ACTA FORESTALIA FENNICA

Vol. 122, 1971

Optimization of Stand Treatment Based on the Marginal Productivity of Land and Growing Stock

Maan ja puuston rajatuottavuuksiin perustuva metsikön käsittelyn optimointi

Pekka Kilkki

SUOMEN METSÄTIETEELLINEN SEURA

Suomen Metsätieteellisen Seuran julkaisusarjat

ACTA FORESTALIA FENNICA. Sisältää etupäässä Suomen metsätaloutta ja sen perusteita käsitteleviä tieteellisiä tutkimuksia. Ilmestyy epäsäännöllisin väliajoin niteinä, joista kukin käsittää yhden tutkimuksen.

SILVA FENNICA. Sisältää etupäässä Suomen metsätaloutta ja sen perusteita käsitteleviä kirjoitelmia ja lyhyehköjä tutkimuksia. Ilmestyy neljästi vuodessa.

Tilaukset ja julkaisuja koskevat tiedustelut osoitetaan Seuran toimistoon, Unioninkatu 40 B, 00170 Helsinki 17.

Publications of the Society of Forestry in Finland

ACTA FORESTALIA FENNICA. Contains scientific treatises mainly dealing with Finnish forestry and its foundations. The volumes, which appear at irregular intervals, contain one treatise each.

SILVA FENNICA. Contains essays and short investigations mainly on Finnish forestry and its foundations. Published four times annually.

Orders for back issues of the publications of the Society, subscriptions, and exchange inquiries can be addressed to the office: Unioninkatu 40 B, 00170 Helsinki 17, Finland.

OPTIMIZATION OF STAND TREATMENT BASED ON THE MARGINAL PRODUCTIVITY OF LAND AND GROWING STOCK

MAAN JA PUUSTON RAJATUOTTAVUUKSIIN PERUSTUVA METSIKÖN KÄSITTELYN OPTIMOINTI

PEKKA KILKKI

HELSINKI 1971

CONTENTS

	Page
1. Timber production function	3
2. Determination of the rational growing density	3
3. Financial maturity and optimum growing density of the stand	5
4. Discussion	7
References	7
Seloste	7

Suomalaisen Kirjallisuuden Kirjapaino Oy Helsinki 1971

1. TIMBER PRODUCTION FUNCTION

The concept production process means the transformation of inputs into outputs, and it can be summarized by a mathematical relation which is called the production function (COHEN and CYERT 1965, p. 110). The timber production of a forest stand, too, can be described by a production function. The inputs of the function consist of land, growing stock, silvicultural measures, etc., the outputs being various assortments of timber. In the present connection, in order to simplify the problem, a production function for only one year will be considered. It is also assumed that land and growing stock are the only variable inputs and the annual value growth, the output. Under these circumstances, the timber production function takes the following form:

(1) $Y = f(X_1, X_2 \mid X_3, \ldots, X_n)$

in which Y = annual value growth of the stand
X_1 = area of the stand
X_2 = growing stock of the stand
$X_3, \ldots, X_n$ = other factors influencing the value growth (here assumed to be constants)

The partial derivative of this function with respect to the area of the stand $(\frac{\partial Y}{\partial X_1})$ gives the marginal productivity of the land. It expresses the increase in annual value growth per unit of marginal increase over the area of the stand when its total growing stock remains unchanged. Consequently, the partial derivative of function (1) with respect to the growing stock $(\frac{\partial Y}{\partial X_2})$ gives the marginal productivity of the growing stock and expresses the increase in annual value growth per unit of marginal increase in the growing stock when the area of the stand remains unchanged.

If we assume that the value growth is linearly dependent on the area of the stand when the growing stock per hectare remains constant, function (1) can be expressed as follows:

(2) $Y = g(W \mid X_3, \ldots, X_n) \cdot X_1$

in which $W = X_2/X_1$

It can be seen that when the area and total growing stock of the stand are multiplied by n, the value growth will also be multiplied by n. Thus, function (2) is a linear and homogeneous production function (SPENCER and SIEGELMAN 1964, p. 267). It can also be noticed that the partial derivatives of function (2) with respect to X_1 and X_2 can be expressed as a function of W only. Furthermore, using its partial derivatives, function (2) can be expressed as follows:

(3) $$Y = \frac{\partial Y}{\partial X_1} X_1 + \frac{\partial Y}{\partial X_2} X_2$$

(BAUMOL 1965, p. 405)

2. DETERMINATION OF THE RATIONAL GROWING DENSITY

Figure 1 shows some of the main features of production function (2) (cf. SPENCER and SIEGELMAN 1964, pp. 260—262). When the density of the growing stock is very low, competition between trees does not markedly reduce the growth of the trees. On the contrary, the trees may support each other's growth by providing protection against wind and other damages. Thus, it can be assumed that in a low density stand, an increase in the growing stock raises the value growth with increasing speed. This means that the marginal productivity of the growing stock goes up as the growing stock increases.

The inflection point of the value growth function is located at the density where the negative effect of competition between the trees nullifies the positive effect of protection. The marginal productivity of the growing stock starts falling after this density. On the other hand, the average productivity of the growing stock increases still further until it

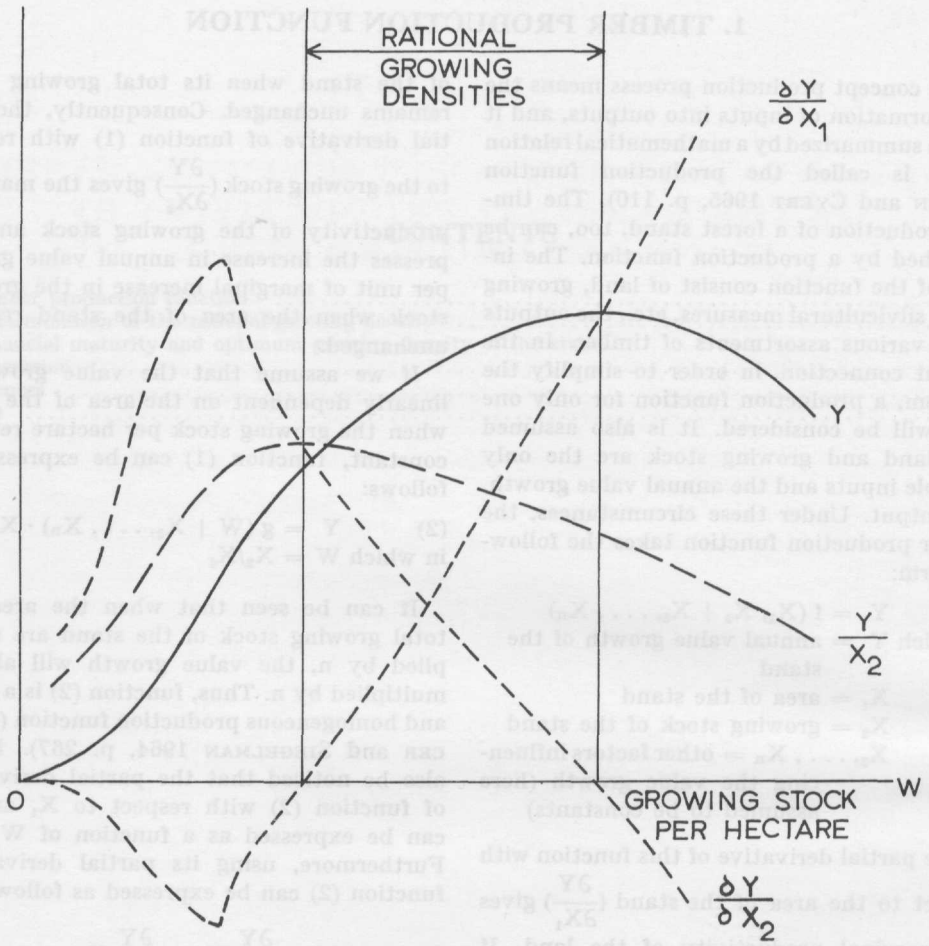

Figure 1. Annual value growth (Y), marginal productivity of the land $(\frac{\partial Y}{\partial X_1})$, marginal productivity of the growing stock $(\frac{\partial Y}{\partial X_2})$, and average productivity of the growing stock $(\frac{Y}{X_2})$ at certain age as a function of the growing stock per hectare (W).

reaches the positive stationary point at a certain density. At the same density the marginal productivity of land becomes positive.

When the density of the growing stock is so high that any increase in it will decrease the value growth, the marginal productivity of the growing stock becomes negative. At this density the marginal productivity of the land equals the value growth per hectare (cf. formula (3)) and continues to ascend with increasing density.

The previous characteristics taken from Figure 1 indicate that the only rational growing densities if only timber production is concerned are those in which the marginal productivities both of the land and of the growing stock are positive. This conclusion becomes even more evident if the isoproduct curves of the production function in figure 1 are examined (Figure 2, solid lines). These indicate that if the growing stock is reduced below the level required for the maximum average productivity of the growing stock, the total growing stock and also the area of the stand must be increased in order to maintain a certain total value growth. Respectively, if the density of the growing stock

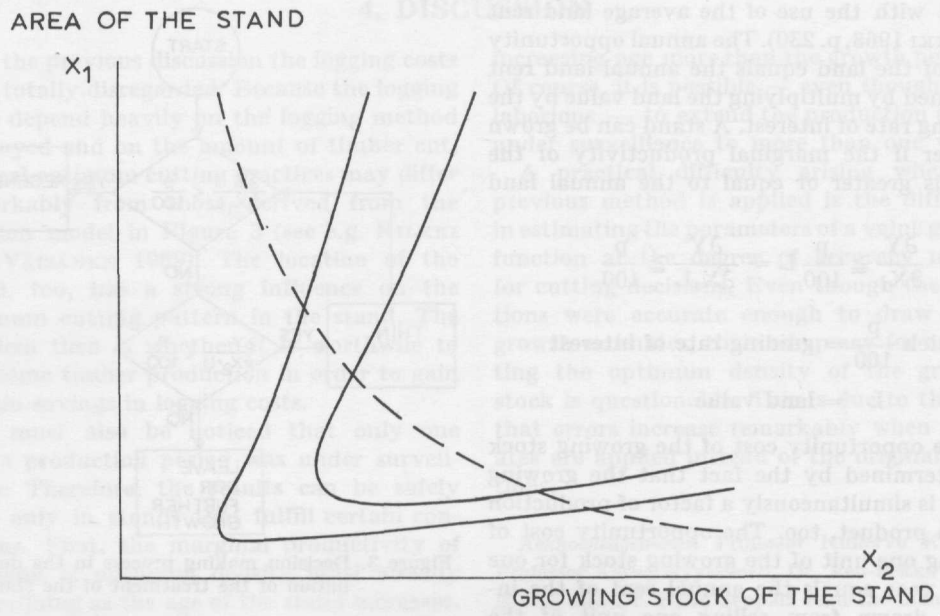

Figure 2. Isoproduct curves of two value growth functions.

exceeds the density where the value growth per hectare is maximized, the area of the stand, and correspondingly also the total growing stock, must be increased in order to reach a certain value growth level.

The value growth function of a stand may, of course, be of a more simple form than that in Figure 1. If there is no inflection point in the value growth function, the marginal productivity of the growing stock decreases at the same time as the growing stock increases and the marginal productivity of the land is never negative. Correspondingly, if the value growth function has no maximum, the marginal productivity of the growing stock remains always positive. If both the inflection point and the positive stationary point are missing, any growing density between zero and infinity may be rational under certain conditions. An isoproduct curve of a value growth function of this kind is presented in Figure 2 with a dotted line.

3. FINANCIAL MATURITY AND OPTIMUM GROWING DENSITY OF THE STAND

In timber production, either land or growing stock, and very often both of them, are scare resources. Therefore, it is not sufficient that the growing density of the stand is within the rational limits defined earlier. It is also required that the marginal productivity both of the land and of the growing stock exceeds or at least equals their opportunity costs.

The opportunity cost of the land in timber production is determined by the fact that land which is now occupied by the present growing stock can also be used to grow further timber generations. The opportunity cost of forest land can be expressed by the land value. In a single forest stand the land value can be calculated by Faustmann's formula, for example, and in a fully regulated

forest with the use of the average land rent (KILKKI 1968, p. 230). The annual opportunity cost of the land equals the annual land rent obtained by multiplying the land value by the guiding rate of interest. A stand can be grown further if the marginal productivity of the land is greater or equal to the annual land rent:

$$\frac{\partial Y}{\partial X_1} \geqq \frac{p}{100} L \Rightarrow \frac{\partial Y}{\partial X_1 L} \geqq \frac{p}{100}$$

in which $\frac{p}{100}$ = guiding rate of interest

L = land value

The opportunity cost of the growing stock is determined by the fact that the growing stock is simultaneously a factor of production and a product, too. The opportunity cost of having one unit of the growing stock for one more year equals the annual rent of the income drawn from selling one unit of the growing stock. The optimum density of the growing stock is obtained when the marginal productivity of the growing stock equals the annual rent of one unit of the growing stock:

$$\frac{\partial Y}{\partial X_2} = \frac{p}{100} U \Rightarrow \frac{\partial Y}{\partial X_2 U} = \frac{p}{100}$$

in which U = unit price of the growing stock.

Thinning and regeneration decisions for the stand can be combined within the same decision process (Figure 3). First, it is examined to see whether the marginal productivity of the land falls below the annual land rent. If this is the case, the stand must be regenerated, because it is impossible momentarily to increase the growing stock so that the marginal productivity of the land would be increased, too.

If the marginal productivity of the land exceeds the annual land rent, the marginal productivity of the growing stock is examined. If this falls below the annual rent of one unit of the growing stock, the stand must be thinned to the density at which the marginal productivity of the growing stock equals the annual rent of one unit of the

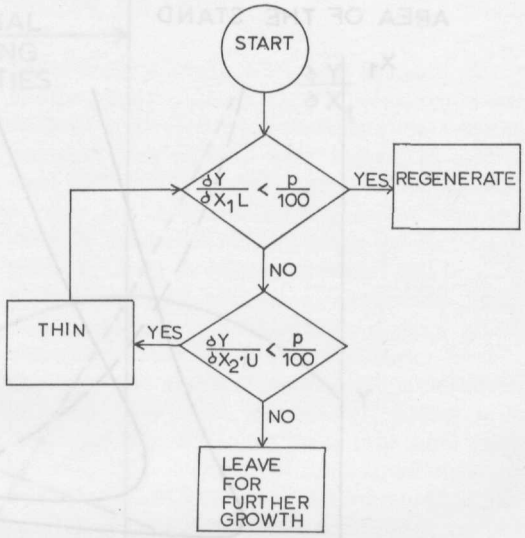

Figure 3. Decision making process in the determination of the treatment of the stand.

growing stock. If the marginal productivity of the growing stock exceeds the annual rent of one unit of the growing stock, the stand should be left for further growth.

If the stand has fallen to the thinning category, it must be re-examined after thinning to see whether the marginal productivity of the land still exceeds the annual land rent. It is fully possible that in an old, heavily stocked stand, the marginal productivity of land is high enough to justify continued growth of the stand: however, after thinning it may appear that the land could be used more efficiently to grow a new tree generation.

It can also be shown by the use of derivation that the optimum density defined as above, guarantees the highest possible v-value (see e.g. JÖRGENSEN and SEIP 1954) that indicates the financial maturity of the stand:

$$v = Y - \frac{p}{100} X_1 L - \frac{p}{100} X_2 U \Rightarrow$$

$$\frac{\partial v}{\partial X_2} = \frac{\partial Y}{\partial X_2} - \frac{p}{100} U = 0 \Rightarrow \frac{\partial Y}{\partial X_2 U} = \frac{p}{100}$$

4. DISCUSSION

In the previous discussion the logging costs were totally disregarded. Because the logging costs depend heavily on the logging method employed and on the amount of timber cut, the real optimum cutting practices may differ remarkably from those derived from the decision model in Figure 3 (see e.g. KILKKI and VÄISÄNEN 1969). The location of the stand, too, has a strong influence on the optimum cutting pattern in the stand. The problem then is whether it is worthwile to lose some timber production in order to gain certain savings in logging costs.

It must also be noticed that only one year's production period was under surveillance. Therefore, the results can be safely used only in stands that fulfill certain conditions. First, the marginal productivity of the land at optimum density must already be declining as the age of the stand increases. Secondly, the density of the stand must be at the optimum level or above it, or clearly under the optimum. Third, the optimum density of the stand may not increase with increasing age more than the growth permits. Of course, it is possible — even though more laborious — to extend the production period under surveillance to more than one year.

A practical difficulty arising when the previous method is applied is the difficulty in estimating the parameters of a value growth function at the degree of accuracy needed for cutting decisions. Even though the functions were accurate enough to draw value growth estimates, their adequacy for estimating the optimum density of the growing stock is questionable. This is due to the fact that errors increase remarkably when derivates are applied instead of the original functions.

Acknowledgements. Professor Kullervo Kuusela, Professor Aarne Nyyssönen, Dr. Veli-Pekka Järveläinen, and Dr. Risto Seppälä read the manuscript and made valuable comments. The English text was checked by Mr. Karl-Johan Ahlsved, B.Sc. The paper is one part of a research project financially supported by the Finnish Natural Resources Foundation.

REFERENCES

BAUMOL, W. J. 1965. Economic theory and operations analysis. Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

COHEN, K. J. and CYERT, R. M. 1966. Theory of the firm: Resource allocation in a market economy. Prentice-Hall Inc., Englewood Cliffs, New Jersey.

JÖRGENSEN, F. and SEIP, H. K. 1954. Hogstmodenhet i glissen granskog. Mimeographed. Vollebekk.

KILKKI, P. 1968. Some economic aspects of growing forest stands. Seloste: Eräitä taloudellisia näkökohtia metsiköiden kasvatuksessa. Silva Fenn. 2. 4: 225–234.

KILKKI, P. and VÄISÄNEN, U. 1969. Determination of the optimum cutting policy for the forest stand by means of dynamic programming. Seloste: Metsikön optimihakkuuohjelman määrittäminen dynaamisen ohjelmoinnin avulla. Acta For. Fenn. 102.

SPENCER, M. H. and SIEGELMAN, L. 1964. Managerial economics. Richard D. Irwin, Inc., Homewood, Illinois.

SELOSTE

MAAN JA PUUSTON RAJATUOTTAVUUKSIIN PERUSTUVA METSIKÖN KÄSITTELYN OPTIMOINTI

Metsikössä tapahtuvaa puuntuotantoa on kuvattu lineaarisella ja homogeenisella tuotantofunktiolla (2), jossa tuotteena on vuotuinen arvokasvu ja muuttuvina tuotannon tekijöinä maa ja puusto. Tämän tuotantofunktion osittaisderivaatat ilmaisevat maan ja puuston rajatuottavuudet. Näitä rajatuottavuuksia on käytetty metsikön uudistamisen ja harventamisen tarvetta määritettäessä (piirros 3). Metsikkö on uudistettava silloin, kun maan rajatuottavuus jää pinta-alayksikön suuruisen alueen vuotuisen koron alapuolelle ja harvennettava silloin, kun puuston rajatuottavuus jää yhden puustoyksikön suuruisen puumäärän vuotuisen koron alapuolelle.

PEKKA KILKKI

O.D.C. 651:624.2

1971. Optimization of stand treatment based on the marginal productivity of land and growing stock. — ACTA FORESTALIA FENNICA 122. 7 p. Helsinki.

Production of timber in forest stands is described by a production function. The variable inputs of the function are land and growing stock and the output is the annual value growth. The partial derivatives of this production function express the marginal productivity of the land and of the growing stock. These marginal productivities can be utilized for determination of the need of regeneration and thinning. The stand should be regenerated when the marginal productivity of the land falls below the annual rent of a unit area of open land and thinned when the marginal productivity of the growing stock falls below the annual rent of one unit of growing stock.

Author's address: Department of Forest Mensuration and Management, University of Helsinki, Helsinki 17, Finland.

PEKKA KILKKI

O.D.C. 651:624.2

1971. Optimization of stand treatment based on the marginal productivity of land and growing stock. — ACTA FORESTALIA FENNICA 122. 7 p. Helsinki.

Production of timber in forest stands is described by a production function. The variable inputs of the function are land and growing stock and the output is the annual value growth. The partial derivatives of this production function express the marginal productivity of the land and of the growing stock. These marginal productivities can be utilized for determination of the need of regeneration and thinning. The stand should be regenerated when the marginal productivity of the land falls below the annual rent of a unit area of open land and thinned when the marginal productivity of the growing stock falls below the annual rent of one unit of growing stock.

Author's address: Department of Forest Mensuration and Management, University of Helsinki, Helsinki 17, Finland.

PEKKA KILKKI

O.D.C. 651:624.2

1971. Optimization of stand treatment based on the marginal productivity of land and growing stock. — ACTA FORESTALIA FENNICA 122. 7 p. Helsinki.

Production of timber in forest stands is described by a production function. The variable inputs of the function are land and growing stock and the output is the annual value growth. The partial derivatives of this production function express the marginal productivity of the land and of the growing stock. These marginal productivities can be utilized for determination of the need of regeneration and thinning. The stand should be regenerated when the marginal productivity of the land falls below the annual rent of a unit area of open land and thinned when the marginal productivity of the growing stock falls below the annual rent of one unit of growing stock.

Author's address: Department of Forest Mensuration and Management, University of Helsinki, Helsinki 17, Finland.

PEKKA KILKKI

O.D.C. 651:624.2

1971. Optimization of stand treatment based on the marginal productivity of land and growing stock. — ACTA FORESTALIA FENNICA 122. 7 p. Helsinki.

Production of timber in forest stands is described by a production function. The variable inputs of the function are land and growing stock and the output is the annual value growth. The partial derivatives of this production function express the marginal productivity of the land and of the growing stock. These marginal productivities can be utilized for determination of the need of regeneration and thinning. The stand should be regenerated when the marginal productivity of the land falls below the annual rent of a unit area of open land and thinned when the marginal productivity of the growing stock falls below the annual rent of one unit of growing stock.

Author's address: Department of Forest Mensuration and Management, University of Helsinki, Helsinki 17, Finland.

ACTA FORESTALIA FENNICA

EDELLISIÄ NITEITÄ — PREVIOUS VOLUMES

Vol. 113, 1971. Leo Heikurainen.
Pohjavesipinta ja sen mittaaminen ojitetuilla soilla. Summary: Ground Water Table in drained Peat Soils and its Measurement.

Vol. 114, 1971. Leo Heikurainen ja Pertti Veijola.
Lannoituksen ja sarkaleveyden vaikutus rämeen uudistumiseen ja taimien kasvuun. Summary: Effect of Fertilization and Ditch Spacing on Regeneration and Seedling Growth in Pine Swamps.

Vol. 115, 1971. Tauno Kallio.
Aerial Distribution of some wood-inhabiting Fungi in Finland. Seloste: Eräiden kuusen puuaineksessa kasvavien sienien Suomessa ilmateitse tapahtuva leviäminen.

Vol. 116, 1971. Aarne Nyyssönen, Pentti Roiko-Jokela and Pekka Kilkki.
Studies on Improvement of the Efficiency of Systematic Sampling in Forest Inventory. Seloste: Systemaattiseen otantaan perustuvan metsän inventoinnin tehokkuudesta.

Vol. 117, 1971. Tauno Kallio.
Protection of Spruce Stumps against *Fomes annosus* (Fr.) Cooke by some Wood-inhabiting Fungi. Seloste: Kuusen kantojen maannousemasieni-infektion estäminen muutamia puussa kasvavia sieniä käyttäen.

Vol. 118, 1971. Eino Oinonen.
The Time Table of Vegetative Spreading in Oak Fern (*Carpogymnia dryopteris* (L.) Löve & Löve) and May-Lily (*Maianthemum bifolium* (L.) F. W. Schmidt) in Southern Finland. Seloste: Kasvullisen leviämisen aikataulu metsäimarteella (*Carpogymnia dryopteris* (L.) Löve & Löve) ja oravanmarjalla (*Maianthemum bifolium* (L.) F. W. Schmidt) Etelä-Suomessa.

Vol. 119, 1971. Leo Ahonen.
Diskonttausarvo ja hakkuitten ajallinen tahdistus. Referat: Diskontierungswert und Zeitliche Einordnung des Einschlags.

Vol. 120, 1971. Matti Keltikangas.
Time Factor and Investment Calculations in Timber Growing. Theoretical Fundamentals. Seloste: Aikatekijä ja investointilaskelmat puunkasvatuksessa. Teoreettisia perusteita.

Vol. 121, 1971. Paavo Havas.
Injury to Pines in the Vicinity of a Chemical Processing Plant in Northern Finland. Seloste: Männyn vaurioista erään Pohjois-Suomen kemiallisen tehtaan läheisyydessä.

KANNATTAJAJÄSENET — UNDERSTÖDANDE MEDLEMMAR

CENTRALSKOGSNÄMNDEN SKOGSKULTUR
SUOMEN PUUNJALOSTUSTEOLLISUUDEN KESKUSLIITTO
OSUUSKUNTA METSÄLIITTO
KESKUSOSUUSLIIKE HANKKIJA
SUNILA OSAKEYHTIÖ
OY WILH. SCHAUMAN AB
OY KAUKAS AB
RIKKIHAPPO OY
G. A. SERLACHIUS OY
TYPPI OY
KYMIN OSAKEYHTIÖ
SUOMALAISEN KIRJALLISUUDEN KIRJAPAINO
UUDENMAAN KIRJAPAINO OSAKEYHTIÖ
KESKUSMETSÄLAUTAKUNTA TAPIO
KOIVUKESKUS
A. AHLSTRÖM OSAKEYHTIÖ
TEOLLISUUDEN PAPERIPUUYHDISTYS R.Y.
OY TAMPELLA AB
JOUTSENO-PULP OSAKEYHTIÖ
TUKKIKESKUS
KEMI OY
MAATALOUSTUOTTAJAIN KESKUSLIITTO
VAKUUTUSOSAKEYHTIÖ POHJOLA
VEITSILUOTO OSAKEYHTIÖ
OSUUSPANKKIEN KESKUSPANKKI OY
SUOMEN SAHANOMISTAJAYHDISTYS
OY HACKMAN AB
YHTYNEET PAPERITEHTAAT OSAKEYHTIÖ