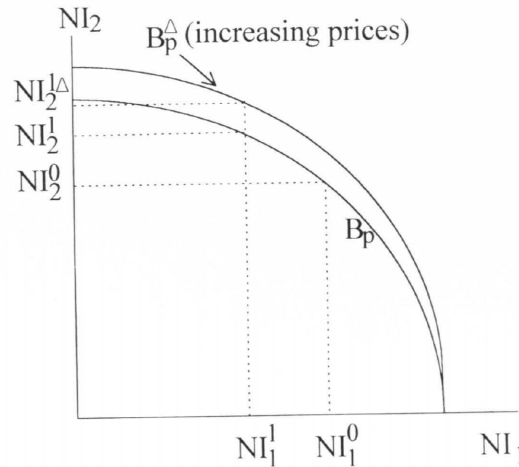


Figure 16. Production possibility boundaries of two successive management periods.

3.4 Utility Model

3.4.1 Compound Utility Functions

The analysis of the resource economic utility model is based on an examination of four different function forms.

Function I

$$U = a_{NI}u_{NI}(NI) + a_{FC}u_{FC}(FC) + a_{ROC}u_{ROC}(ROC) + a_{ITI}u_{ITI}(ITI) \quad (129)$$

Function II

$$U = u_{NI}(NI)^{a_{NI}} u_{FC}(FC)^{a_{FC}} u_{ROC}(ROC)^{a_{ROC}} u_{ITI}(ITI)^{a_{ITI}} \quad (130)$$

Function III

$$U = \sum_{i=1}^{n_p} a_i u_i(NI_i) + a_{FC}u_{FC}(FC) + a_{ROC}u_{ROC}(ROC) \quad (131)$$

Function IV

$$U = \left(\prod_{i=1}^{n_p} u_i(NI_i)^{a_i} \right) u_{FC}(FC)^{a_{FC}} u_{ROC}(ROC)^{a_{ROC}} \quad (132)$$

Functions III and IV can alternatively be written in a more hierarchical form. For III this means

$$U = a_{TNI}u_{TNI}(TNI) + a_{FC}u_{FC}(FC) + a_{ROC}u_{ROC}(ROC) \quad (133)$$

$$u_{TNI}(TNI) = \sum_{i=1}^{n_p} a_{TNI_i}u_{TNI_i}(NI_i)$$

and for IV

$$U = u_{TNI}(TNI)^{a_{TNI}} u_{FC}(FC)^{a_{FC}} u_{ROC}(ROC)^{a_{ROC}} \quad (134)$$

$$u_{TNI}(TNI) = \prod_{i=1}^{n_p} u_{TNI_i}(NI_i)^{a_{TNI_i}}$$

Functions II and IV can be made separable by taking logarithms, that is, for II

$$\ln(U) = a_{NI} \ln(u_{NI}(NI)) + a_{FC} \ln(u_{FC}(FC)) + a_{ROC} \ln(u_{ROC}(ROC)) + a_{ITI} \ln(u_{ITI}(ITI)) \quad (135)$$

and for IV

$$U = \sum_{i=1}^{n_p} a_i \ln(u_i(NI_i)) + a_{FC} \ln(u_{FC}(FC)) + a_{ROC} \ln(u_{ROC}(ROC)) \quad (136)$$

The inclusion of four different function forms has the following purposes:

- (1) To find out the differences between additive (I,III) and multiplicative (II,IV) functions.
- (2) To compare the presentation of income flow by a distinct variable (I-II) with that of an implicit trajectory determination (III-IV).

The second purpose originates from a fundamental difference between functions I-II and III-

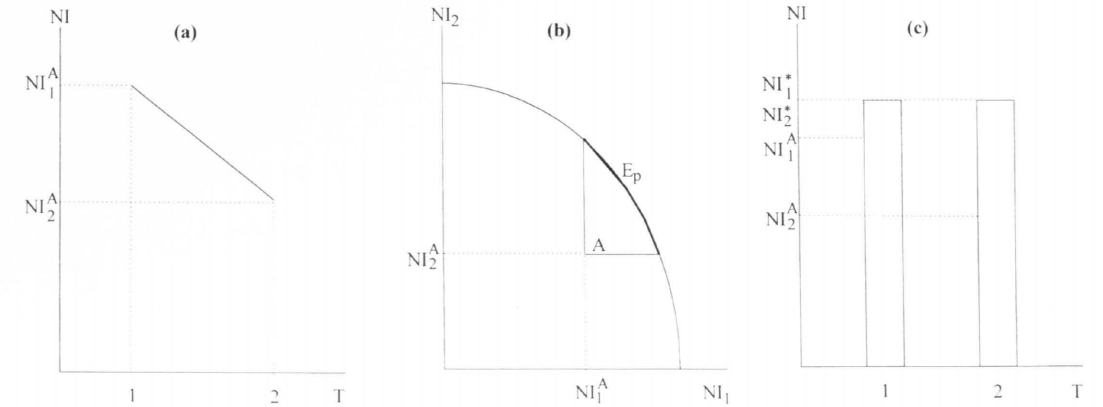


Figure 17. Desired income trajectory (a), corresponding aspiration point (b) and one of the solutions (c).

IV: in the latter functions, the absolute level of net income and the income trajectory are determined simultaneously. The decision-maker specifies the desired net income trajectory in monetary units, and NI is computed from

$$NI = \sum_{i=1}^{n_p} NI_i \quad (137)$$

Furthermore, FC can be interpreted as the income of the period n_p+1 .

In the above compound utility functions, the term NI has a general interpretation. In other words, the computation method – e.g., whether it includes compounding or not (CONI 1,...,CONI 3) – is not specified.

3.4.2 Partial Utility Functions

Partial utility functions are formed according to the rules introduced in Section 2.3. For functions I and II this means the specification of four aspiration and reservation levels. Functions III and IV require the setting of $n_p + 2$ levels.

The fundamental difference between functions I-II and III-IV is in the presentation of income trajectories. It is, therefore, useful to examine more closely a few theoretic aspects concerning the choices of the decision-maker with respect to intertemporal preferences. This need is based on the theoretic conclusion that, in principle, the

method of TNI (III-IV) may fail to yield solutions consistent with a desired trajectory. The possibility of this failure depends on the location of the aspiration point. The resulting income flow may be substantially different from the desired (Fig. 17). The verification of this intuitively derived hypothesis is left to the numerical analysis to be presented later in the study.

3.4.3 Maximization of Utility

The utility maxima for functions I-IV can be obtained by any appropriate method. As is customary in problem solving, one single method is rarely superior. Simplicity of implementation and suitability for describing the actual problem can be regarded as the most crucial criteria for selection.

Linear programming is a widely used method in solving management problems. Linearity requirements, inability to guarantee results for non-divisible compartments, and difficulties in dealing with trajectory variables were found problematic in this study. Goal programming – a branch of linear programming – has inherently more or less the same problems. This method is, however, appropriate for function III, which can be considered a classic goal programming problem.

In order to produce integer solutions, a direct search algorithm by Kilkki et al. (1986) was chosen (see also Pukkala and Kangas 1993). In addition,

tion to being computationally simple, this method allows the original forms of the utility functions to be retained. The method also has the advantage of being capable of solving all the functions I–IV without modifications.

The algorithm was modified such that a predetermined treatment schedule for each stand compartment was chosen for the first overall solution. The reason for this modification was stabilization: the original algorithm yields different local solutions in successive optimizations with equal premises. The global optimum can be solved with repeated optimizations. In the study of Kilkki et al. (1986), even the worst local optima were above 99 % of the global optima in the case of linear partial utility functions.

Direct search contains the following steps:

(1) Select a predetermined starting-point $S \in Q$.

(2) $r = 1$
 $U = U(S)$

(3) Compute total utility from

$$U_{\Delta} = \max \left\{ U_j^* = U \left(u_1(Q_{1rj} + \sum_{i=1}^{n_c} S_{1i}), \dots, u_p(Q_{prj} + \sum_{i=1}^{n_c} S_{pi}) \right), j = 1, \dots, n_r^s \right\} \quad (138)$$

Replace the previous schedule in S_{1r}, \dots, S_{pr} by the optimal one.

(4) $r = r + 1$
if $r \leq n_c$, return to 3
(5) If $U_{\Delta} > U$, return to 2
Otherwise deliver S as a local optimum.

4 Numerical Testing of the Model

4.1 Purpose and Basic Data

Chapter 2 involved the presentation of several properties of aspiration-based utility functions, choices of the decision-maker, and differences between the function forms. Rules and formulas derived mainly through analytic mathematics can be checked by simulating different production possibility sets, economic circumstances, and objectives of the decision-maker. This testing will be completed here by means of the model introduced in Chapter 3.

The numerical testing has the following aims:

- (1) Based on a numerical analysis of the choices of the decision-maker, to compare the solutions yielded by the additive and the multiplicative model.
- (2) To compare the different methods to allow for income trajectories.
- (3) To examine the differences and economic trade-offs between exogenously and endogenously determined income trajectories.

Numerical tests are based on an examination of three randomly chosen nonindustrial forest holdings (FH1, FH2 and FH3) situated in eastern Finland (Table 2). The description of the forest resource stock is based on data measured by a standwise inventory. A high percentage of old forest is characteristic of FH1 and FH3, whereas the age distribution for FH2 is more even (Fig. 18).

Table 2. Basic data on forest holdings used in numerical analysis.

	FH1	FH2	FH3
Number of compartments	34	53	52
Area, ha	32.4	67.1	53.4
Volume, m ³	4662	7603	5861
Management schedules	281	286	360

A planning horizon of ten years with a management period of one year was used in the computations. The average prices and silvicultural costs of the statistics of the Finnish Forest Research Institute (Aarne 1994) were applied to value the management schedules (Table 3).

4.2 Choices of the Decision-Maker

By definition, the optimal choice of the decision-maker is determined by the tangency of an indifference curve and the production possibility boundary. Consequently, the curvatures of the production possibility boundary and indifference curves play a key role in the search for a solution to the forest management planning problem.

Economic theory usually presumes the production possibility boundaries to be strictly concave. This condition does not necessarily hold in numerical analysis (e.g. Nautiyal 1988). The same conclusion is also evident from the computations of the present numerical study (Figs. 19–22). An examination of the graphical illustrations does not support the concavity of the boundaries; rather, the curves are at most (nonstrictly) quasiconcave.

Table 3. Initial capital and bound values of objective variables ($i = 3.5\%$). Bound values have been computed from alternatives involving compartmental minima and maxima.

		FH1	FH2	FH3
IC,	FIM	659099	1019065	795804
FNI _{min} ,	FIM	0	-5052	0
FNI _{max} ,	FIM	658687	833311	635643
FC _{min} ,	FIM	198794	700217	511720
FC _{max} ,	FIM	857988	1573102	1134380
ROC _{min} ,	%	1.87	3.87	2.99
ROC _{max} ,	%	3.24	4.75	4.18

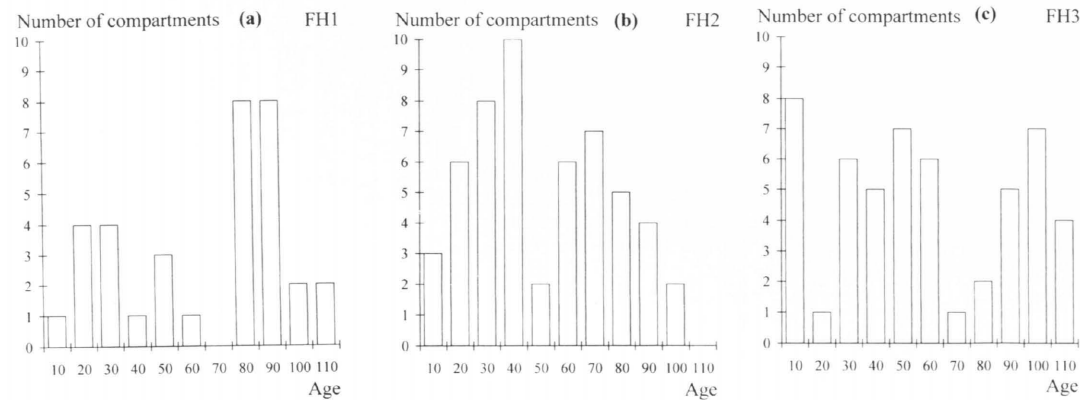


Figure 18. Age distributions of stand compartments in forest holdings.

Scanning of the boundary revealed only approximately linear segments. The interpretation of linearity depends on the numerical accuracy applied in the computations. It follows that the existence of nonhorizontal plane portions is possible, giving rise to the nonstrict quasiconcavity of the curves.

The linear or convex segments of quasiconcave boundaries are problematic especially from the standpoint of additive aspiration-based functions. This is because explicitly quasiconcave partial utility functions give rise to linearity in indifference curves. Multiplicative forms, on the other hand, can yield unique solutions on quasiconcave boundaries insofar as the convex segments do not coincide with convex indifference curves.

To examine the solving procedure more closely, a few numerical solutions with different premises are presented below. Only those situations in which the aspiration point exceeds the production possibility boundary are selected for study. This is because solutions derived from low aspiration points mainly characterize the search algorithm, not the features of the utility functions. If the aspiration points are beyond the boundary, the solution is affected by either relative importances or slopes of the partial utility functions, or both. This examination applies the reduced forms of functions I and II, i.e.

$$U = a_{FNI}u_{FNI}(FNI) + a_{FC}u_{FC}(FC) \quad (I) \quad (139)$$

$$U = u_{FNI}(FNI)^{a_{FNI}} u_{FC}(FC)^{a_{FC}} \quad (II) \quad (140)$$

and

$$U = a_{ROC}u_{ROC}(ROC) + a_{ITI}u_{ITI}(ITI) \quad (I) \quad (141)$$

$$U = u_{ROC}(ROC)^{a_{ROC}} u_{ITI}(ITI)^{a_{ITI}} \quad (II) \quad (142)$$

For the analysis of FNI and FC, a production possibility boundary with an interest rate of 3.5% (Fig. 19a) acts as an equality constraint in optimization. The lowest acceptable values of each objective are always zero. The first solution involves setting the aspiration levels equal to single-objective maxima (see Table 3 for details). The slopes of the partial utility functions are computed from (14) and the slopes of the indifference curves from (26) and (42) (Table 4).

The solution produced by the additive functions differs substantially from that produced by the multiplicative form (Table 5, row 1). The result is self-explanatory (Fig. 19c–d): the strictly convex indifference curve of the multiplicative function bends strongly towards the approximately linear production possibility boundary. The tangency point is at the center of the boundary. In contrast, the linear indifference curve of the additive function has a slope such that the tangency point is located near the maximum net income.

By using the utility values presented in Table 4, the solutions can be checked analytically. Let us examine FH1 more closely. By virtue of (55), the strictly convex indifference curve is tangent to the linear one at the point

$$FNI = \frac{U}{c_{FNI}} = \frac{0.623}{1.52 \times 10^{-6}} = 409868 \quad (143)$$

$$FC = \frac{c_{FNI}}{c_{FC}} FNI = 1.3 \times 409868 = 532828$$

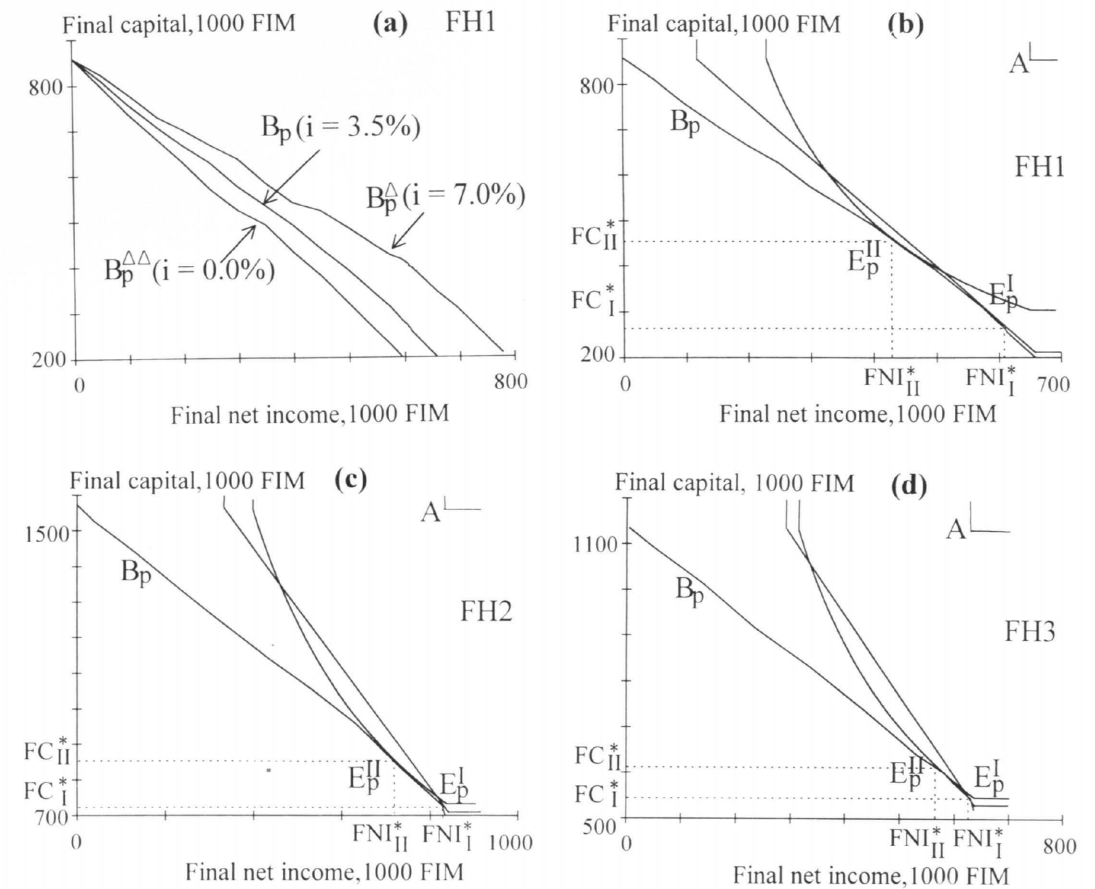


Figure 19. Production possibility boundaries of FH1 with different interest rates (a) and solutions produced by functions I and II when aspiration levels equal single-objective maxima (b–d).

This point is not attainable because of its location beyond the production possibility boundary. Consequently, the solution offered by the multiplicative function has to produce a lower utility level. The numerically computed value of the multiplicative form supports this observation.

The indifference curve corresponding to the utility value 0.601 passes through the point

$$P_{I/U=0.601}(FNI, FC) = (605054, 243173) \quad (144)$$

where FC has been computed from formula (37). The difference in FC is 281 653 FIM – 243 173 FIM = 38 480 FIM when compared to the optimum point of the additive model.

If the aspiration level of FNI is decreased, the solution of II remains unchanged. For FH1, func-

Table 4. Slopes of partial utility functions, indifference curves and utility values of solutions.

Assumptions:
 $a_{FNI} = a_{FC} = 0.5$, $FNI_A = FNI_{max}$, $FC_A = FC_{max}$

	FH1	FH2	FH3
c_{FNI}	1.52×10^{-6}	1.20×10^{-6}	1.57×10^{-6}
c_{FC}	1.17×10^{-6}	6.36×10^{-7}	8.81×10^{-7}
$G_I^{FC/FNI}$	-1.30	-1.89	-1.78
$G_{II}^{FC/FNI}$	-FC/FNI	-FC/FNI	-FC/FNI
U_I	0.623	0.728	0.735
U_{II}	0.601	0.689	0.699

Table 5. Values of parameters, aspiration levels of FNI and FC, and corresponding solutions yielded by functions I and II.

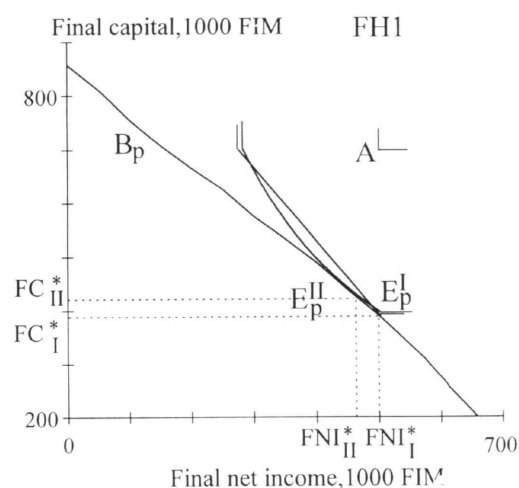
a_{FNI}	a_{FC}	FNI_A	FC_A		I		II	
					FNI*	FC*	FNI*	FC*
0.50	0.50	FNI_{max}	FC_{max}	FH1	605054	281653	439055	465050
				FH2	827679	727721	713148	873600
				FH3	629869	544114	573982	613889
0.50	0.50	$0.95FNI_{max}$	FC_{max}	FH1	615400	262196	439055	465050
				FH2	791386	766736	713148	873600
				FH3	603650	576128	573982	613889
0.45	0.55	FNI_{max}	FC_{max}	FH1	518418	383639	394926	510619
				FH2	827679	727721	664859	929839
				FH3	624807	552407	512337	682316
0.30	0.70	FNI_{max}	FC_{max}	FH1	2399	857275	279046	620791
				FH2	47625	1542695	457705	1157143
				FH3	45344	1115500	373148	824274
0.50	0.50	$0.76FNI_{max}$	$0.76FC_{max}$	FH1	500502	398444	487619	417471
				FH2	632230	946126	631763	961051
				FH3	483362	666753	480533	703615

tion I yields a solution with a higher net income (Table 5, row 2), because the slope of the corresponding partial utility function becomes steeper. This has no effect on II, but in I, the indifference curves become steeper forcing the tangency point towards a higher compounded net income. For FH2 and FH3, the corresponding increase in FNI_A results in a corner solution.

Changes in relative importances result in different solutions, the additive model being more sensitive to alterations (Table 5, rows 3 and 4). This is a direct consequence of the linearity of the indifference curves: a small change in the slope of a linear indifference curve can cause a considerable shift of the tangency point.

Corner solutions are common when function I is applied (Table 5, row 5, Fig. 20). When the indifference curves contain corners, the probability of a corner solution is high if the production possibility boundary is not strictly concave. The multiplicative model with equal premises yields a solution in which the convex part of the indifference curve is tangent to the boundary.

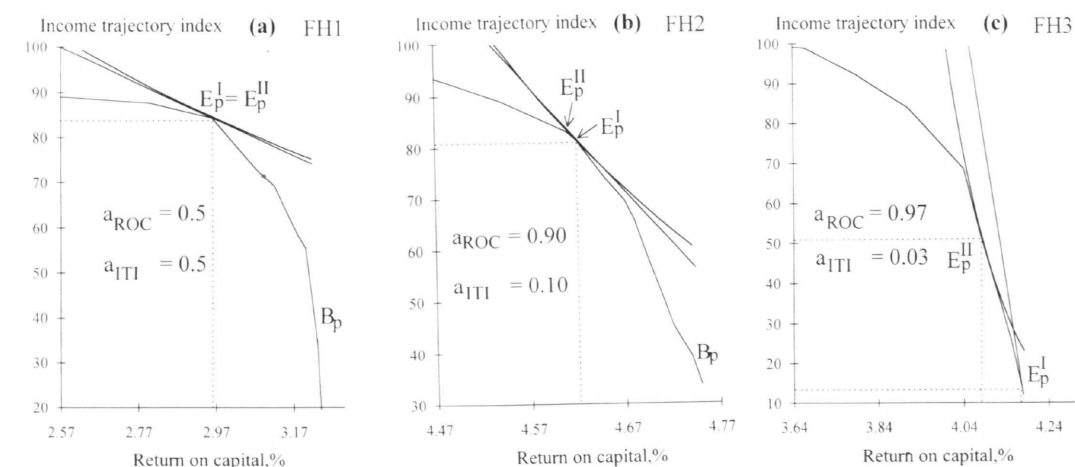
In the case of ROC and ITI of an even income trajectory, the production possibility boundaries are steeper than those of FNI and FC, although not necessarily strictly concave (Fig. 21). The results show that if the return requirement is low,

**Figure 20.** Example of a corner solution of I and corresponding solution of II.

the boundaries are flat. It follows that an increase in ROC can be required without causing a substantial degradation in the evenness of income flow. The curves have a sharp edge at the point at which ROC exceeds 3.20 % (FH1) and 4.10 % (FH3). In that area, already a marginal increase in ROC has a considerable effect on evenness. For

Table 6. Values of parameters, aspiration levels of ROC and ITI, and corresponding solutions yielded by functions I and II.

a_{ROC}	a_{ITI}	ROC_A	ITI_A		I		II	
					ROC	ITI	ROC	ITI
0.50	0.50	ROC_{max}	ITI_{max}	FH1	2.96	84.3	2.96	84.3
				FH2	4.46	87.6	4.38	93.4
				FH3	3.68	97.2	3.72	96.7
0.70	0.30	ROC_{max}	ITI_{max}	FH1	2.99	80.8	2.97	83.8
				FH2	4.54	87.2	4.41	89.2
				FH3	3.79	92.2	3.79	92.2
0.90	0.10	ROC_{max}	ITI_{max}	FH1	3.18	58.2	3.12	66.0
				FH2	4.62	81.4	4.61	83.6
				FH3	4.04	68.9	3.98	77.8
0.97	0.03	ROC_{max}	ITI_{max}	FH1	3.24	24.0	3.19	51.8
				FH2	4.74	39.2	4.70	56.8
				FH3	4.18	12.1	4.08	51.7

**Figure 21.** Solutions to problems including ROC and ITI, when $ROC_A = ROC_{max}$ and $ITI_A = ITI_{max}$.

all three forest holdings, the steeper curvature implies that the differences between the two function forms remain smaller than what was observed in the examination of FNI and FC.

The first solution comprises equal relative importances as well as aspiration levels that are equal to single-objective maxima (Table 6, row 1). In the case of FH1, functions I and II end up in the same solution. This point is presented by the sharp corner in the production possibility boundary (Fig. 21a). For FH2 and FH3 the solutions yielded by I

and II are different. When the relative importances are changed to yield higher values of ROC, the effects are smaller for function II (Table 6, rows 2 and 3). Especially after the sharp edge, the differences between the functions are considerable (Table 6, row 4).

The production possibility boundaries of periodical net income (function IV) were computed for a hypothetical case of two management periods (Fig. 22). These curves are approximately linear for all cases.

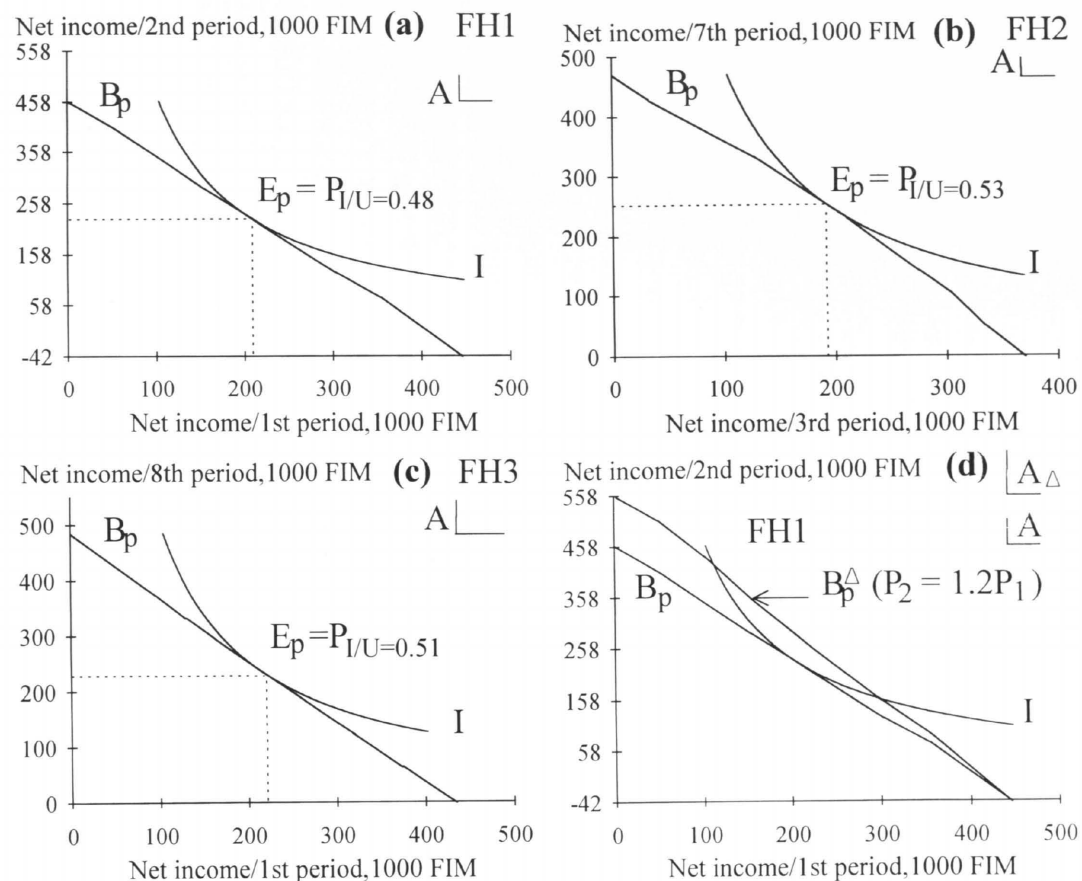


Figure 22. Production possibility boundaries of two management periods and solutions of the multiplicative model to problems involving single-objective maxima as aspiration levels (a–c), as well as production possibility boundaries with respect to two price levels (d).

4.3 Optimizing Income Trajectories

4.3.1 Methods to Control Income Flow

Functions I and II include an objective variable ITI to control the income flow directly. This variable allows trajectory optimization, i.e., the use of procedures to search for an implementable income distribution that matches the desired one. For functions III and IV, strict income trajectory optimization is not possible, because the shape of the income flow is not regarded as crucial. Only shortages are disadvantageous. This raises a theoretic suspicion concerning the efficiency of the latter method, above all because of the difficulty to manage the production possibil-

ity surface composed of incomes per several management periods.

If models I or II are applied, alternative income trajectories can be ranked by summing up either squared or unsigned deviations from the objective trajectory. Total deviation is used to measure the overall discrepancy between the desired and a feasible trajectory.

The numerical analysis of income trajectory optimization contains two parts. A comparison is first made of the trajectory optimization methods of functions I and II, followed by an evaluation of the efficiency of the method of periodical net income (functions III and IV). The results are compared to find out the differences between the methods.

Table 7. Results of trajectory optimizations with summing of squared deviations (SSD) and summing of unsigned deviations (SUD). The figures are averages of optima from five test computations. $|D|_{\max}$ denotes the maximal single deviation from the objective trajectory.

		Optimization criterion	
		SSD	SUD
$ D _{\max}$	FH1	0.201	0.381
	FH2	0.300	0.508
	FH3	0.104	0.169
SSD	FH1	0.336	0.352
	FH2	0.470	1.226
	FH3	0.047	0.123
SUD	FH1	1.129	0.860
	FH2	1.376	1.765
	FH3	0.455	0.674
Iterations	FH1	65	94
	FH2	66	78
	FH3	108	126

4.3.2 Comparison of Methods Using the Income Trajectory Index

To compare the methods containing the objective ITI, five deliberate income trajectories were formed. The purpose was to generate trajectories with different shapes. An even income trajectory was also included.

The results were computed consecutively with two algorithms. In the first computation, the sum of the squared deviations (referred to as SSD) was used as an optimization criterion. In the second computation, unsigned deviations (referred to as SUD) were used instead of squared ones. As a general conclusion, it appears that optimization based on SSD is more efficient. This statement holds regardless of whether efficiency is measured by maximal single deviations, number of iterations, SSD, or SUD (Table 7). As assumed, the algorithm based on the minimization of SSD usually resulted in trajectories with smaller maximal single deviations. This seems to imply that squaring efficiently eliminates alternative trajectories with sharp peaks.

The differences between the methods incorporating SSD and SUD are evident, although the

importance of the discrepancies may be negligible. Nevertheless, if smooth trajectories are preferred, the procedure applying SSD seems to be a relevant choice. This is because the optimal solution is always a compromise: a trajectory with a small SUD may contain a single high peak. In practical formulations, the net income level is usually restricted by other competing objectives, so that unconstrained trajectory optimization is rarely possible.

4.3.3 Methods Using Periodical Net Income

The performance of functions III and IV in income trajectory optimization were evaluated with the same five objective trajectories as in the previous section. This time, however, the objective trajectories were expressed in monetary units describing the aspiration levels of periodical net incomes (TNI). Results were computed for three total net income levels.

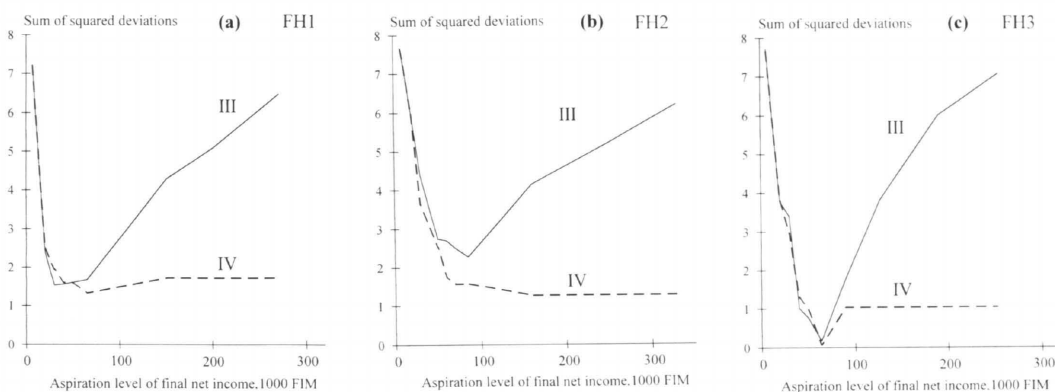
Let us first compare the results (Table 8) with those produced by optimization based on ITI (Table 7). Regardless of the income level, the optimization of TNI yields more fluctuating income distributions than strict trajectory optimization. The values of maximal deviations, SSD and SUD exceeded those of optimization based on SSD and SUD. The number of function evaluations (iterations) is smaller in TNI, which is in accordance with the low values of the test variables.

If the level of final net income is low, trajectories tend to deviate considerably from the desired ones. As FNI is raised, fluctuations decrease. Although the final net income of 350 000 FIM was intended to represent an approximation of the production possibility boundary of FH1 and FH3, the results show a substantial discrepancy at that point. Deviations – whether they are measured by maximal single deviation, SSD or SUD – seem to increase if the aspiration levels define an unattainable point, i.e., if the net income level is high. Measured by SSD of ITI_c, the results (Fig. 23) show the multiplicative model to be insensitive to increases in unattainable aspiration levels. The number of iterations is strongly dependent on the specified net income level, although it varies also according to the desired shape of the income flow.

This particular experimentation justifies only

Table 8. Results of optimization of periodical net income with functions III and IV. $|D|_{\max}$ denotes the maximal single deviation from the objective trajectory.

		FNI=100 000		FNI=350 000		FNI=600 000	
		III	IV	III	IV	III	IV
$ D _{\max}$	FH1	0.832	0.839	0.541	0.515	0.739	0.519
	FH2	0.885	0.885	0.627	0.589	0.531	0.479
	FH3	0.832	0.874	0.574	0.601	0.333	0.303
SSD	FH1	3.282	3.341	0.897	0.784	0.951	0.724
	FH2	4.350	4.350	1.932	1.705	1.109	0.856
	FH3	4.354	4.272	1.629	1.544	0.395	0.210
SUD	FH1	4.781	4.848	2.355	2.140	2.083	1.973
	FH2	5.663	5.663	3.615	3.478	2.455	2.305
	FH3	5.574	5.555	3.355	3.203	1.522	1.035
Iterations	FH1	15	14	41	39	58	55
	FH2	8	8	30	28	49	47
	FH3	27	28	56	48	82	87

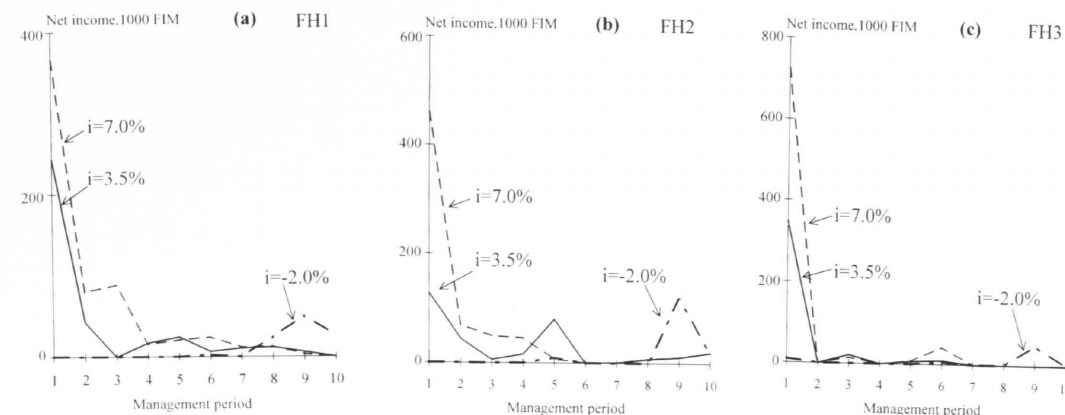
**Figure 23.** Sum of squared deviations from an even income trajectory presented as a function of aspired yearly net income levels.

cautious interpretations about the differences between the additive and multiplicative models. The figures seem to imply that the multiplicative model results in smaller fluctuations, especially if the aspiration levels are high. In addition, the multiplicative model is likely to converge more efficiently than the additive one. Definite conclusions would, however, require more comprehensive test data.

4.4 Endogenous vs. Exogenous Determination of Income Trajectories

4.4.1 Profitability Maximization

The maximization of profitability means finding a management plan in which treatments are implemented so that the total return on capital reaches its highest value. This approach takes into account both the returns from felling incomes via opportunity cost as well as the rate of value growth of forest. Profitability maximization equals profit maximization, if final capital is regarded as hypothetical income.

**Figure 24.** Income trajectories as a result of optimization based on maximization of ROC.**Table 9.** Values of objective variables in maximization of ROC when different interest rates are applied.

		$i = 3.5\%$	$i = -2.0\%$	$i = 7.0\%$
CNI, FIM	FH1	363424	103602	623103
	FH2	309534	146276	668598
	FH3	386914	67055	790063
FC, FIM	FH1	543517	759042	400600
	FH2	1310860	1439164	1079083
	FH3	811129	1081344	578091
ROC, %	FH1	3.24	2.73	4.50
	FH2	4.75	4.52	5.54
	FH3	4.18	3.74	5.57

To formulate a problem of profitability maximization, the partial utility function of ROC was defined such that the maximum value of ROC produces maximum utility. The lowest acceptable value of ROC was set at zero.

The results – which are equal for functions I–IV – show that most of the cuttings occur in the first year (Fig. 24, $i=3.5\%$). The net income level of this management plan means that approximately half of the potential cutting possibilities should be used. This regime results in a declined value of the forest capital of FH1 (Table 9).

The maximization of ROC leads to an uneven distribution of income. The temporal peak in the cuttings depends on the applied interest rate of

the alternative investments. Lower interest rates generally result in delayed cuttings, while high interest rates lead to immediate cuttings. In this respect, two additional tests were carried out to find out whether the results of the model match the capital-theoretic basis.

In the first test, the interest rate was set at -2.0% . This usually means conditions of high inflation provided that only financial investments are considered. As expected, the timing of the cuttings changes so that the emphasis is on the last years of the planning period (Fig. 24), and at the same time, both the total return on capital and the net income decline (Table 9). For some of the over-dense stand compartments of FH1 and FH3 (peaks in the first years), it is profitable to thin the growing stock despite the disadvantageous economic circumstances. This is because the accelerated growth caused by thinning compensates for the financial loss produced by the negative interest rate.

In the second test, the interest rate was set at 7.0% . This clearly exceeds the return on capital of timber growing (value growth percentage) for all three forest holdings. It follows that the interest rate of 7.0% should yield solutions favoring early cuttings. As expected, the solutions are characterized by a high level of FNI, a considerable increase in return on capital (Table 9), and immediate cuttings (Fig. 24).

Table 10. Values of objective variables in sustained yield formulation with different objectives.

		$FC_A = IC$ $ITI_A = ITI_{max}$				$CNI_A = 0.45IC$ $FC_A = IC$ $ITI_A = ITI_{max}$			
		I	II	III	IV	I	II	III	IV
CNI, FIM	FH1	220760	284968	241955	260415	268579	324980	293948	308616
	FH2	533714	482214	563089	557265	533714	544341	557580	559796
	FH3	367604	395879	368289	405043	367877	395879	367210	405043
FC, FIM	FH1	653528	590815	633003	605752	599727	549992	570229	560441
	FH2	1045178	1085619	1019259	1017848	1045178	1025833	1021242	1021652
	FH3	787282	762940	783824	744463	786870	762940	785193	744463
ITI	FH1	95.9	87.3	82.1	84.9	86.1	82.9	91.8	90.1
	FH2	90.3	86.1	75.1	92.6	90.3	90.9	75.1	75.1
	FH3	71.2	80.8	88.1	85.3	71.2	80.8	86.8	85.3

4.4.2 A Sustained Yield Formulation

As an example of the exogenous determination of income flow, the formulation of a sustained yield model is presented below. The term “sustained yield” here refers to a situation in which the primary goals are the conservation of forest capital and evenness of temporal income distribution. To stress the aspects of the exogenous determination of income trajectories, return on capital is not dealt with as an objective in these calculations.

For function I, the sustained yield problem involves the formulation

$$\begin{aligned} \text{Max } U &= a_{FC}u_{FC}(FC) + a_{ITI}u_{ITI}(ITI) \\ RNI_1 &= RNI_2 = \dots = RNI_{10} \end{aligned} \quad (145)$$

For function III, the formulation is

$$\begin{aligned} \text{Max } a_{TNI}u_{TNI}(TNI) + a_{FC}u_{FC}(FC) \\ u_{TNI}(TNI) &= \sum_{i=1}^{10} a_{TNI_i}u_{TNI_i}(FNI_i) \\ FNI_1^A &= FNI_2^A = \dots = FNI_{10}^A = k \end{aligned} \quad (146)$$

where k represents the chosen yearly net income level. The multiplicative forms (II and IV) include the same variables.

Because a maximum evenness of income distribution is required, the corresponding partial utility function is linear. The partial utility function of final capital, in turn, is piecewisely linear: the aspiration level is set equal to initial capital.

To solve (145), the relative importances a_{FC} and a_{ITI} are set at 0.5. For (146) it is assumed that

$$\begin{aligned} a_{TNI} = a_{FC} = 0.5 \wedge a_{11} = a_{12} = \dots = a_{110} = 0.1 \\ \Rightarrow a_{TNI} + a_{FC} = 1 \wedge \sum_{i=1}^{10} a_{TNI_i} = 1 \end{aligned} \quad (147)$$

The relevant value for the constant k was determined as follows.

- (1) The production possibility boundary was approximated with the weighting technique.
- (2) The parameter k was computed by setting

$$FC = IC \quad (148)$$

and searching for the corresponding CNI from the boundary. After that

$$k = \frac{CNI}{10} \quad (149)$$

to obtain the values for the vector **FNI**.

All the functions yield an even income distribution without considerable intertemporal deviations. This applies also to functions III and IV, which do not strictly optimize the trajectories. The final capital varies falling below the initial capital in all the solutions of FH1 and FH3 (Table 10).

Additional requirements can be incorporated alongside ITI and FC. A typical one is the aspiration to guarantee a certain net income level. If the aspiration level of compounded net income is set

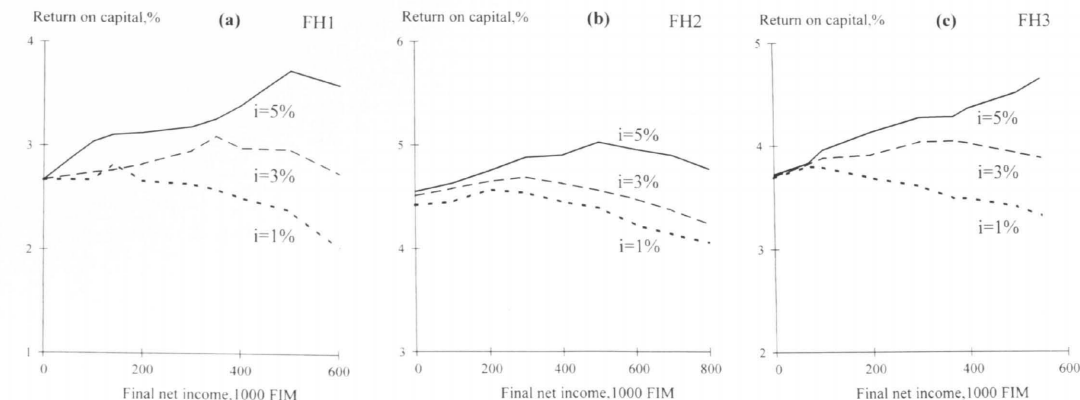


Figure 25. Relationships between final net income and return on capital when different interest rates are applied.

Table 11. Values of ROC in case of maximization of either ROC or ITI_c .

	$U = U(ROC)$ $ROC_A = ROC_{max}$ ROC	$U = U(ITI_c)$ $ITI_A = ITI_{max}$ ROC	Difference
FH1	3.24	2.57	0.67
FH2	4.75	4.30	0.45
FH3	4.18	3.60	0.58

equal to 0.45IC, for which $FNI_i = 0.045IC$ for all $i=1, \dots, 10$, losses have to be accepted for FH1 and FH3 compared to the initial capital. For FH2, this income level along with other requirements falls short of the production possibilities: the values of CNI in the solutions exceed the required level (Table 10).

4.4.3 Profitability and Evenness

Tradeoffs between profitability (production optimum) and evenness (exogenously determined income pattern) can be compared by computing plans for both objectives separately. The difference in profitability between these two approaches can be detected by subtracting the corresponding values of ROC from each other. The results (Table 11) imply that there is a strong conflict between these two objectives. It follows that the inclusion of both variables with maximum aspiration levels inevitably results in a compromise solution.

If the opportunity cost exceeds the average value growth percent of forest, management plans with abundant cuttings become more profitable (Fig. 25). Therefore, if evenness is required regardless of the net income level, then losses in profitability have to be accepted.

5 Discussion

5.1 Main Results

From a methodological standpoint, the present study concentrates on two primary topics. The first is the derivation of a theory of aspiration-based utility models. This approach is characterized by aspiration levels defined as those values which the decision-maker accepts as reasonable or satisfactory. Because the application of aspiration levels differs from the conventional utility theory, the main emphasis in this analysis was on the development of a mathematical framework describing the choice situation of the decision-maker. The rules for characterizing the choice situation were derived mainly by differential calculus.

The second major topic involves the formulation of a planning model for timber flow management: a model based on the derived theory of aspiration-based utility functions. The notions of natural resource economics were used in model formulation. Special attention was paid to the examination of income trajectories resulting either from exogenous determination or capital-theoretic optima.

The formulation of the utility model was based on a study of four different compound utility functions. The function forms were chosen to represent the most frequently applied models: the additive (I, III) and the multiplicative (II, IV). An additional combination was formed by altering the structure of the utility model (I–II vs. III–IV). Aspiration levels were interpreted as the determinants of the culmination points of the partial utility functions for each objective.

A direct search algorithm was incorporated into the model to facilitate utility maximization in numerical computations. Numerical analysis was used to test the essential elements of the theoretic framework.

The following general conclusions can be drawn from this study:

- (1) A multiobjective optimization problem which is based on the use of aspiration levels guarantees feasible solutions. The uniqueness of the solution depends on the aspiration levels.
- (2) The valid part of the indifference curves is linear (additive model) or strictly convex (multiplicative model). Aspiration levels affect the domain of the negatively sloped segment (multiplicative model) and the slope of the segment (additive model).
- (3) The multiplicative model gives rise to a higher marginal rate of substitution between objective variables. It follows that the use of the multiplicative model implies stronger dependence on forest management than the additive model.
- (4) Due to the shape of the indifference curves, the multiplicative form is more stable than the additive one. Especially in the case of approximately linear noninferior sets, the additive model is sensitive to changes in relative importances of objectives. Corner solutions contribute to the stability of the additive model.
- (5) If the aspiration point has a location inside the noninferior set, the properties of the indifference curves have no effects on the solution.
- (6) If the profile of income flow is considered crucial, more efficient results can be obtained by including an income trajectory variable than by using the aspiration levels of income per period. Squaring of the relative deviations from the objective trajectory guarantees smooth trajectories without large single deviations.
- (7) The presented flow model functions consistently in a theoretic sense. The approximation of production possibility boundaries is possible by applying the weighting or the aspiration level technique. Numerically produced results are verifiable by means of graphic and analytic solutions.

5.2 Evaluation

5.2.1 Resource Model for Timber Flow Economics

The approach of resource economics is a natural starting-point in modeling the use of forest resources. The general concepts of consumption and conservation are always the same, although the interpretation and modeling methods may vary (e.g. Kasanen 1982, Valsta 1993).

If the terms of consumption and conservation are described in monetary units, the problem of time preference is inevitable. The problem arises because the decision-maker is expected to be able to express the time preference in the form of an interest rate: the opportunity cost for growing forest. The determination of the interest rate is especially important because the opportunity cost affects both the alternative set – provided that monetary objectives are examined and intertemporal money flows include time compounding – and the composition of the optimal solution.

Monetary variables were chosen for this model on the grounds of the presumed financial purposes of planning and decision-making. This approach is based on the assumption that the ultimate goal of the decision-maker is the need to control money flows. Monetary terms are assumed to be preferred to objectives expressed, e.g., in cubic meters. In this framework, timber flows act as decision variables.

The net income from treatments is described as terminal value, i.e., the computation includes compounding. The stumpage value of the remaining growing stock describes the terminal value of forest and is referred to as final capital. By virtue of this interpretation, the sum of net income and final capital can be seen as the value of forest capital, as either invested or grown on. Profitability examinations can be based on the computation of the total returns yielded by this capital.

The approach of compounding can be regarded as somewhat controversial if the felling income is used for consumption: consumption preferences are more naturally described by means of discounting. However, both valuation methods convey the time preference of the decision-maker. It follows that a deviation from a capital-theoretic optimum results in losses irrespective of which valuation

method is applied.

The stumpage value of the growing stock is assumed to describe the value of the forest resource stock. Strictly speaking, this approach fails to describe the real value of the forest. This is because stumpage value ignores the future growth potential of the stock. Moreover, forest land – the value of which is excluded – is also part of the forest resources. These deficiencies were not, however, regarded as crucial, because the model yields the stumpage values for the initial and the final state, which can always be compared. Nonetheless, stumpage value suffers from the problems of statics: it includes little information on future action possibilities.

The inclusion of the profitability term (ROC) emphasizes the importance of the final state. This favors management regimes that produce a high terminal value, whether obtained through profitable further investments or through correctly implemented cuttings. An example of the latter case are thinnings which accelerate the growth of the most valuable part of the growing stock.

In functions I and II, the profitability term ROC can be used to stress the importance of the terminal state. If economic timing is based on compounded net income – which, in terms of results, is the same as the method of present net value – the final state is ignored. In functions III and IV, ROC is the only possible way to incorporate the economic timing of treatments. This is because these functions are designed to reach an aspiration level described as income per period: the total net income with its compound elements is only the sum of periodical net incomes. The effects of the different formulations can be measured, e.g., by the sawtimber-pulpwood percentage (Fig. 26), which can be computed from the results produced by the model.

Simulation of forest development was used to generate alternatives. From the viewpoint of the utility model, the properties and structure of the simulator need not be known, because the information on forest development is conveyed by means of the alternative set. This aspect also makes it possible to apply any other relevant simulation model for generating alternatives to facilitate numerical analysis.

Functions I and II are based on a static examination, i.e., dynamic events are described in a static

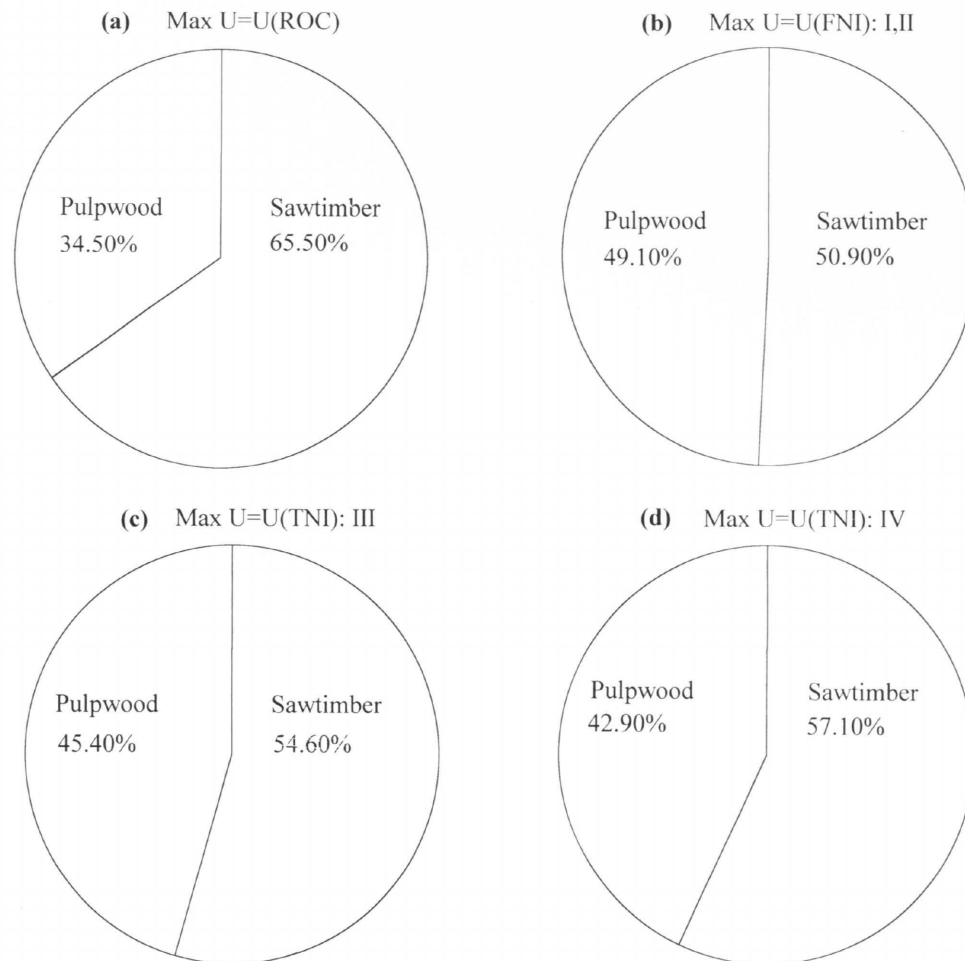


Figure 26. Structure of final growing stock as a result of the application of different function forms.

manner by ignoring the dynamic features of forest development and planning. The incorporation of dynamics would require, e.g., the use of the optimum control theory to solve forest management problems (see Lyon and Sedjo 1983). Dynamics is, however, recognized in functions III and IV, which allow for direct intertemporal decision-making concerning income in each period.

5.2.2 Utility Model

5.2.2.1 Compound Utility Functions

The introduction of four different compound utility function forms was based on the underlying idea of mutual comparison. The additive and multiplicative forms were chosen because of their clear theoretic background and frequent use in various applications. Because the control of income trajectories was regarded as an essential ingredient, this aspect was incorporated in two ways. The first method uses a distinct trajectory variable, while the second one involves the si-

multaneous determination of income trajectories and total net income. This leads to four function forms.

The implicit trajectory determination of functions III and IV of RM_2 has two clear advantages compared to functions I and II. First, the decision-maker has one variable less to evaluate and second, the elements of periodical net income can be weighted. The weighting possibility enables, e.g., the incorporation of time preference to describe the importances of elements of periodical net income. This method was applied by Kilkki et al. (1986) with the exception that the utility functions contained nonmonetary objectives. If I or II are applied, only the importance of the trajectory as a whole can be specified. In this study, the weighting of periodical net income was not experimented. This is a topic for further research.

A drawback of III and IV is that they fail to produce a net income distribution having the shape of a desired trajectory, unless the trajectory is a close approximation of the production possibility boundary. This is, however, not a serious problem because functions III and IV are based on the idea that only shortages are disadvantageous. The same utility is always obtained after the aspiration level has been achieved.

5.2.2.2 Partial Utility Functions

The cornerstone of this study is the incorporation of aspiration levels into the traditional utility theoretic framework. Aspiration levels (Wierzbicki 1979, 1980a) have a specific interpretation in the realm of multiobjective optimization, especially in goal programming. Although utility functions are frequently mentioned in the context of aspiration levels (see Vetschera 1994), those interpretations are primarily concerned with scalarizing functions seen as a generalization of compound utility functions (Wierzbicki 1979). They also apply the concept of a threshold utility function. The present study, on the other hand, develops a synthesis of aspiration levels and the multiattribute utility theory.

A basic assumption regarding aspiration levels is that they determine the point which produces the highest utility on each objective. This was assumed to correspond with the definition of a

reasonable or satisfactory value. On the other hand, after that particular point there is no need to optimize (Lewandowski and Wierzbicki 1989). The lower part of a partial utility function is fixed by the lowest acceptable value of each objective.

Partial utility functions originating from these definitions can be seen as piecewise linear approximations of strictly quasiconcave utility functions. The horizontal plane segments are characterized by a zero value of marginal utility. In this area, an increase in the quantity of an objective variable does not increase utility. Although marginal utility equal to zero is a reality in some cases (Lipse and Steiner 1972), the assumption may be debatable when monetary units are used.

A drawback arising from zero marginal utility concerns the shape of the indifference curves: unique solutions are not easily guaranteed and, moreover, corners in the curves can cause surprises. The nonsmoothness of functions as such does not create difficulties. Although the differentiability condition is not met, piecewise examination rules out this theoretic problem.

The applied compound utility model has a crucial effect on the shape of indifference curves. In the case of the additive form, the valid part of the indifference curves is linear. In addition, the slope of the partial utility functions affects both the domain of this segment and the slope of the indifference curves. The slope of the indifference curves is further affected by relative importances.

In the case of the multiplicative form, the slope of the indifference curves is independent of the aspiration levels. This is due to the linearity of the partial utility functions. In the partial differentiation of the logarithmic form, the linear term drops out, giving rise to a ratio of partial derivatives dependent only on the absolute values of the objectives. This independence feature contributes to the stability of the multiplicative model. If the positively sloped segment of the partial utility functions is nonlinear, the slopes of the corresponding indifference curves are affected by the slopes of the partial utility curves.

As is evident by virtue of the strict convexity of the indifference curves, the multiplicative model generates a higher marginal rate of substitution between objectives. This means that if deviations from the optimum occur, greater utility losses are encountered if preferences are expressed by means

of the multiplicative model. From the standpoint of objectives, the multiplicative model requires a higher compensation to attain an equivalent utility level (Fig. 27).

The properties of indifference curves presented here are also applicable to situations in which the partial utility functions have several linear segments (Pukkala and Kangas 1993). A few additional formulas and examinations are, however, required to create a more general mathematical basis. This may be necessary also if an aspiration level is given an interpretation other than that of a partial utility maximum.

An examination of the features of indifference curves is of substantial importance especially when production possibility boundaries are far from the well-behaving curves of economic theory. Considering this, the multiplicative form has the salient feature of producing indifference curves with strictly convex negatively sloped segments. This shape actually implies a diminishing marginal rate of substitution, which is often a basic assumption in utility models.

The theory of indifference curves makes it possible to solve limited forest management problems analytically in cases when the noninferior set can be approximated. In addition, graphic and analytical tools are useful in sensitivity analysis and in the examination of differences between function forms.

5.2.2.3 The Search for the Solution

In this study, the utility maxima of functions I–IV were sought by applying an algorithm which enables the use of multiple objectives and nonlinear objective functions. The structure of the algorithm also makes it possible to take trajectory variables into account. This approach has the advantage of allowing all the studied models to be solved with the same procedure.

Function III can be solved by means of goal programming. This requires the use of one-sided goal variables (e.g. Buongiorno and Gilles 1987). The multiplicative forms II and IV are more difficult because they generate nonlinear objective functions. Besides, function II has a trajectory variable ITI which is difficult to deal with in ordinary mathematical programming. Multiplicative

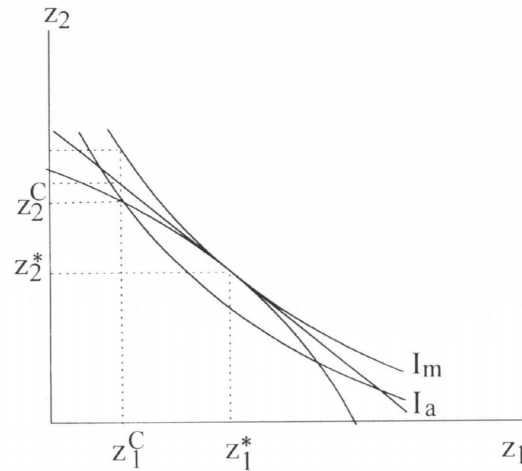


Figure 27. Effects of deviations from the optimum.

functions can be changed into additive forms by logarithmic transformations; the problem of nonlinearity remains.

The interpretation of compartments as nondivisible management units was another crucial factor when considering a relevant optimization method. The use of a direct search method in problems involving discrete choices and interactivity is strongly supported by the observations of Vetschera (1994). Usually this kind of procedure combines a relatively small computational effort with good solution quality. Solving similar problems, e.g., by means of mixed integer programming results in unacceptably long computing times.

Finding an appropriate optimization method was not a major issue in this study. Instead, the uniqueness of the solution was considered to be the decisive aspect. Uniqueness proved to be a problem regardless of the optimization method used.

In a theoretic sense, nonunique solutions are not harmful. This is because the aspiration points within the production possibility boundary reflect a choice which the decision-maker finds acceptable. It follows that the decision-maker is indifferent to points beyond the satisfactory levels of the objectives.

Another problem is the locality and randomness of solutions yielded by the direct search algorithm. In repeated computations, the discretization of the decision space and the nonconcavity

of the noninferior sets result in different local solutions. In the computations made in this study, the initial solution was fixed such that a predetermined schedule was chosen for a basic feasible solution. This always yields the same solution provided that the premises remain unchanged. A change in the compartmentwise treatment schedule to be chosen for the initial solution thus results in a different final solution.

The approach of fixed initial solutions contributes only to stabilization; the problem of local solutions remains. To attain true global optima, the computations of the numerical analysis should have been repeated hundreds of times. This aspect has to be kept in mind when evaluating the results of the analysis.

Reference point optimization involves finding the point from the noninferior set nearest to the reference point (e.g. Kallio et al. 1980). On the other hand, in utility-theoretic optimizations, the efficient solution has to be consistent with the preferences of the decision-maker. This difference is the reason for not applying the approach of reference point optimization in the present study. For further illustration, let us examine two cases related to different locations of the aspiration point.

In the first case, the aspiration point is assumed to lie outside the production possibility boundary. The optimum point is determined by indifference curves and, ultimately, by aspiration levels. It is, thus, a solution based on the utility theory. In the second case, the aspiration point is assumed to lie within the production possibility boundary. In the approach applied in this study, this nonunique solution is predetermined and dependent on the initial solution, though not known. A solution consistent with partial utility functions could be found by searching for an efficient solution in the direction indicated by the slope of the indifference curves.

In the latter case, the applied optimization method has the same drawback as goal programming: it fails to guarantee noninferior solutions. Reference point optimization – as a generalization of goal programming – has the salient feature of being able to yield efficient solutions also for aspiration points located within the noninferior set.

The solution found by the algorithm is a compromise between conflicting objectives. The so-

lution is partly dependent on the method by which the partial utilities are combined, i.e., compound utility functions. The additive model is sensitive even to slight changes in relative importances, especially if the aspiration levels are equal to the single-objective maxima and the production possibility boundary is not strictly concave. The multiplicative model produces well-predictable results even when the production possibility boundary is nearly linear.

Three different methods were introduced to optimize the income trajectories in functions I and II. Two of these are capable of dealing with nonlinear trajectories, whereas the third one is aimed at finding candidates for a linear and constant objective trajectory. The first two methods measure discrepancies between an objective income trajectory and feasible trajectories by examining relative deviations; the third one makes use of absolute deviations. Other measurement techniques were not considered because there is actually no consensus about what measure to use (Steuer 1986).

Trajectory optimization based on the squaring of relative deviations was found to result in smooth trajectories with a rapid convergence. If summing of unsigned deviations is used, the characteristics of the trajectories may show good correspondence, but single peaks are customary. The method of absolute deviations was not tested because of its limited capabilities. Trajectory optimization with a distinct variable (I–II) is usually more efficient than one using periodical net income (III–IV).

5.2.3 Aspects of Income Flow Determination

The last part of the numerical analysis concentrated on the endogenous and exogenous determination of income flows. Tradeoffs between these two approaches were of primary concern. The succinct material of the analysis justifies only limited conclusions. Therefore, the following remarks are valid only in this context and should be seen as supporting the findings in respect of the capital-theoretic basis.

Maximization of profitability along with additional requirements leads to uneven income distributions. The peaks of these distributions de-

pend on prices and interest rate. As a rule, increased prices and interest rates suggest intensified cuttings. Thinnings may be profitable in over-dense forest stands even if the opportunity cost is negative. If evenness of income distribution is required, losses in profitability are inevitable (see e.g. Kilkki 1989). The losses depend on the required net income level and the interest rate.

Functions I and II enable profitability maximization with a total net income requirement. If the net income requirement is included in III or IV, finding the maximum profitability presupposes *a priori* information about the corresponding income trajectory. If evenness is required, the net income is implicitly specified as a sum of the incomes per period.

The calculations in this study are based on the assumptions of determinism: all events are assumed to occur in circumstances of perfect certainty. In reality, uncertainty prevails, implying that conclusions drawn from single computations may be misleading. In forest management planning, uncertainty concerns both natural processes and the economic environment. A solution to this problem has been presented by Valsta (1992b): the formation of scenarios to describe the stochastic elements in a decision situation. The same approach could also be applied to the model presented in this study. Subjective scenarios regarding, e.g., timber prices could be formed to describe the expectations of the decision-maker.

5.3 Conclusions and Future Work

The first aim of this study was to develop a theoretic basis concerning the application of aspiration levels in multiattribute utility models.

The approach is a general one, allowing the formulation of models with different objective variables. Provided that applications are planned to meet the assumptions of multiattribute models, the aspiration-based method is applicable to any decision problems involving utility considerations. The general theory offers analytic tools to deal with production possibility boundaries of different shapes.

The method presented here is theoretically well-based although all the conditions founded on microeconomic theory are not met. This offers a good starting-point for further research as well as for development towards applications of practical management models. In this respect, studies measuring the relevancy and simplicity of the method in practical planning situations would be especially important.

The model resulting from the second aim of this study can be regarded as a theoretic framework of a planning model for timber flow management. In the realm of forest management planning, the standpoint of timber flow economics is extremely limited. The model as such is, therefore, not intended for use in practical planning situations. In addition, the model contains only selected variables, meaning that the application is not fully comprehensive even for timber flow economics. This is, however, in accordance with the underlying idea of modeling: an abstraction of the real world.

Further research and practical tests should cover subjects related to the inquiry of preferences and the relevancy of the results as a basis for timber management. Sufficient research in this area is the only method for providing information about the applicability of the model to the ultimate goal: producing feasible timber management plans.

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Total of 119 references

Appendix. Generating alternatives with simulation

The following forest simulation model was developed to generate a finite number of possible management schedules needed in numerical analysis. The model was composed to cover the most common forest treatment methods. The simulation procedure consists of two primary elements: growth and yield functions and generation of treatments.

If the description of the forest resource stock is based on a standwise inventory, the diameter distributions can be predicted with the Weibull function (see, e.g. Kilkki et al. 1989). A separate distribution is estimated for each tree species and storey. The probability density function for the Weibull random variable x is (Rennols et al. 1985)

$$f_W(x) = \frac{c}{b} \left(\frac{x-a}{b}\right)^{c-1} \exp\left(-\left(\frac{x-a}{b}\right)^c\right), \quad a \leq x \leq \infty \quad (150)$$

$$= 0, \quad x < a$$

and the cumulative distribution function of the Weibull is

$$F_W(x) = 1 - \exp\left(-\left(\frac{x-a}{b}\right)^c\right), \quad a \leq x \leq \infty \quad (151)$$

$$= 0, \quad x < a$$

Parameters a , b and c determine the shape and location of the Weibull distribution. Estimates of these parameters are computed from stand characteristics by regression functions. For pine, parameter a is derived from (Mykkänen 1986)

$$\ln(a) = -1.306454 + 1.154433 \ln(D) \quad (152)$$

and parameter c from

$$\ln(c) = 0.647888 - 0.005558G + 0.025530D \quad (153)$$

Parameter b can be solved from

$$b = \frac{D-a}{(-\ln(0.5))^{1/c}} \quad (154)$$

For spruce, parameters a and c are solved from (Maltamo 1988)

$$a = -0.784428 + 0.9767681 \ln(D) \quad (155)$$

$$c = 0.642605 + 0.021394D - 0.007945G + 0.000980A \quad (156)$$

The models for pine are used for the other tree species also.

The diameter range of the cumulative distribution function is divided into a fixed number of diameter classes. Six classes are used in this simulation model. The stem in the center of each class is chosen to represent the whole class. The number of stems is calculated from (Kilkki 1984)

$$n_T = 12732.4 \frac{G}{D^2} \quad (157)$$

to give an estimate of number of stems in each diameter class

$$n_i = (F_W(d_i^U) - F_W(d_i^L)) n_T, \quad i = 1, \dots, 6 \quad (158)$$

For volume calculations, the heights of the stems are estimated with the models of Veltheim (1987).

Tree growth is simulated with the functions that predict the alterations in the basal area and height of the stem during periods of five years. Growth for shorter periods is linearly interpolated. The basal area growth models are (Ojansuu et al. 1991)

$$d_5^{ga} = \beta_0(1 + \beta_1 DD)^{\beta_2} (d_{ref}^{ga} t^{\beta_3} + \beta_4 d) \quad (159)$$

$$d_5^{gp} = \gamma_1 \gamma_2 (1 + \beta_1 DD)^{\beta_2} (d_{ref}^{gp} + \beta_4 d) \quad (160)$$

Height growth is determined as the difference between the predicted heights of a tree at two points in time. Height is assumed to develop according to (ibid.)

$$h = A_h (1 - e^{-\omega t})^{1/(1-\omega_2)} \quad (161)$$

in which the asymptote A_h is computed from

$$A_h = \kappa_1 (\kappa_2 + DD)^{\kappa_3} \left(\frac{g}{g_{max}}\right)^{\kappa_4} \quad (162)$$

A deterministic estimate of tree mortality is predicted by a logistic function (Haapala 1983)

$$P_d = \frac{1}{1 + \exp(\zeta_0 + \zeta_1 d + \zeta_2 G + \zeta_3 G_d)} \quad (163)$$

The volumes of different timber assortments are calculated with the taper curve models of Laasasenaho (1982), which give estimates of sawtimber, pulpwood and wastewood (Laasasenaho and Snellman 1983). The growth estimates for each timber assortment can be calculated as the difference between the total volumes at two points in time.

Treatment simulation is based on rules, called control variables, which originate partly from the Finnish law (Yksityismetsälaki 1967, Yksityismetsäasetus 1991) and from silvicultural recommendations (Metsänhoitosuositukset 1989), but mostly from heuristics. Treatment rules cover, e.g., felling methods, time intervals between consecutive treatments, number of plants in planting, minimum removals, and number of trees remaining in felling for natural regeneration.

The forest treatment methods can be divided into cuttings and silvicultural treatments. The category of cuttings includes

- cutting of hold-overs
- thinning
- clear cutting
- shelterwood felling
- seeding felling

and that of silvicultural treatments covers

- seeding
- planting
- cleaning of sapling stands
- soil preparation
- clearing

Rotation ages are calculated by means of marginal analysis. During simulation, marginal changes in the stumpage value of growing stock are computed from

$$r_i^F = T_E - T_I^T \sqrt{\frac{V_i^E}{V_i^T}} - 1, \quad i = 1, \dots, n_p \quad (164)$$

On economic grounds, forest can be regenerated when the rate of growth falls below the opportunity cost. Rotations defined by this type of procedure depend not only on interest rate, but on timber price scenarios as well. Expectations of price increases result in longer rotations, and vice versa.

The profitability of thinning is used as a thinning criterion. This variable is composed of two parts: the value growth of the thinned growing stock and the return on felling income if invested. Hence, a thinning decision involves the comparison

$$T_E - T_I^T \sqrt{\frac{V_i^E}{V_i^T}} > T_E - T_I^T \sqrt{\frac{TV_i^E + I_i^T (i+1)^{T_E - T_I^T}}{V_i^T}}, \quad i = 1, \dots, n_p \quad (165)$$

If the return produced by thinning (right side) falls short of the future value growth of the growing stock (left side), the forest stand should be left to grow on. As the growing stock becomes denser, the relative value growth tends to decrease, making thinning more profitable.

The thinning model presupposes that the removal percentage, i.e., the intensity of thinning, is defined. This figure affects not only the thinning income but also the growth potential of the remaining growing stock. For this reason, the structure of the model allows the incorporation of several thinning rates with different intensities for simulating, e.g., light, medium and heavy thinnings.

The timing of thinning as described by the profitability criterion depends on the interest rate and the relative timber prices. A high interest rate results in early thinnings. This is because the critical density of forest is reached earlier if other investments yield high returns. The effects of prices are more difficult to observe because price increase expectations lead to higher values of both unthinned and thinned growing stock. As a rule, an increase in logwood price corresponds with early thinnings.

The simulation of thinning and other corresponding treatments is based on an iterative removal of trees (for comparison, see e.g. Jämsä 1991, Saramäki 1992, Valsta 1992a). The number of removed trees in each iteration is expressed by removal percentages, which determine the type of treatment (e.g. thinning from below). These iteration values are defined separately for each diameter class and tree species.

During the iterations, the variables depicting the growing stock are tested against the control variables. If the basal area is used as a control variable, the test involves a comparison

$$\sum_{j=1}^6 (N_{ij} - (R_{ij}/100)N_{ij}) \pi\left(\frac{D_{ij}}{2}\right)^2 \leq G_i^G, \quad i = 1, \dots, 3 \quad (166)$$

in the case of six diameter classes and three tree species.

Along with the basal area, the number of stems and the volume of removal can be used as a stopping rule. For each iteration, the total removal is computed from

$$\sum_{i=1}^3 \sum_{j=1}^6 \sum_{k=1}^3 ((R_{ij} / 100) N_{ij}) V_{ik} \quad (167)$$

where k is an index of timber assortment. The structure of the removal is calculated from modifications of (167).

Simulation proceeds in an iterative manner. The development of the growing stock is calculated at the end of each management period. After that, the algorithm examines the structure of the growing stock and infers the possible treatment. If no treatment alternative is valid, the simulation continues to the end of the

next management period. If any of the treatments can be implemented, the growing stock is manipulated according to the rules described in the control variables. After treatment, the simulation continues in the same way as without treatment. The simulation ceases at the end of the planning horizon.

The above procedure describes the first round of the simulation. In the second round, the starting-point is the year of the first treatment (next-event incrementing, see Buongiorno and Gilles 1987) added by the length of the management period. The feasible treatment is generated by this procedure, and the remaining part of the planning horizon is simulated. The rounds with shifted starting-points are repeated until the end of the planning horizon. The number of treatment schedules for an arbitrary compartment i is

$$n_i^s = \sum_{j=1}^{n_p} n_{ij}^s \quad (168)$$

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- 243 **Rauno Väisänen** and **Kari Heliövaara**: Assessment of insect occurrence in boreal forests based on satellite imagery and field measurements.
- 244 **Eero Kubin** and **Lauri Kemppainen**: Effect of soil preparation of boreal spruce forest on air and soil temperature conditions in forest regeneration areas.
Instructions to Authors.
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