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SAMPLE TREES IN TIMBER VOLUME ESTIMATION

KOEPUUT PUUSTON TILAVUUDEN ESTIMOINNISSA

Pekka Kilkki

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Seloste

KOEPUUT PUUSTON TILAVUUDEN ESTIMOINNISSA

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Analysis of three sample tree data showed that the major part of the residual variation of the stem volume estimates occurs within the forest stands. The division of the residual variation into the variation within and between populations is the basis for the mean-square error formulae of the volume estimators. Efficiency of different sample tree measurement combinations has been studied by comparing the errors of the volume estimates to the sampling costs. The measurement of the upper diameter (d_6) is of less value than is generally suggested.

Kolmen koepuuaineiston analysointi osoitti, että suurin osa puun rungon tilavuusestimaatin jäännösvirheestä on metsikön sisäistä vaihtelua. Jäännösvaihtelun jako metsikön sisäiseen ja metsiköiden väliseen vaihteluun on perustana tilavuusestimaattien keskineliövirheen kaavoille. Erilaisten koepuumittausyhdistelmien tehokkuutta on tutkittu vertaamalla tilavuusestimaattien virhettä mittauskustannukseen. Yläläpimitan mittaus ei ole niin edullista kuin on yleisesti luultu.

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PREFACE

This study was carried out in the Department of Forest Mensuration and Management, University of Helsinki. The help and patience of my colleagues Messrs Risto Ojansuu, Risto Päivinen, and Markku Siitonen gave me invaluable encouragement. Professors Matti Kärkkäinen, Kullervo Kuusela, Aarne Nyyssönen and Simo Poso read the manuscript making valuable comments and suggestions. Discussions with Messrs Juha Lappi, Timo Pekkonen and Juha Puranen helped to clarify several problems. Mr Ashley Selby checked the English text. Ms Helena Henttonen and Ms Tiina Pitkänen drew the figures.

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Joensuu December, 1982

Pekka Kilkki

1. INTRODUCTION

The stem volume of the tree population is generally the parameter of greatest interest in forest mensuration. If the cross sections of the stem at all heights of the tree are circular and if the taper curve is exactly known, the volume of the stem can be determined via integration of the squared taper curve. Then, the error of the volume estimate is equal to zero.

In most cases, the cross sections of the stem segments are sufficiently close to a circle, but the exact measurement of the taper curves of all trees in the population of interest is practicable only in some limited research projects. Usually, more or less erroneous taper curve estimates must suffice.

The major task in most practical forest mensuration projects is to determine the taper curves of the trees in the population as a function of the measured variables, such as trunk diameter at breast height and tree height. The taper curve can be expressed either explicitely by a large number of diameters or implicitely by few stem dimensions and volume of the tree (Demaerschalk 1972).

Kilkki et al. (1978; 1979; 1981) have introduced the use of simultaneous equations for the determination of taper curves. In this multi-equation model all variables measurable from the tree are potential endogenous variables and those measurable from the environment of the tree, exogenous variables. In practice, however, some tree characteristics may be treated as exogenous variables. When the model is applied, the exogenous variables and some of the endogenous variables are given their measured values. The solution of the simultaneous model then yields estimates for the other endogenous variables. The multi-equation approach also makes it possible to derive the error formulae for the diameter and volume estimates (Kilkki and Varmola 1981).

The major advantage of the multi-equation model is its versatility. Practically any combination of the diameter measurements can be included in the set of the predicting variables. However, the multi-equation models – as with any other tree models – are directly applicable only in those cases where the same endogenous variables are measured from all trees of the population under survey. In a normal situation the tree population is divided into tallied trees and sample trees. Multiphase sampling is also frequently applied, i.e. different sets of endogenous variables are measured from different sets of sample trees.

Usually there is a priori information available of the value of the sample tree characteristics. This information is based on the preceding measurements of the tree population and its environment. Furthermore, sampling and measurement errors reduce the information of the sample tree measurements. Consequently, the employment of the sample tree measurements calls for a detailed analysis of their informational value. In order to find the optimal sampling design, the measurement costs have also to be taken into account.

The purpose of this paper is to analyse the stem volume estimation problem and to derive error formulae which can be employed in the comparison of the efficiency of different sample tree measurement combinations.

The study is based upon the idea that the stem volume estimates are derived via estimation of the stem dimensions and other easily measurable characteristics of the tree (see Kilkki 1979). Thus, the estimation of the stem volume is divided into several phases which begin, for instance, from the determination of the tree species and diameter at breast height and end to the taper curve and stem volume estimates.

As a basis for the study, simultaneous equation taper curve models for Scots pine, as well as a number of regression models for other tree characteristics, are constructed. The residual variation of tree characteristic estimates is then studied mainly with Scots pine (*Pinus sylvestris*) data but some results are also given for Norway spruce (*Picea abies*) and birch (*Betula pendula* & *B. pubescens*). The residual variation is divided into two components: variation within populations and variation between populations.

The mean-square error (see Cochran 1963, p 15) formulae for the volume estimators are determined and they are employed to demonstrate the precision of different sample tree measurement combinations. Finally, the efficiency of different sampling designs is studied in order to discover the combination of the sample tree measurements which yields the minimum error of the volume estimate with a given sampling cost.

2. TAPER CURVE MODELS

The data employed in the derivation of the taper curve models are described in a previous paper (Kilkki and Varmola 1981). The simultaneous models now used are slightly different from those presented in the earlier paper.

In order to reduce the computational work only half of the measured relative-height diameters were included in the simultaneous equation model as endogenous variables. Instead of the logarithms and squared logarithms of the height and crown ratio, the height (h), its square, the relative crown height (h_c/h), and its square were employed as exogenous variables of the models.

Two simultaneous equation models, one without the crown height (equations 1, . . ., 12) and one with it (equations 13, . . ., 24) as an exogenous variable, are given in tables 1 and 2. The diameters are expressed in millimeters and the heights in meters. The models are called model I and model II, respectively.

The residual errors of the equations in the simultaneous models decrease when the size of the tree increases. Therefore, regression equations were calculated to derive these errors. These regression equations which yield the square root of the absolute residuals of these equations as a function of the height of the tree are of the same form as in the earlier paper (Kilkki and Varmola ibid, p 11).

$$y = a + bh \tag{2.25}$$

where

$y = \sqrt{|\ln d_{xh} - \widehat{\ln} d_{xh}|}$

h = height of the tree

a, b = regression coefficients

$\ln d_{xh}$ = logarithm of the measured diameter at the relative height x

$\widehat{\ln} d_{xh}$ = estimate of the logarithmic diameter at the relative height x

Table 1. Regression coefficients and residual standard errors of the regression equations for estimating the relative-height diameters when the crown height is unknown.

Taulukko 1. Suhteellisia osakorkeuksia edustavia läpimittoja ennustavien regressioyhtälöiden kertoimet ja keskivirheet, kun latvusrajaa ei tunneta.

Independent variables *Selittävät muuttujat*	(2.1) $\ln d_{.01h}$	(2.2) $\ln d_{.05h}$	(2.3) $\ln d_{.1h}$	(2.4) $\ln d_{.15h}$	(2.5) $\ln d_{.2h}$	(2.6) $\ln d_{.3h}$	(2.7) Dependent variable – *Selitettävä muuttuja* $\ln d_{.4h}$	(2.8) $\ln d_{.5h}$	(2.9) $\ln d_{.6h}$	(2.10) $\ln d_{.7h}$	(2.11) $\ln d_{.8h}$	(2.12) $\ln d_{.9h}$
$\ln d_{.01h}$		.339+00	−.655−02	−.527−01	.355−01	−.213−01	.427−01	−.176−01	−.133−01	.565−01	−.941−01	.394−01
$\ln d_{.05h}$	.108+01		.452+00	.130+00	−.130+00	.294−01	−.797−01	.473−02	.519−01	−.770−01	.296+00	−.471+00
$\ln d_{.1h}$	−.325−01	.703+00		.271+00	223+00	−.132−01	.779−01	−.723−01	.181−01	.220−01	−.188+00	.465−01
$\ln d_{.15h}$	−.289+00	.223+00	.299+00		.558+00	.112+00	.473−01	.252−01	−.120+00	.135+00	−.306+00	.523+00
$\ln d_{.2h}$	.190+00	−.218+00	.240+00	.545+00		.373+00	.633−01	.546−01	−.281−01	−.219+00	.319+00	−.225+00
$\ln d_{.3h}$	−.847−01	.366−01	−.106−01	.812−01	.278+00		.499+00	.142+00	.457−01	−.789−01	−.480−01	−.149+00
$\ln d_{.4h}$	.151+00	−.880−01	.554−01	.304−01	.418−01	.443+00		.469+00	.170+00	.703−01	−.158+00	−.140−01
$\ln d_{.5h}$	−.385−01	.324−02	−.318−01	.100−01	.223−01	.780−01	.291+00		530.+00	.209+00	−.585−01	−.526−01
$\ln d_{.6h}$	−.178−01	.218−01	.488−02	−.293−01	−.706−02	.154−01	.646−01	.325+00		.467+00	.106+00	.430−01
$\ln d_{.7h}$	.505−01	−.215−01	.396−02	.220−01	−.365−01	−.177−01	.178−01	.851−01	.311+00		.886+00	.648−01
$\ln d_{.8h}$	−.384−01	.318−01	−.155.−01	−.228−01	.243−01	−.493−02	−.183−01	−.109−01	.322−01	.405+00		.111+01
$\ln d_{.9h}$	.346−02	−.130−01	.823−03	.840−02	−.371−02	−.330−02	−.348−03	−.212−02	.282−02	.638−02	.238+00	
h	.383−02	−.176−02	.895−04	−.706−03	−.530−03	.120−02	−.754−03	.139−02	.252−02	.540−02	.304−02	−.293−01
h^2	−.177−04	−.142−04	.516−05	.309−04	.638−05	−.146−04	.156−04	−.314−04	−.676−04	−.135−03	−.314−04	.799−03
constant	.230+00	−.946−01	.304−01	.551−01	−.294−01	.297−01	−.477−01	−.220−01	−.327−01	−.138−01	−.120−01	.277+00
standard error	.471−01	.263−01	.211−01	.201−01	.204−01	.236−01	.251−01	.318−01	.407−01	.498−01	.737−01	.159+00

Table 2. Regression coefficients and residual standard errors of the regression equations for estimating the relative-height diameters when the crown height is known.

Taulukko 2. Suhteellisia osakorkeuksia edustavia läpimittoja ennustavien regressioyhtälöiden kertoimet ja keskivirheet, kun latvusraja (h_c) tunnetaan.

Independent variables *Selittävät muuttujat*	(2.13) ln $d_{.01h}$	(2.14) ln $d_{.05h}$	(2.15) ln $d_{.1h}$	(2.16) ln $d_{.15h}$	(2.17) Dependent variable – *Selitettävä muuttuja* ln $d_{.2h}$	(2.18) ln $d_{.3h}$	(2.19) ln $d_{.4h}$	(2.20) ln $d_{.5h}$	(2.21) ln $d_{.6h}$	(2.22) ln $d_{.7h}$	(2.23) ln $d_{.8h}$	(2.24) ln $d_{.9h}$
ln $d_{.01h}$		.339+00	.502−02	−.539−01	.280−01	−.287−01	.359−01	−.227−01	.119−01	.839−01	−.670−01	.316−01
ln $d_{.05h}$	.106+01		.436+00	.133+00	−.127+00	.332−01	−.797−01	−.834−03	.344−01	−.814−01	.297+00	−.354+00
ln $d_{.1h}$	.252−01	.698+00		.272+00	.240+00	.797−02	.956−01	−.548−01	−.495−01	−.613−01	−.257+00	.408−01
ln $d_{.15h}$	−.294+00	.230+00	.295+00		.548+00	.980−01	.498−01	.425−01	−.701−01	.149+00	−.325+00	.233+00
ln $d_{.2h}$	.149+00	−.215+00	.253+00	.534+00		.354+00	.506−01	.427−01	.166−01	−.156+00	.360+00	−.203+00
ln $d_{.3h}$	−.113+00	.418−01	.627−02	.711−01	.264+00		.488+00	.141+00	.925−01	−.299−01	−.270−01	−.253+00
ln $d_{.4h}$	.125+00	−.886−01	.664−01	.319−01	.333−01	.430+00		.449+00	.178+00	.988−01	−.115+00	.574−01
ln $d_{.5h}$	−.495−01	−.578−03	−.237−01	.170−01	.175−01	.774−01	.280+00		495.+00	.215+00	−.185−01	.100+00
ln $d_{.6h}$	.166−01	.153−01	−.137−01	−.179−01	.437−02	.326−01	.711−01	.317+00		.397+00	.839−01	.187+00
ln $d_{.7h}$	.764−01	−.236−01	−.111−01	.250−01	−.268−01	−.689−02	.258−01	.898−01	.259+00		.836+00	.791−01
ln $d_{.8h}$	−.274−01	.387−01	−.209−01	−.244−01	.277−01	−.280−02	−.135−01	−.347−02	.246−01	.375+00		.967+00
ln $d_{.9h}$	.295−02	−.105−01	.758−03	.400−02	−.357−02	−.599−02	.154−02	.431−02	.126−01	.812−02	.221+00	
h	.430−02	−.190−02	−.202−03	−.449−03	−.347−03	.151−02	−.669−03	.121−02	.129−02	.434−02	.288−02	−.233−01
h^2	−.260−04	−.132−04	.975−05	.289−04	.336−05	−.180−04	.131−04	−.320−04	−.518−04	−.117−03	−.222−04	.721−03
h_c/h	−.120+00	.607−01	.606−01	−.107+00	−.362−01	−.103+00	.750−02	.114+00	.355+00	.205+00	−.132+00	−.178+01
$(h_c/h)^2$	.503−01	−.558−01	−.216−01	.995−01	.120−01	.763−01	−.293−01	−.132+00	−.270+00	−.103+00	.217+00	.175+01
constant	.303+00	−.113+00	−.117−01	.860−01	−.390−02	.728−01	−.346−01	−.385−01	−.176+00	−.132+00	−.350−01	.719+00
standard error	.467−01	.264−01	.208−01	.200−01	.203−01	.235−01	.250−01	.316−01	.396−01	.489−01	.730−01	.153+00

Table 3. Regression coefficients (a and b) and standard errors of equation (2.25) when the crown height is unknown.

Taulukko 3. Kaavan (2.25) regressiokertoimet (a ja b) ja keskivirheet (s), kun latvusrajaa ei tunneta.

Diameter *Läpimitta*	a	b	s
$d_{.01h}$	.20007	−.00156	.07621
$d_{.05h}$	.15084	−.00128	.05741
$d_{.1h}$	.14506	−.00198	.05263
$d_{.15h}$	.14290	−.00204	.05165
$d_{.2h}$	.13736	−.00175	.05278
$d_{.3h}$	.14518	−.00162	.05587
$d_{.4h}$	.15706	−.00212	.05571
$d_{.5h}$	.17349	−.00238	.06581
$d_{.6h}$	.20051	−.00292	.07182
$d_{.7h}$	.20458	−.00210	.08325
$d_{.8h}$	.24457	−.00225	.10232
$d_{.9h}$	.37164	−.00421	.14691

Table 4. Regression coefficients (a and b) and standard errors of equation (2.25) when the crown height is known.

Taulukko 4. Kaavan (2.25) regressiokertoimet (a ja b) ja keskivirheet (s), kun latvusraja tunnetaan.

Diameter *Läpimitta*	a	b	s
$d_{.01h}$	.19337	−.00122	.07608
$d_{.05h}$	.15147	−.00129	.05666
$d_{.1h}$	.14384	−.00189	.05159
$d_{.15h}$	.14150	−.00196	.05149
$d_{.2h}$	.13723	−.00173	.05236
$d_{.3h}$	.14270	−.00148	.05536
$d_{.4h}$	.15808	−.00220	.05573
$d_{.5h}$	.17451	−.00247	.06500
$d_{.6h}$	.19819	−.00293	.07123
$d_{.7h}$	.20464	−.00209	.08098
$d_{.8h}$	.24713	−.00239	.09930
$d_{.9h}$	.35738	−.00334	.14001

The regression coefficients and the residual standard errors of the equations are given in tables 3 and 4. To derive unbiased residual variances from equation (2.25), Taylor's expansion is required (see Kilkki and Varmola ibid).

The coefficients of correlation between the residuals of the regression equations of models I and II are given in tables 5 and 6, respectively. The figures for the coefficients of correlation between the residuals of the adjacent diameter estimates are significant and negative; the other coefficients are close to zero. The negative correlation between the residuals of the adjacent diameters decreases the errors in taper curve estimates (see Kilkki and Varmola 1979, p 299).

The use of the simultaneous equation taper curve model is described in the earlier publication (Kilkki and Varmola ibid). It should be noticed that Taylor's expansion is needed in the derivation of unbiased volume estimates from the estimated taper curve.

The new models gave slightly better results than the earlier models. Thus, it is apparent that it is not necessary to employ as many relative-height diameters as in the earlier study. On the contrary, a smaller number or endogenous relative-height diameters led to a smoother taper curve, especially with several measured diameters. The illogical features discernible in the estimated taper curves of the short trees also disappeared when the new transformations of the height and the crown height were introduced.

Table 5. Correlation coefficients between the residuals of the regression equations (2.1) ... (2.12).

Taulukko 5. Regressioyhtälöiden (2.1) ... (2.12) residuaalien väliset korrelaatiokertoimet.

	(2.2)	(2.3)	(2.4)	(2.5)	(2.6)	(2.7)	(2.8)	(2.9)	(2.10)	(2.11)	(2.12)
(2.1)	−.606	.016	.127	−.087	.044	−.082	.029	.014	−.053	.060	−.011
(2.2)		−.564	−.176	.174	−.032	.083	−.004	−.033	.040	−.105	.078
(2.3)			−.280	−.230	.007	−.063	.046	−.008	−.008	.053	−.006
(2.4)				−.560	−.090	−.039	−.015	.057	−.055	.085	−.067
(2.5)					−.321	−.051	−.033	.013	.089	−.089	.029
(2.6)						−.471	−.104	−.025	.036	.016	.022
(2.7)							−.370	−.105	−.034	.053	.002
(2.8)								−.416	−.131	.025	.011
(2.9)									−.382	−.058	−.011
(2.10)										−.599	−.020
(2.11)											−.513

Table 6. Correlation coefficients between the residuals of the regression equations (2.13) ... (2.24).

Taulukko 5. Regressioyhtälöiden (2.13) ... (2.24) residuaalien väliset korrelaatiokertoimet.

	(2.14)	(2.15)	(2.16)	(2.17)	(2.18)	(2.19)	(2.20)	(2.21)	(2.22)	(2.23)	(2.24)
(2.13)	−.601	−.010	.129	−.069	.058	−.069	.036	−.015	−.079	.042	−.009
(2.14)		−.552	−.181	.171	−.037	.084	−.000	−.022	.043	−.106	.060
(2.15)			−.278	−.245	−.011	−.077	.034	.028	.027	.072	−.005
(2.16)				−.549	−.079	−.041	−.026	.034	−.061	.091	−.031
(2.17)					−.304	−.040	−.026	−.009	.065	−.101	.027
(2.18)						−.459	−.103	−.054	.013	.009	.039
(2.19)							−.355	−.113	−.049	.039	−.010
(2.20)								−.397	−.137	.007	−.020
(2.21)									−.322	−.045	−.049
(2.22)										−.560	−.025
(2.23)											−.462

3. VARIATION OF TREE CHARACTERISTICS

31. Formulae

The tree population can be described by a multidimensional distribution, the dimensions repserenting various characteristics of the trees (see Kilkki 1979). In this chapter, estimates of the parameters of the conditional distribution of the tree characteristics with respect to the preceding variables in some sample populations are given. The differences between the true values and the values derived from the superpopulation models are examined.

The parameters under survey are the mean residual, total residual variance, residual variance within the populations, and residual variance between the populations. A population refers to a forest stand or to a homogeneous group of stands. The parameter estimates are calculated by the following formulae.

Mean residual

$$\bar{x} = \left(\sum_{i=1}^{k} \sum_{j=1}^{m_i} x_{ij}\right)/M , \qquad (3.1)$$

where

x_{ij} = deviation of the estimated value j in population i from the true value

k = number of populations

m_i = sample size in population i

$M = \sum_{i=1}^{k} m_i$.

Total variance

$$s_t^2 = SST/(M-1) \qquad (3.2)$$

Variance within populations

$$s_w^2 = SSE/(M-k) \qquad (3.3)$$

Variance between populations

$$s_b^2 = ((SST-SSE)/(k-1) - SSE/(M-k))/t , \qquad (3.4)$$

where

$$SST = \sum_{i=1}^{k} \sum_{j=1}^{m_i} x_{ij}^2 - \left(\sum_{i=1}^{k} \sum_{j=1}^{m_i} x_{ij}\right)^2/M$$

$$SSE = \sum_{i=1}^{k} \sum_{j=1}^{m_i} x_{ij}^2 - \left(\sum_{i=1}^{k} \sum_{j=1}^{m_i} x_{ij}^2/m_i\right)$$

$$t = \left(M^2 - \left(\sum_{i=1}^{k} m_i^2\right)\right)/((k-1)M)$$

(see Hicks 1973, p 186)

32. Taper curve data

The first results refer to the tree data upon which the taper curve models (see section 2) were based. The 29 forest stands from which the sample trees were measured were employed as populations. The results from taper curve model I are presented in tables 7 and 8 and the results from model II in tables 9 and 10. The variances have been transformed into standard errors.

The mean residuals and the total standard errors are close to the figures presented in the earlier study (Kilkki and Varmola 1981). The slight decreases are due to the new transformations of the height and crown height and to the smoother taper curve (see section 2). The total standard errors of the stem volume estimates have been reduced by 0.1 . . . 0.3 percent units.

Table 7. Standard errors and mean residuals of the relative-height diameter and volume estimates, percent. Model I. All sample trees.

Taulukko 7. Mallilla I estimoitujen läpimittojen ja tilavuuksien prosentuaaliset keskipoikkeamat ja keskivirheet. Kaikki koepuut.

Diameter *Läpimitta*	Measured information – *Mitatut tiedot* d,h			
	$\bar{x}$	s_t	s_w	s_b
$d_{.01h}$	–.1	6.0	5.7	1.9
$d_{.05h}$	–.1	3.7	3.5	1.1
$d_{.1h}$	–.1	3.0	2.9	.7
$d_{.15h}$	–.0	3.3	3.1	1.1
$d_{.2h}$	–.0	3.8	3.6	1.1
$d_{.3h}$	–.0	4.5	4.3	1.2
$d_{.4h}$	–.0	5.3	5.0	1.8
$d_{.5h}$	–.0	7.0	6.5	2.5
$d_{.6h}$	–.1	9.2	8.1	4.6
$d_{.7h}$	–.1	13.5	11.7	6.9
$d_{.8h}$	–.1	19.5	17.1	9.5
$d_{.9h}$	–.6	31.8	29.0	13.4
Volume *Tilavuus*	–.1	7.9	7.2	3.3

Table 8. Standard errors and mean residuals of the relative-height diameter and volume estimates, percent, and of the diameter at 6 meters' height, millimeters. Model I. Trees taller than 8.6 meters.

Taulukko 8. Mallilla I estimoitujen läpimittojen ja tilavuuksien prosentuaaliset keskipoikkeamat ja keskivirheet sekä läpimitan d_6 vastaavat luvut millimetreinä. Yli 8.6 metrin puut.

Diameter *Läpimitta*	Measured information – *Mitatut tiedot* d ,h				d,d_6, h			
	$\bar{x}$	s_t	s_w	s_b	$\bar{x}$	s_t	s_w	s_b
$d_{.01h}$	–.4	5.5	5.2	1.7	–.4	5.5	5.2	1.8
$d_{.05h}$	–.4	2.9	2.8	.7	–.4	2.9	2.8	.7
$d_{.1h}$	–.3	2.2	2.1	.6	–.2	1.8	1.8	.0
$d_{.15h}$	–.2	3.1	2.9	.9	–.0	2.3	2.3	.1
$d_{.2h}$	–.3	3.7	3.5	1.1	–.1	2.4	2.4	.4
$d_{.3h}$	–.3	4.4	4.2	1.2	–.0	2.6	2.6	.4
$d_{.4h}$	–.4	5.0	4.7	1.7	.0	3.0	2.9	.6
$d_{.5h}$	–.4	6.4	6.1	1.9	.1	3.9	3.8	1.0
$d_{.6h}$	–.5	8.4	7.5	3.9	.0	6.0	5.3	2.9
$d_{.7h}$	–.5	12.2	10.5	6.3	.2	9.6	8.3	5.0
$d_{.8h}$	–.5	17.8	15.4	9.1	.3	15.4	13.5	7.6
$d_{.9h}$	–.9	27.9	27.4	11.8	.1	25.0	23.0	10.0
d_6	–.4	11.6	11.0	3.6				
Volume *Tilavuus*	–.6	7.5	6.9	3.1	–.0	4.3	3.9	1.8

Table 9. Standard errors and mean residuals of the relative-height diameter and volume estimates, percent. Model II. All sample trees.

Taulukko 9. Mallilla II estimoitujen läpimittojen ja tilavuuksien prosentuaaliset keskipoikkeamat ja keskivirheet. Kaikki puut.

Diameter *Läpimitta*	Measured information – *Mitatut tiedot* d,h,h_c			
	$\bar{x}$	s_t	s_w	s_b
$d_{.01h}$	–.1	6.0	5.6	1.9
$d_{.05h}$	–.1	3.6	3.4	1.0
$d_{.1h}$	–.1	2.9	2.8	.7
$d_{.15h}$	–.0	3.3	3.1	1.1
$d_{.2h}$	–.0	3.8	3.6	1.1
$d_{.3h}$	–.0	4.5	4.3	1.2
$d_{.4h}$	–.0	5.2	5.0	1.6
$d_{.5h}$	–.0	6.5	6.3	1.6
$d_{.6h}$	–.1	7.8	7.5	2.1
$d_{.7h}$	–.1	10.4	10.0	2.8
$d_{.8h}$	–.1	14.1	13.5	4.1
$d_{.9h}$	–.2	22.2	21.3	6.6
Volume *Tilavuus*	–.1	7.2	6.9	2.3

The results show that the major part of the residual variation occurs within the stands. The ratios of the diameter variances between and within the stands for trees taller than 8.6 meters are given in Fig. 1. As one could already see from the figures in tables 7 . . . 10, the residual variance between the stands is only a small fraction of the total residual variance; the variance ratio is generally below .15. Only when the crown height is unknown the variance ratio exceeds .25 in the upper half of the stem.

The total residual variation expressed as standard error in $\hat{d}_6$ is 11.6 mm when only d and h are known, and 11.0 mm when h_c is also known. The standard errors between, the stands are 3.6 and 3.2 mm, respectively. Thus, the information derivable from the crown height only slightly decreases the error in the upper diameter estimate.

To get information on the importance of d, d_6, and h_c/h as predicting variables of the height, the following regression equations were calculated.

All trees

$$\hat{h} = \exp(1.533 + .007308d - .000007816d^2) \quad (3.5)$$
$$R^2 = .712$$

$$\hat{h} = \exp(.7973 + .006640d - .000006853d^2 + 2.755(h_c/h) - 2.086(h_c/h)^2) \quad (3.6)$$
$$R^2 = .759$$

Trees taller than 8.6 meters

$$\hat{h} = \exp(2.093 + .003732d - .000002606d^2) \quad (3.7)$$
$$R^2 = .595$$

$$\hat{h} = \exp(1.374 + .003827d - .000002741d^2 + 2.251(h_c/h) - 1.652(h_c/h)^2) \quad (3.8)$$
$$R^2 = .656$$

$$\hat{h} = \exp(2.334 - .008359d + .000005970d^2 + .01356d_6 - .00001108d_6^2) \quad (3.9)$$
$$R^2 = .829$$

$$\hat{h} = \exp(2.115 - 007286d + .000004763d^2 + .01239d_6 - .000009458d_6^2 + .6120(h_c/h) - .4392(h_c/h)^2) \quad (3.10)$$
$$R^2 = .833$$

The residual variation of these height models is presented in table 11. Due to the way the data has been selected (see Kilkki and Varmola 1981), the level of the variation gives little valuable information. On the other hand, the decrease of the residual variation attributable to the information d_6 and h_c/h yield is to be noticed.

To study the variation of the crown height, regression equations were calculated from the taper curve data. In the first equation, the predicting variables comprise d and h

$$\widehat{h_c/h} = \exp(-1.151 - .001367\,d + .6990\,h - .001107\,h^2)$$
$$R^2 = .174 \quad (n=492) \quad (3.11)$$

In the second equation, d_6 also is a predicting variable

$$\widehat{h_c/h} = \exp(-.1743 - .01282\,d + .0000163\,d^2 - .02186\,h + .0009653\,h^2 + .01398\,d_6 - .0000242\,d_6^2) \quad (3.12)$$
$$R^2 = .195 \quad (n=426)$$

As the coefficients of determination indicate, only a small fraction of the total variation in the relative crown height could be explained by the models. The residual variation of the relative crown height is presented in table 12. The figures indicate that the residual variation of the relative crown height is almost equally large between the stands and within the stands. This result is quite different when compared to the variation of the upper diameter (see tables 7 . . . 10).

Table 10. Standard errors and mean residuals of the relative-height diameter and volume estimates, percent, and of the diameter at 6 meters' height, millimeters. Model II. Trees taller than 8.6 meters.

Taulukko 10. Mallilla II estimoitujen läpimittojen ja tilavuuksien keskipoikkeamat ja keskivirheet prosentteina sekä läpimitan d_6 vastaavat luvut millimetreinä. Yli 8.6 metrin puut.

Diameter *Läpimitta*	Measured information – *Mitatut tiedot*: d,h,h_c $\bar{x}$	s_t	s_w	s_b	d,d_6,h,h_c $\bar{x}$	s_t	s_w	s_b
$d_{.01h}$	–.4	5.5	5.2	1.9	–.4	5.5	5.2	1.8
$d_{.05h}$	–.4	2.9	2.8	.7	–.4	2.9	2.8	.6
$d_{.1h}$	–.2	2.2	2.1	.6	–.2	1.8	1.8	.0
$d_{.15h}$	–.2	3.0	2.9	.9	–.1	2.2	2.2	.1
$d_{.2h}$	–.3	3.6	3.5	1.1	–.1	2.3	2.3	.2
$d_{.3h}$	–.3	4.4	4.2	1.2	–.0	2.5	2.5	.1
$d_{.4h}$	–.3	4.9	4.7	1.7	–.0	3.0	2.9	.6
$d_{.5h}$	–.4	6.1	5.9	1.6	–.0	3.8	3.8	.8
$d_{.6h}$	–.4	7.2	6.9	2.2	–.0	5.2	4.9	1.7
$d_{.7h}$	–.3	9.6	9.0	3.3	.1	7.7	7.3	2.7
$d_{.8h}$	–.3	13.2	12.4	4.5	.1	12.0	11.3	4.1
$d_{.9h}$	–.4	20.9	19.7	7.1	.1	19.9	18.8	6.8
d_6	–.3	11.0	10.6	3.2				
Volume *Tilavuus*	–.5	6.9	6.6	2.2	–.1	3.9	3.7	1.1

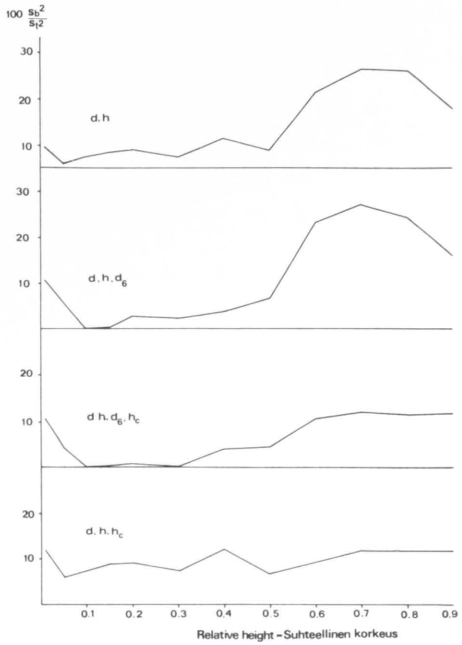


Fig. 1. Ratio of the diameter estimate's error variance between the stands and total error variance as a function of the predicting variable combination and relative height. Trees taller than 8,6 meters.

Kuva 1. Läpimittaestimaatin metsiköiden välisen virhevarianssin ja kokonaisvirhevarianssin suhde mittausyhdistelmän ja suhteellisen mittauskorkeuden funktiona. Yli 8.6 metrin pituiset puut.

33. Inventory data

The second data comprises the pine, spruce and birch sample trees taller than 7.5 meters measured in the sixth Finnish National Forest Inventory in the Forestry Board District Pohjois-Savo in the eastern part of central Finland. The diameter at breast height, height, and the diameter at the height of 6 meters had been measured from the sample trees. The 3688 sample trees were employed to examine the residual variation of the height and upper diameter. Unpublished height curve models were employed for height. The upper diameters were calculated by the models where d, h, and a few stand characteristics were employed as predicting variables (Päivinen 1978). The results of these calculations are presented in tables 13 and 14.

When it is taken into account that the standard error within the populations also contains the random measurement error, the true residual variation within the populations

Table 11. Standard errors of the height estimates in the taper curve model material, meters

Taulukko 11. Pituusestimaattien keskivirheet runkokäyräaineistossa, m.

Measured information *Mitatut tiedot*	s_t	s_w	s_b
All trees *Kaikki puut*			
d	3.7	2.5	2.8
d,h_c/h	3.3	2.2	2.5
h > 8.6 m			
d	3.5	2.3	2.8
d,h_c/h	3.3	2.2	2.5
d,d_6	2.4	1.9	1.4
d,d_6,h_c/h	2.3	1.9	1.4

Table 12. Standard errors of the relative crown height estimate in the taper curve material.

Taulukko 12. Suhteellisen latvusrajan korkeuden estimaatin keskivirheet runkokäyräaineistossa.

Measured information *Mitatut tiedot*	s_t	s_w	s_b
All trees *Kaikki puut*			
d,h	.134	.104	.086
h > 8.6 m			
d, h	.126	.098	.082
d,h,d_6	.121	.092	.081

Table 13. Standard errors and mean residuals of the height estimates in the sample tree material of the Finnish National Forest Inventory, meters.

Taulukko 13. Pituusestimaattien keskivirheet ja keskipoikkeamat valtakunnan metsien inventoinnin aineistossa, m.

Tree species *Puulaji*	$\bar{x}$	s_t	s_w	s_b	M
Pine–Mänty	.2	2.2	1.4	1.6	1052
Spruce–*Kuusi*	–.3	2.0	1.5	1.2	2015
Birch–*Koivu*	–.3	2.3	1.7	1.5	621
All–*Yhteensä*	–.2	2.1	1.7	1.3	3688

Table 14. Standard errors and mean residuals of the upper diameter (d_6) estimates in the sample tree material of the Finnish National Forest Inventory, millimeters.

Taulukko 14. Läpimitan d_6 estimaattien keskivirheet ja keskipoikkeamat valtakunnan metsien inventoinnin aineistossa, mm.

Tree species *Puulaji*	$\bar{x}$	s_t	s_w	s_b	M
Pine–Mänty	– .3	11.3	9.8	5.8	1052
Spruce–*Kuusi*	–1.6	12.5	10.7	6.4	2015
Birch–*Koivu*	– .8	13.4	11.9	7.3	621
All–*Yhteensä*	–1.1	12.3	11.0	5.6	3688

is less than the figures in table 13 indicate. If the standard deviation of the random measurement error is 1.0 meter, for example, the standard error within populations for all tree species is 1.4 meters instead of 1.7 meters. Similarly, the possible systematic measurement errors between the field workers have increased the standard error between the populations. The mean residuals may be attributable both to the inadequacy of the height model and to the systematic measurement errors.

The total residual variation in the upper diameter estimate of pines is about equal to that observed in the first material (11.3 vs. 11.6). The residual variation within the plots is somewhat smaller (9.8 vs. 11.0) and the residual variation between the plots markedly greater (5.8 vs. 3.6). The residual variation of spruce and birch is slightly greater than that of pine. The residual variation between the plots is, quite expectedly, smaller for all tree species than the respective figures of the tree species separately.

It is, however, difficult to make direct comparisons between the two materials. The stands of the taper curve material have been subjectively chosen from the whole southern part of Finland and the trees almost randomly sampled from the whole stand marked for cutting. No marked measurement errors probably exist in the taper curve data.

The sample trees in the National Forest Inventory represent small relascope plots (factor=2) and the total residual variance comprises both the random and systematic measurement errors. If the measurement error is eliminated from the inventory data the residual variation within the plots will markedly fall. On the other hand, the models of Päivinen (1978) to some extent reduce the variation between the stands, since they employ stand characteristics as predicting variables.

34. Forest areas marked for cutting

The third material comprised 194 forest areas marked for cutting. The results from the forest measurements were employed in the determination of the sales prices and logging wages. During the field work, part of the areas had been divided into two or more sample tree areas in order to reduce the variation between the trees. As a result 242 sample tree areas had been formed.

The material comprised pine, spruce, and birch sample trees and it was used to study the variation of h and d_6. Only trees taller than 7.5 meters were utilized. Unpublished models (cf. section 33) were employed for height estimation and the models of Päivinen (1978) were used for the upper diameter. The stand characteristics employed by the models were given constant values. This led to relatively large mean residuals in the height estimates. The residual variation of the upper diameter estimate was practically the same when either 194 or 242 populations were used. Therefore, only the results representing the 242 populations are given. The same populations were employed for the analysis of the height variation. The results are given in tables 15 and 16.

Table 15. Standard errors and mean residuals of the height estimates in the sample tree material of the forest areas marked for cutting, meters.

Taulukko 15. Pituusestimaattien keskivirheet ja keskipoikkeamat pystymittaleimikoiden koepuuaineistossa, m.

Tree species *Puulaji*	$\bar{x}$	s_t	s_w	s_b	M
Pine–Mänty	– .3	2.9	2.3	1.9	17623
Spruce–*Kuusi*	–1.0	2.5	2.1	1.3	28337
Birch–*Koivu*	– .8	2.6	2.2	1.3	12887
Al–*Yhteensä*	– .8	2.7	2.3	1.4	58855

Table 16. Standard errors and mean residuals of the upper diameter (d_6) estimates in the sample tree material of the forest areas marked for cutting, millimeters.

Taulukko 16. Läpimitan d_6 estimaattien keskivirheet ja keskipoikkeamat pystymittaleimikoissa, mm.

Tree species *Puulaji*	$\bar{x}$	s_t	s_w	s_b	M
Pine–Mänty	–2.2	11.7	11.2	3.6	17623
Spruce–*Kuusi*	–2.5	11.6	11.0	3.8	28337
Birch–*Koivu*	–3.7	12.7	12.1	3.9	12887
All–*Yhteensä*	–2.7	11.9	11.5	3.0	58855

The residual variation of the height estimates is of the same order of magnitude as in the inventory data (see table 13). The mean residuals are due to the missing information on stand characteristics.

The total standard errors of the upper diameter estimates (table 16) are of about the same magnitude as the respective figures for the inventory data (see table 14). However, the standard errors between the populations are much lower and they are even lower than in the taper curve data (see table 8). This result demonstrates the tendency of decreasing error variances between the populations with increasing size of the populations.

4. MEASUREMENT ERRORS AND COSTS

The magnitude of the measurement errors of the tree characteristics is influenced by several factors, the quality of the instruments, forest conditions, and training of the workers being the most important ones. For this study no original investigations on the measurement errors were made. The following discussion of the errors in the measurement of h and d_6 is based both on literature and on the results from the Finnish National Forest Inventory.

Hyppönen and Roiko-Jokela (1978) have studied the accuracy of the sample tree measurements in Finnish Lapland. All measured trees were Scots pines. The measurement conditions can be regarded quite favourable since the stands were sparse and the trees short and well pruned naturally.

In the case of the commonly used handheld Suunto altimeter the standard deviation was .8 m and the bias −.4 m. The Suunto altimeter attached to a camera tripod gave the standard deviation of .6 m and bias of −.1 m. The standard deviation of the height measurements with the Lönnroth altimeter was 1.0 m and bias .3 m.

The common curved caliber yielded a standard deviation of 8.0 mm and bias of −.0 mm in the measurement of d_6. The measurements were made at the precision of 10 mm. The classification error is included into the standard deviation. The standard deviation of the precision caliber was 8.4 mm and its bias −4.0 mm. The standard deviation of the tested optical caliber was 34.7 mm and its bias 20.6 mm.

Some information on the biases of the height and upper diameter measurements could also be obtained by comparing the estimates of these variables derived by measurements and by models in the National Forest Inventory in Pohjois-Savo. The standard error of the height measurements between the crews was 0.0 m for pine, 0.2 m for spruce, 0.4 m for birch, and 0.2 m for all tree species. The standard error of d_6 measurements between the crews was 1.9 mm for pine, 1.2 mm for spruce, 3.5 mm for birch, and 1.5 mm for all tree species.

These results suggest small differences between the crews in height and upper diameter measurements. Furthermore, it cannot be concluded that all observed variation between the crews indicates systematic measurement errors. Part of it may be due to differences between the crew areas.

No data was available on the accuracy of the measurement of the crown height. Its measurement error is evidently smaller than that of the height of the tree. However, there may be errors in the determination of the crown limit.

According to the results of Hyppönen and Roiko-Jokela (ibid) the height measurement required an average of 1.07 min/tree when hand-held caliber was used. The measurement of the upper diameter took 0.3 min/tree with the curved caliber.

5. ACCURACY OF STEM VOLUME ESTIMATION METHODS

51. Pure sample tree utilization

The stem volume estimate can be expresses as a function of its predicting variables

$$\hat{v} = v_i(x), \qquad (5.1)$$

where

x = an ordered vector of predicting variables $x_1, \ldots, x_i$

It is first assumed that the values of the predicting variables are estimated either via models derived from the sample tree measurements or via a priori models. It can be assumed that given an infinite number of predicting variables the volume model (5.1) yields correct volumes. Thus, the difference between the true volume and that estimated by the model is attributable to the predicting variables' erroneous or missing information.

Given uncorrelated measurement errors and relatively small error variances in the estimates of the predicting variables, the mean-square error of the mean volume estimate of a tree population ($\hat{\bar{v}}$) can be calculated from the following approximate formula based upon the first terms of Taylor's series (see e.g. Kilkki 1979)

$$\begin{aligned} MSE(\hat{\bar{v}}) &= E(\bar{v} - \hat{\bar{v}})^2 = E(\bar{v}^2) - 2E(\bar{v}\hat{\bar{v}}) + E(\hat{\bar{v}}) \\ &= \bar{v}^2 - 2\bar{v}^2 + \bar{v}^2 + \sum_{i=1}^{\infty} \left(\frac{\partial v_i}{\partial x_i}\right)^2 m_{2i} \\ &= \sum_{i=i}^{\infty} \left(\frac{\partial v_i}{\partial x_i}\right)^2 m_{2i}, \end{aligned} \qquad (5.2)$$

where m_{2i} = second central moment of $\bar{x}_i$ about its expected value $\hat{\bar{x}}_i = x_i(\bar{x}_1, \ldots, \bar{x}_{i-1})$

$\frac{\partial v_i}{\partial x_i}$ = partial derivative of the volume function v_i with repect to x_i

If $\hat{\bar{x}}_i$ is obtained from measurements we get

$$m_{2i} = \frac{s_w\hat{x}_i^2}{f_i} + \frac{s_r\hat{x}_i^2}{n_i} + b\hat{x}_i^2$$

and if $\hat{\bar{x}}_i$ is derived from an a priori model

$$m_{2i} = s_b\hat{x}_i^2,$$

where

$s_w\hat{x}_i^2$ = residual variance of $\hat{x}_i$ within the sampling population

$s_b\hat{x}_i^2$ = residual variance of $\hat{x}_i$ between the sampling populations

$s_r\hat{x}_i^2$ = variance of the random measurement errors of x_i

$b\hat{x}_i$ = bias in the measurement of x_i

f_i = $n_i/(1-n_i/N)$

n_i = number of sample trees with measured predicting variable x_i; $n_i \leqq n_{i-1}$ and x_i is measured only from trees where variable x_{i-1} is measured

N = size of the population

Now we can regroup the terms in formula (5.2)

$$MSE(\hat{\bar{v}}) = A+B+C+D+E, \qquad (5.3)$$

where

$A = \sum_{i=1}^{k} \left(\frac{\partial v_i}{\partial x_i}\right)^2 s_w\hat{x}_i^2/f_i$ (=sampling error)

$B = \sum_{i=k+1}^{l} \left(\frac{\partial v_i}{\partial x_i}\right)^2 s_b\hat{x}_i^2$ (= bias in model $\hat{x}_i = x_i(x_1, \ldots, x_{i-1})$)

$C = \sum_{i=1}^{k} \left(\frac{\partial v_k}{\partial x_i}\right)^2 s_r\hat{x}_i^2/n_i$ (= random measurement error)

$D = \sum_{i=1}^{k} \left(\frac{\partial v_k}{\partial x_i}\right)^2 b\hat{x}_i^2$ (= measurement bias)

$E = \sum_{i=l+1}^{\infty} \left(\frac{\partial v_i}{\partial x_i}\right)^2 s_b\hat{x}_i^2 = s_e^2$ (= bias in model v_l)

= error variance of the residuals of the volume function v_l between the sampling populations

k = number of measured predicting variables

l = maximum number of predicting variables in volume functions

l–k = number of unknown predicting variables

A+C = variable part of the mean-square error

B+D+E = fixed part of the mean-square error.

The errors in the predicting variables attributable to sampling, and to the models, are treated differently from the measurements errors (cf. A and B vs. C and D). The difference is based upon the fact that in the case of the sampling and model errors the covariation of the errors is included in the errors of the preceding predicting variables. Contrary to this, the measurement errors were assumed to be uncorrelated and hence the effect of the measurement error of a predicting variable must be weighted only by the partial derivative of the volume function which includes all measured predicting variables.

The divisor in term C is n_i instead of f_i since the effect of the random measurement error does not disappear even though all the trees in the population are taken as sample trees.

If the number of sample trees equals one, the sum A+B indicates the error variance of volume function v_1 within the sampling population.

The employment of formula (5.3) presupposes that there are volume functions available for each predicting variable combination in the given order. Five volume functions were tested.

$$\hat{v} = v_1 (d) \quad (5.4)$$
$$\hat{v} = v_2 (d, h) \quad (5.5)$$
$$\hat{v} = v_3 (d, h, d_6) \quad (5.6)$$
$$\hat{v} = v_4 (d, h, d_6, h_c) \quad (5.7)$$
$$\hat{v} = v_5 (d, h, h_c) \quad (5.8)$$

Table 17. Partial derivatives of the volume functions with respect to the predicting variables for a sample tree with d=250 mm, h=20 m, d_6=200 mm, and h_c=10 m.

Taulukko 17. Tilavuusfunktioiden osittaisderivaatat koepuutunnusten suhteen. Esimerkkipuun d=250 mm, h=20 m, d_6=200 mm ja h_c=10 m.

Function	$\frac{\partial v_i}{\partial d}$ l/mm	$\frac{\partial v_i}{\partial h}$ l/m	$\frac{\partial v_i}{\partial d_6}$ l/mm	$\frac{\partial v_i}{\partial h_c}$ l/m
v_1	4.64			
v_2	3.49	23.6		
v_3	1.03	18.5	3.30	
v_4	1.09	17.4	3.28	5.03
v_5	3.53	22.6		4.80

These functions facilitate the use of two sets of ordered predicting variables d,h,d_6,h_c and d,h,h_c. A model from the study by Laasasenaho (1976) is used as function v_1 Taper curve models I and II have been employed for functions $v_2 \ldots v_5$. Volume functions $v(d,d_6)$, $v(d,h_c)$, and $v(d,d_6,h_c)$ were omitted because preliminary tests showed that sample tree measurement combinations (d,h), (d,h,d_6), and (d,h,h_c) were far superior to combinations (d,d_6), (d,h_c), and (d,d_6,h_c) in volume estimation.

As an example, the mean-square error formula (5.9) for volume model (5.6) is given. The size of the population is assumed to be infinite and the same sample tree measurements made on all trees, i.e. $n_i = n_1$ for i = 2, . . . , k.

$$\begin{aligned} MSE(v) = {} & (\frac{\partial v_1}{\partial d})^2 s_w \hat{d}^2/n_1 + (\frac{\partial v_2}{\partial h})^2 s_w \hat{h}^2/n_1 \\ & + (\frac{\partial v_3}{\partial d_6})^2 s_w \hat{d}_6{}^2/n_1 + (\frac{\partial v_4}{\partial h_c})^2 s_b \hat{h}_c{}^2 \\ & + (\frac{\partial v_3}{\partial d})^2 s_r \hat{d}^2/n_1 + (\frac{\partial v_3}{\partial h})^2 s_r \hat{h}^2/n_1 \\ & + (\frac{\partial v_3}{\partial d_6})^2 s_r \hat{d}_6{}^2/n_1 + (\frac{\partial v_3}{\partial d})^2 b\hat{d}^2 + (\frac{\partial v_3}{\partial h})^2 b\hat{h}^2 \\ & + (\frac{\partial v_3}{\partial d_6})^2 b\hat{d}_6{}^2 + s_e{}^2 \end{aligned} \quad (5.9)$$

Numerical values of the partial derivatives from different volume functions are presented in table 17 for a sample tree. The volumes are expressed in litres.

In function v_4 the partial derivative with respect to h is calculated under the assumption that the relative crown height (h_c/h) remains constant.

The values of the variation parameters in the ensuing examples correspond to the figures in table 18, if not notified otherwise. These parameters represent a common situation in Finnish pine forests. It is assumed that the number of tallied trees is so large and the accuracy of the diameter measurements so hight that the measurement errors and the variation parameters of the diameter at breast height can be given the value zero.

The examples refer to an infinite tree population with E(d)=250 mm, E(h)=20 m, $E(d_6)$=200 mm, and $E(h_c)$=10m. It is further assumed that in each combination of the predicting variables their values are measured from all sample trees, i.e. $n_i = n_1$ for i = 2, . . . , k.

Table 18. Basic values for the variation parameters.

Taulukko 18. Vaihteluparametrien oletusarvot.

Error component	Value	Error component	Value
$s_w\hat{d}$	0.0 mm	$s_r\hat{d}$	0.0 mm
$s_w\hat{h} \mid d$	1.5 m	$s_r\hat{h}$	0.8 mm
$s_w\hat{d}_6 \mid d, h$	8.0 mm	$s_r\hat{d}_6$	8.0 mm
$s_w\hat{h}_c \mid d, h, d_6$	1.8 m	$s_r\hat{h}_c$	0.8 m
$s_w\hat{h} \mid d, h$	2.0 m		
$s_b\hat{h} \mid d$	1.0 m	$b\hat{d}$	0.0 mm
$s_b\hat{d}_6 \mid d, h$	3.0 mm	$b\hat{h}$	0.0 m
$s_b\hat{d}_6 \mid d, h, h_c$	2.7 mm	$b\hat{d}_6$	0.0 mm
$s_b\hat{h}_c \mid d, h, d_6$	1.6 m	$b\hat{h}_c$	0.0 m
$s_b\hat{h}_c \mid d, h$	1.6	s_e	0.011v cu.m.

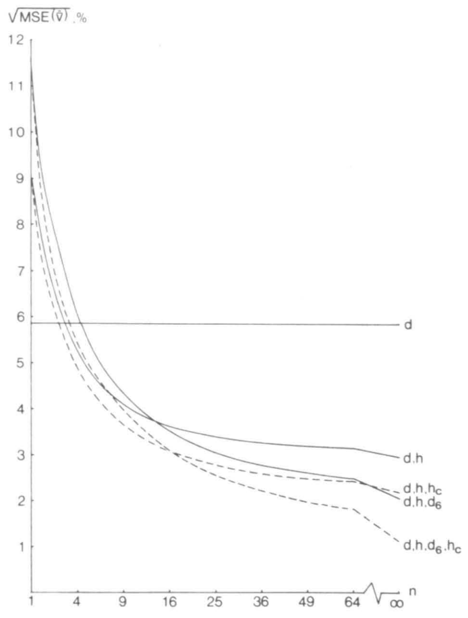


Fig. 2. Relative root-mean-square error of the volume estimate as a function of the measured predicting variable combination and number of sample trees.

Kuva 2. Tilavuusestimaatin keskineliövirheen neliöjuuri koepuiden mittausyhdistelmän ja koepuiden lukumäärän funktiona.

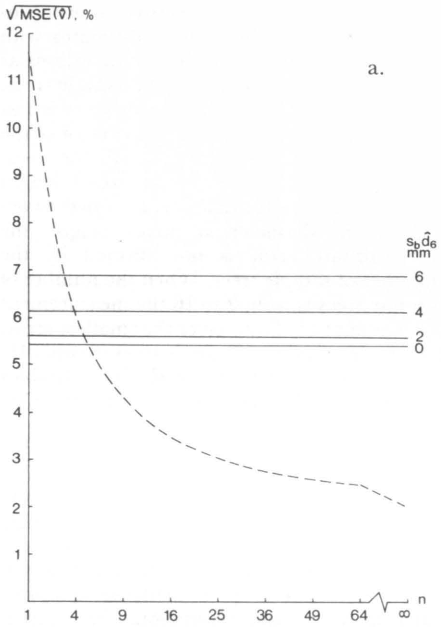
Figure 2 gives the relative root-mean-square errors of the volume estimates with different predicting variable combinations as a function of the number of the sample trees. When the number of the sample trees is less than 2, the smallest mean-square error of the volume estimate is attained with the measurement of the diameter at breast height only. Since no variation was assumed to occur in the diameter at breast height, the mean-square error is not affected by the number of sample trees. When the number of sample trees is from 2 to 16 the measurement of combination d,h,h_c gives the smallest error. When the number of sample trees exceeds 16, the measurement of all predicting variables (d,h,d_6,h_c) yields the lowest mean-square error of the volume estimate.

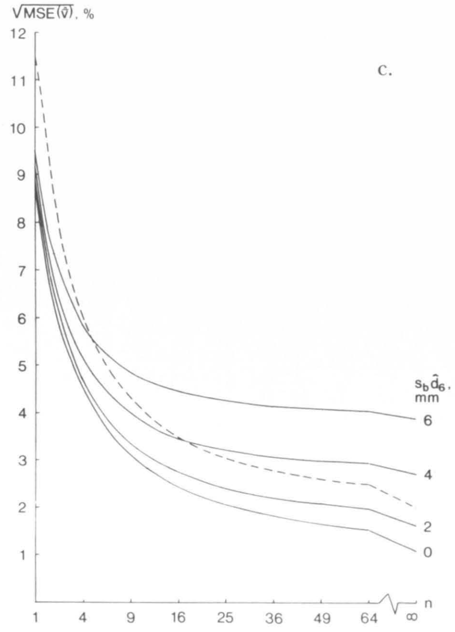
When the number of sample trees approaches infinity, the mean-square error approaches the fixed mean-square error of the volume estimate of the respective predicting variable combination (see formula 5.3).

A more thorough look was taken at the importance of the upper diameter measurement in the volume estimation. The relative root-mean-square errors of the volume estimates with measurement combinations (d), (d,h), and (d,h,h_c), in which the upper diameter has not been measured, are given as a function of $s_b\hat{d}_6$ in Fig. 3. The results demonstrate the rapidly increasing importance of the upper diameter measurement with increasing $s_b\hat{d}_6$ values.

52. Mixed utilization of sample trees

It is reasonable to assume that additional measured information never impairs the volume estimates. The results given in the previous section contradict this assumption. With a small number of sample tree measurements additional information frequently increases the mean-square error. Pekkonen (1982) has shown that the mixed utilization of the a priori models and sample tree measurements improves the accuracy of the volume estimates. In this mixed approach the value of each predicting variable is estimated as a weighted average of its estimates, one based upon the a priori model and the other upon the sample tree measurements. The mean-square error for the weighted estimate is





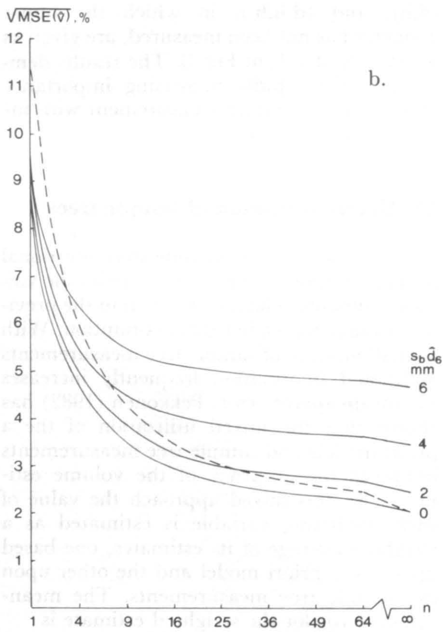


Fig. 3. Root-mean-square error of the volume estimate as a function of the deviation of d_6 ($s_b\hat{d}_6$) from its expected value (solid lines) compared to the error of the value estimates of combination d,h,d_6 (broken line).

Kuva 3. Tilavuusestimaatin keskineliövirheen neliöjuuri yläläpimitan odotusarvosta poikkeaman funktiona (yhtenäiset viivat) verrattuna mittausyhdistelmällä d,h,d_6 saatuun virheeseen (katkoviiva).

a. Measured predicting variable: d
Mitattu d

b. Measured predicting variables: d,h
Mitattu d ja h

c. Measured predicting variables: d,h,h_c
Mitattu d, h ja h_c

$$\begin{aligned} MSE(\hat{x}_i) &= E(p_i\hat{y}_i + (1-p_i)\hat{z}_i - \hat{x}_i)^2 \qquad (5.10)\\ &= s_b\hat{x}_i^2 - 2p_i\, s_b\hat{x}_i^2 + \\ &\quad p_i^2\,(s_w\hat{x}_i^2/f_i + s_b\hat{x}_i^2 + s_r\hat{x}_i^2/n_i + b\hat{x}_i^2), \end{aligned}$$

where

p_i = weight of $\hat{y}_i$

$\hat{y}_i$ = sample tree estimate of the predicting variable x_i

$\hat{z}_i$ = a priori model estimate of the predicting variable x_i

$\hat{\hat{x}}_i$ = true value of predicting variable x_i.

The optimum weights are derived through minimization of the mean-square error of the estimate of the predicting variable. The optimum value for p_i is

$$p_i = \frac{s_b\hat{x}_i^2}{s_w\hat{x}_i^2/f_i + s_b\hat{x}_i^2 + s_r\hat{x}_i^2/n_i + b\hat{x}_i^2}\,. \qquad (5.11)$$

The mean-sguare error of the mean volume estimate of a tree population is derived from formula

$$MSE(\hat{v}) = \sum_{i=1}^{k} \left(\frac{\partial v_i}{\partial x_i}\right)^2 (p_i^2\,(s_w\hat{x}_i^2/f_i + s_r\hat{x}_i^2/n_i + b\hat{x}_i^2) + (1-p_i)^2 s_b\hat{x}_i^2) + \sum_{i=k+1}^{l} \left(\frac{\partial v_i}{\partial x_i}\right)^2 s_b\hat{x}_i^2 + s_e^2 \qquad (5.12)$$

Fig. 4 points out the merits of the mixed approach. The parameter values are the same as applied in Fig. 2. The comparison of these two figures shows that with a small number of sample tree measurements, the mixed approach is clearly superior to the pure sample tree utilization. When the number of sample trees exceeds 20 . . . 30, the differences between these two methods become small. Fig. 4 also demonstrates the fact that additional sample tree measurements, when properly applied, never increase the mean-square error of the volume estimate.

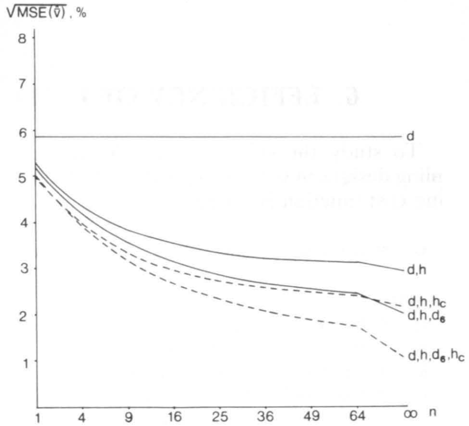


Fig. 4. Relative root-mean-square error of the mixed volume estimate as a function of the measured predicting variable combinations and number of sample trees.

Kuva 4. Yhdistetyn tilavuusestimaatin keskineliövirheen neliöjuuri koepuiden mittausyhdistelmän ja koepuiden lukumäärän funktiona.

6. EFFICIENCY OF DIFFERENT SAMPLING DESIGNS

To study the efficiency of different sampling designs in volume estimation the following cost function is suggested

$$C = c_1 + c_2 n + \sum_{i=1}^{k} c_{3i} n_i , \qquad (6.1)$$

where

c_1 = fixed cost of preparing the sample
c_2 = fixed cost per sample tree
k = number of measured predicting variables
n = total number of sample trees
c_{3i} = marginal cost of one measurement of predicting variable x_i
n_i = number of sample trees with measured predicting variable x_i.

Given a fixed total cost (C'), the optimal numbers (n_i) of sample tree measurements can be determined via solution of the following nonlinear programming problem

$$\min \mathrm{MSE}(\hat{\hat{v}}) \qquad (6.2)$$

$$\text{s.t.} \quad C \leqq C' \qquad n_i \leqq n_{i-1} \quad i = 2, \ldots, k$$

In order to simplify the problem it was assumed that $n_i = n_1$ for all $i = 2, \ldots, k$. The cost data are largely based upon the study of Hyppönen and Roiko-Jokela (1978). The measurement costs are expressed in timber volume units. It was assumed that the cost of the height measurement equals 0.01 cu.m. of timber, the marginal cost of d_6 measurement 0.0028 cu.m., and that of h_c measurement 0.001 cu.m. The fixed costs, c_1 and c_2, were assumed to be zeros. Thus, the cost per sample tree with different measurement combinations were

combination	cost
d	0.0
d,h	0.01
d,h,d_6	0.0128
d,h,d_6,h_c	0.0138
d,h,h_c	0.011

The root-mean-square errors of different sample tree measurement combinations were plotted as a function of the sampling cost. Fig. 5 demonstrates these relationships when the variation parameters from table 18 are employed. The curves do not markedly differ from those given in Fig. 2. With relatively low sampling costs the measurement combination (d,h,h_c) gives the best result; thereafter the measurement of all sample tree variables yields the lowest root-mean-square error with

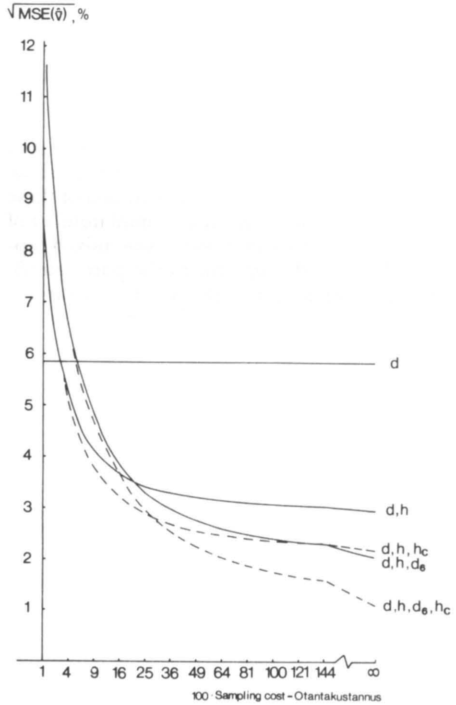


Fig. 5. Relative root-mean-square error of the volume estimate as a function of the measured predicting variable combination and sampling cost.
Kuva 5. Tilavuusestimaatin keskineliövirheen neliöjuuri koepuiden mittausyhdistelmän ja otantakustannuksen funktiona.

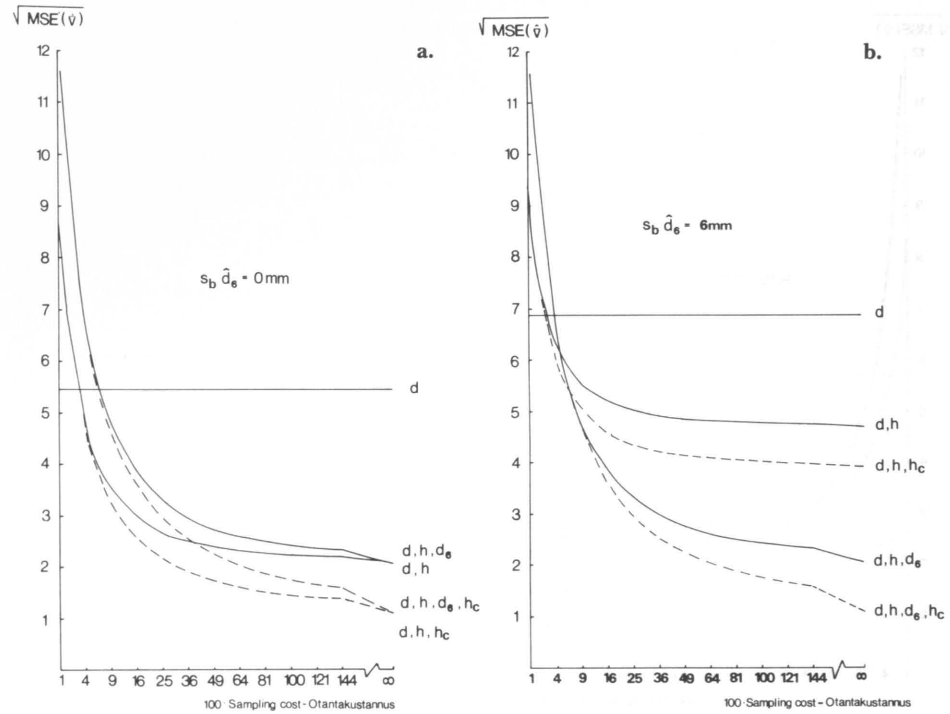


Fig. 6. Relative root-mean-square error of the volume estimate as a function of the measured predicting variable combination and sampling cost with two different values of $s_b\hat{d}_6$.
Kuva 6. Tilavuusestimaatin keskineliövirheen neliöjuuri koepuiden mittausyhdistelmän ja otantakustannuksen funktiona kahdella $s_b\hat{d}_6$:n arvolla.

a given sampling cost. If the available funds would cover the measurement of only 1 or 2 sample trees it is best to estimate all sample tree variables from a priori models.

Figs. 6a and 6b demonstrate the influence of the value of parameter $s_b\hat{d}_6$ upon the error-cost curves. If $s_b\hat{d}_6$ assumes value 0 mm (Fig. 6a), combination (d,h,h_c) is the best one except the case with a very small measurement cost budget. Then, again, it does not pay to measure any sample trees. In the other extreme with $s_b\hat{d}_6$ having a value 6 mm (Fig. 6b) the measurement combination (d,h,d_6,h_c) is the most advantageous one when the measurement costs exceed 0.06 units, i.e. the measurement cost of approximately six sample trees. With lower measurement costs it is best to measure combination (d,h,h_c) and with very small funds available not to measure any sample trees.

Since the information of the crown height variation is based only upon the taper curve data, the sensitivity of the results on changes in $s_b\hat{h}_c$ was tested. The other variation parameters assumed the values given in table 18. The results of these calculations are given in Figs. 7a and 7b. If the difference in the crown height between the populations is only 1 m, the usefulness of the crown height measurement clearly decreases but combinations (d,h,d_6,h_c) and (d,h,h_c) still result to the most efficient sampling designs (Fig. 7a). Their edge is clearly increased when the value of parameter $s_b\hat{h}_c$ is increased to 2 m (Fig. 7b).

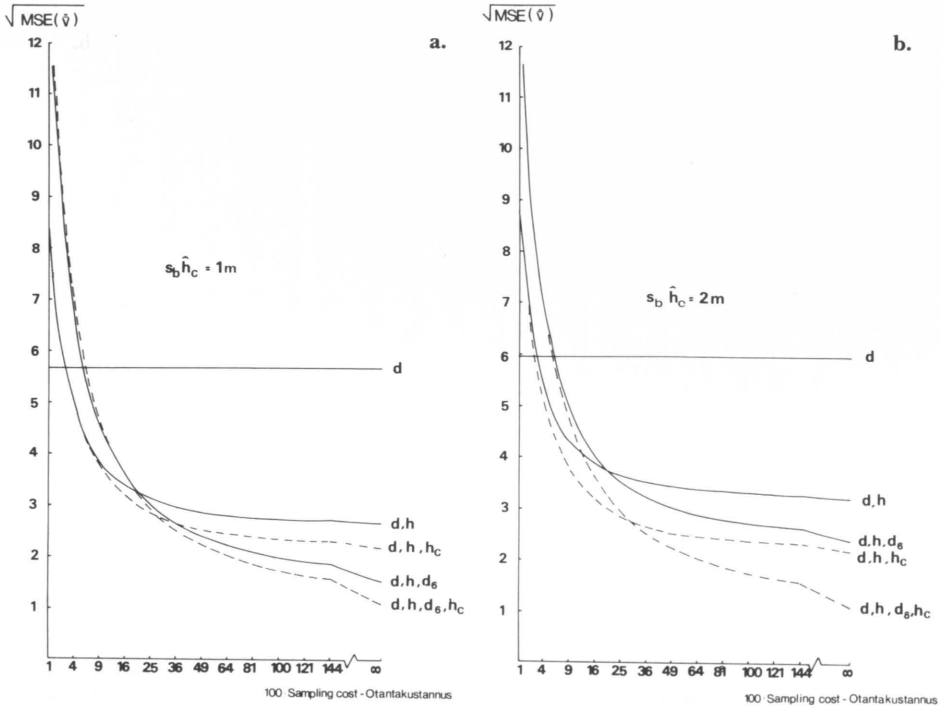


Fig. 7. Relative root-mean-square error of the volume estimate as a function of the measured predicting variable combination and sampling cost with two different values of $s_b\hat{h}_c$.

Kuva 7. Tilavuusestimaatin keskineliövirheen neliöjuuri koepuiden mittausyhdistelmän ja otantakustannuksen funktiona kahdella $s_b\hat{h}_c$:n arvolla.

The rating of the predicting variable combinations (d,h,d_6) and (d,h,h_c) is clearly dependent upon the magnitude of $s_b\hat{h}_c$ even though no dramatic changes occur when the values of $s_b\hat{h}_c$ vary. Further studies are needed on the value of $s_b\hat{h}_c$ in different circumstances. However, it looks probable that in most cases the optimum predicting variable combination calls for measurement of h_c. The only exceptions occur with very low sampling costs. Then either combination (d,h) or bare (d) yields the best result.

Fig. 8 demonstrates the influence of the mixed utilization of the sample trees upon the efficiency of the sampling designs. The variation parameters correspond to those given in table 18. The error-cost curve of the optimal pure sample tree utilization is also plotted in Fig. 8 with a dotted line. Mixed utilization yields the greatest gains with low sampling costs. Measurement combinations (d,h,d_6,h_c) and (d,h,h_c) are the most efficient ones. The reduction in the root-mean-square error is over 20 percent when the number of sample trees is small. With a large number of sample trees the gains due to the mixed utilization of the sample trees become negligible.

It should be noticed that measurement combinations (d,h) and (d,h,d_6), which are most commonly applied in Finland, never appear to be the most effivient ones.

The efficiency of sampling might increase if the number of sample tree measurements would be unequal for different sample tree characteristics (cf. problem 6.2).

The optimum sample size in each design can be determined by minimizing the total expected cost which is the sum of the cost of the errors involved using an estimate for the value of the parameter of interest and of the cost of the sampling (see e.g. Ackoff et al. 1962, p 234).

The total expected cost can be derived from formula

$$\mathrm{TEC} = c_1 + c_2 n + \sum_{i=1}^{k} c_{3i} n_i + c_4 f(\mathrm{MSE}(\hat{v})), \quad (6.3)$$

where

c_4 = parameter

Quite often it is assumed the cost of errors involved using an estimate be a linear function of the error variance (see e.g. Cochran 1963, p 83). Another simple function suggested is the linear function of the standard error (see e.g. Blythe 1945).

Fig. 9 demonstrates some iso-cost curves derived from formula (6.3) with different c_4 values. It was assumed that the cost of errors is a linear function of the mean-square error. The error – cost curves in Fig. 9 are the same as in Fig. 5.

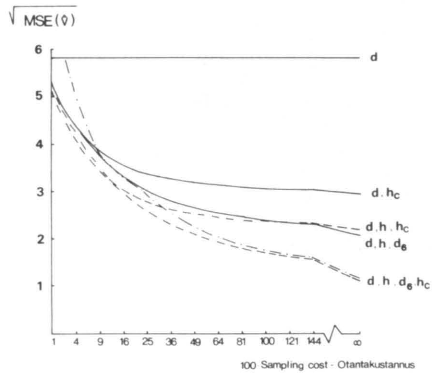


Fig. 8. Relative root-mean-square error of the mixed volume estimate as a function of the masured predicting variable combination and sampling cost. The minimum errors derived from pure sample tree utilization are expressed by a dotted line.

Kuva 8. Tilavuusestimaatin keskineliövirheen neliöjuuri koepuiden mittausyhdistelmän ja otantakustannuksen funktiona yhdistetyssä koepuiden käytössä. Pelkkään koepuiden käyttöön perustuvan menetelmän antamat minimivirheet on kuvattu pisteviivalla.

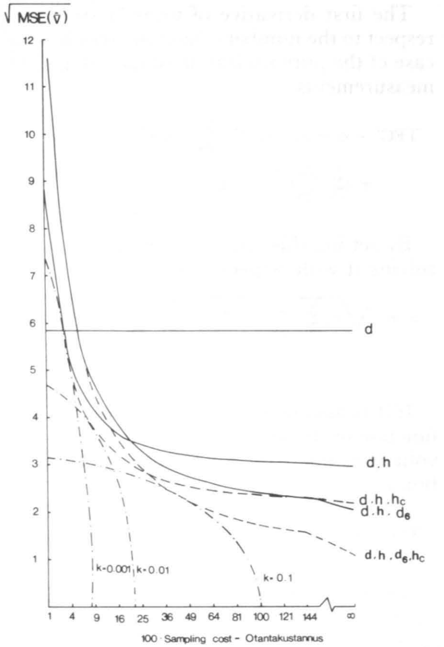


Fig. 9. Iso-cost curves derived from the total expected cost formula (6.5).

Kuva 9. Kaavasta (6.5) johdettuja samakustannuskäyriä.

The total expected cost can be minimized by solving the following nonlinear programming problem

$$\min \mathrm{TEC} \quad (6.4)$$

s.t.

$$n_i \leqq n_{i-1} \quad l = 2, \ldots, k .$$

If it is assumed that $n_i = n_l$ for all i, the total expected cost can be minimized without any constraints. The following examples refer to this case.

First it is assumed that f(MSE($\hat{v}$)) is a linear function of the mean-square error. Then, the total expected cost can be calculated from formula

$$\mathrm{TEC} = c_1 + c_2 n + c_3 n + c_4 (A + C) + c_4 (B + D + E), \quad (6.5)$$

where

$$c_3 = \sum_{i=1}^{k} c_{3i} .$$

The first derivative of formula (6.5) with respect to the number of sample trees is in the case of the pure utilization of the sample tree measurements

$$TEC' = c_2 + c_3 - c_4 \left(\sum_{i=1}^{k} \left(\frac{\partial v_i}{\partial x_i}\right)^2 s_w\hat{x}_i^2 + \left(\sum_{i=1}^{k} \left(\frac{\partial v_k}{\partial x_i}\right)^2 s_r\hat{x}_i^2\right) n^{-2} \qquad (6.6)$$

By setting this equation equal to zero and solving it with respect to n we get

$$n = \sqrt{c_4\left(\sum_{i=1}^{k} \left(\frac{\partial v_i}{\partial x_i}\right)^2 s_w\hat{x}_i^2 + \left(\sum_{i=1}^{k} \left(\frac{\partial v_k}{\partial x_i}\right)^2 s_r\hat{x}_i^2\right)/c_2+c_3\right)} \, . \qquad (6.7)$$

If it is assumed that $f(MSE(\hat{\bar{v}}))$ is a linear function of the root-mean-square error of the volume estimate the total expected cost function is

$$TEC = c_1 + c_2 + c_3 n + c_4 \sqrt{MSE(\hat{\bar{v}})} \, . \qquad (6.8)$$

No simple analytical solution is available for the minimization of this function, but the optima have to be derived iteratively.

The assumption that function $f(MSE(\hat{\bar{v}}))$ is linear with respect to root-mean-square error may be quite relevant when timber is measured for sale. The linear function gains some justification from the reasoning that in a sale of timber the increased possibility of negative error due to reduced number of sample tree measurements may be compensated by increased price of the timber. Two cases are examined.

In the first case it is assumed that the purchaser (seller) of timber wants to minimize the sum of the measurement costs and over(under)payments in a timber lot with volume V. If the estimates, $\hat{\bar{v}}$, are normally distributed around the true value, $\bar{v}$, the total expected cost is

$$TEC = c_1 + c_2 + c_3 n + c5\, V/100 \sqrt{MSE(\hat{\bar{v}})} \, , \qquad (6.9)$$

where

$c_5 = 0.5\, E(|\bar{v} - \hat{\bar{v}}| / \sqrt{MSE(\hat{\bar{v}})}) = 0.4$

(cf. Ackoff et al. 1962, p. 240).

From formulae (6.8) and (6.9) we get

$$c_4 = 0.4\, V/100 = 0.004\, V.$$

Thus the values of parameter c_4 can be expressed as a function of the the timber volume (V):

c_4	V, cu.m.
0.01	2.5
0.1	25
1	250
10	2500
100	25000

If the purchaser (seller) accepts these values for parameter c_4 he shows his willingness to take a 34.5 percent risk of employing too few sample trees.

Fig. 10 indicates critical $s_b\hat{d}_6$ values between four different combinations of predicting varables as a function of the volume. If the volume of the sale is 1000 m^3, for example, it pays to measure combination ($d,h,d_{,6}$) instead of combination (d,h) if $s_b\hat{d}_6$ is greater than 1.4 mm. For the same amount of timber combination (d,h,d_6) is superior to combination (d,h,h_c) if $s_b\hat{d}_6$ exceeds 3.1 mm. Combination ($d,h,d_{,6},h_c$) is superior to combination (d,h) for all $s_b\hat{d}_6$ values and it is superior to combination (d,h,h_c) if $s_b\hat{d}_6$ exceeds 1.5 mm.

In the second case it is assumed that the purchaser (seller) increases the number of sample trees to the point where the additional cost of the measurement of one sample tree equals the reduction in the maximum over-(under)estimation (cf. Blythe 1945). Blythe proposes that the maximum error be twice the standard error. If this proposition is accepted it means that the purchaser (seller) is ready to accept a 2.5 percent risk, and we get the following relationships between parameter c_4 and timber volume:

c_4	V, cu.m.
0.01	0.5
0.1	5
1	50
10	500
100	5000

Consequently, the curves in Fig. 10 will move to the left and the measurement of d_6 is profitable with lower $s_b\hat{d}_6$ values than indicated by the curves in Fig. 10. The opposite is true if the decision maker evaluates the possible losses below their expected value. This way of thinking may be relevant for a decision maker with an inclination to gambling.

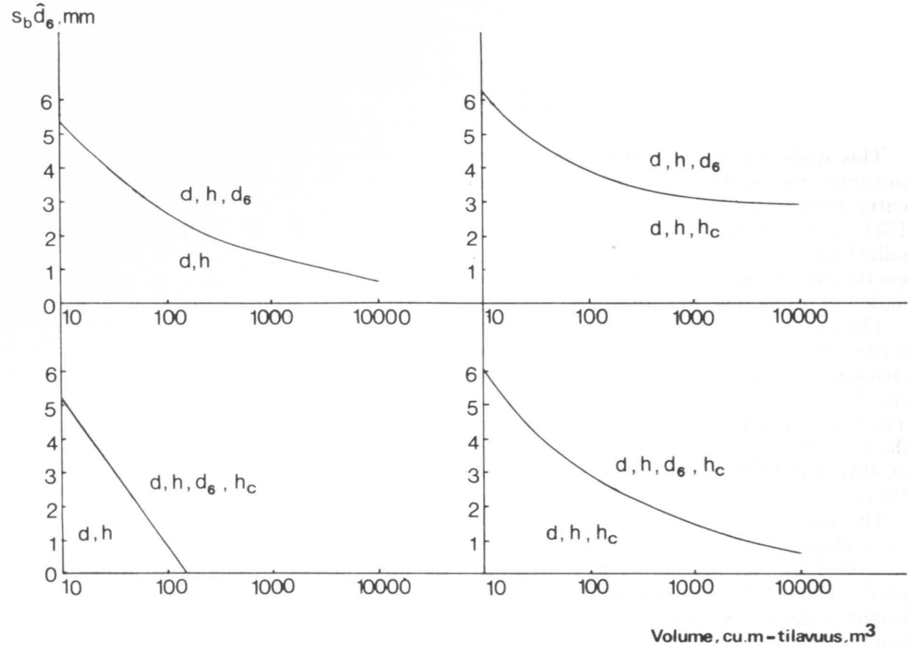


Fig. 10. Limit for profitable d_6 measurement between four pairs of measurement combinations as a function of the volume of the tree population.

Kuva 10. Yläläpimitan mittaamisen kannattavuusraja neljän mittausyhdistelmän välillä puujoukon tilavuuden funktiona.

7. DISCUSSION

This study was initiated from the need to integrate the simultaneous equation taper curve model (see e.g. Kilkki and Varmola 1981) into a measurement system involving tallied trees and sample trees. From this origin the study grew into a more general examination of the sample tree utilization.

The functions applied in this study to describe the relationships between the tree characteristics have been mainly based upon simultaneous equation taper curve models. The construction of the models has followed the lines of the previous studies of the author (Kilkki et al. 1978; Kilkki and Varmola 1979, 1981).

The models would have been substantially more elegant if the polar coordinate system suggested by Sloboda (1976) had been applied. Then both the diameters and the height of the tree would be expressed as the lengths of the rays in a polar coordinate system with its pole in the middle point of the butt of the tree. A scheme for this approach is given in Fig. 11. With this approach the separate derivation of the height models might be unnecessary and the height measurements could be employed in the calibration of the parameters of the simultaneous equation model. This possibility is available also with the use of the present models but its effectuation would call for the employment of the nonlinear simultaneous equation model which again makes the computations more complex (see Kilkki and Varmola 1979).

There was some indication that the position of the tree in the stand correlates with the taper curve. Better data is required to verify this hypothesis. It is also possible to derive separate regression equations for the residuals with respect to the exogenous variables. This approach would decrease the random variation.

The variation studies (section 3) pointed out that if the diameter at breast height and the height of the tree are known, the vast majority of the residual variation of the taper curve estimates occurs within the stand. The small variation between the populations (and/or a large variation within the population) leads to a situation where a great number of sample tree measurements is needed to achieve the same precision as attained with the use of a priori models.

The numerical examples in sections 5 and 6 serve only as to demonstrate the ideas of the study. However, the parameter values employed have been chosen in such a way that they represent a common situation in Finnish Scots pine forests.

The results would change to some degree if the numerical values of the tree dimensions were changed. The value of the upper diameter measurement for instance, is greatly dependent on the tree size.

A special emphasis is laid upon the use of the upper diameter (d_6) in the estimation of the stem volume of the tree population. This emphasis is justified by the common use of this predicting variable in Finland and other countries (see e.g. Schmid-Haas and Wintzeler 1981) and by the cost and inconveniences involved in its measurement.

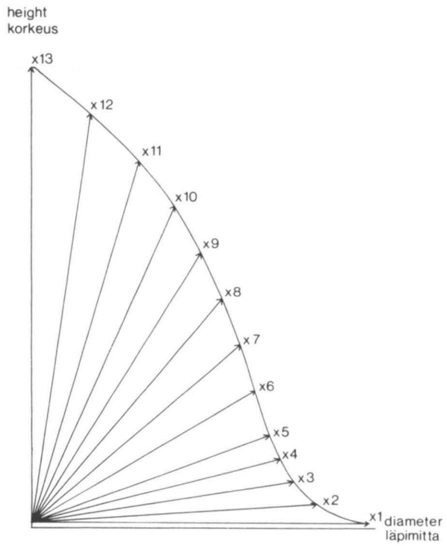


Fig. 11. Endogenous variables of the simultaneous equation model in the polar coordinate system.

Kuva 11. Simultaaniyhtälömallin endogeeniset muuttujat napakoordinaatistossa.

The results of the calculations indicate that the measurement of the crown height is quite competitive with the measurement of the upper diameter. If a great accuracy is required the measurement of both of them yields the best result.

The costs applied for d_6 measurements in the sample calculations may be underestimates since usually an extra member is required in the measurement crew just to carry the upper diameter measurement instrument.

More emphasis should be laid upon the development of such tree characteristic models in which the set of the predicting variables would comprise all easily measurable environmental variables. Then, the importance of the measurement of d_6 might be further reduced.

Mixed utilization of a priori models and sample tree measurements gave promising results. The method seems especially suitable to such forest inventory tasks where accurate information at relatively low cost is needed for a large number of strata. This is the situation in standwise inventories for forest management planning, for example. Then, even a small number of sample trees per each stand may suffice.

In this paper the examination is limited only to the stem volume of the tree population. The study could be expanded to other variables of interest, such as value of the timber and volume increment of the growing stock. Another expansion would be the inclusion of the error in the estimation of the diameter distribution at breast height. If the errors in stem diameter distribution and sample tree measurements can be considered independent of each other the combination of these two errors is quite straightforward.

8. SUMMARY

The purpose of the study has been to optimize the utilization of the sample tree measurements in the estimation of the stem volume of a tree population.

Two simultaneous equation taper curve models for Scots pine were developed. These models constitute the main basis for the variation studies. Furthermore, a few regression equations to predict the height and crown height of the tree were developed.

One pine material and two materials comprising pine, spruce, and birch have been employed to analyse the residual variation of the models which predict different characteristics of the tree. This analysis showed convincingly that the major part of the residual variation of the stem dimension estimates and accordingly of the stem volume estimates occurs within the populations and the minor part between the populations; a population typically containing the trees of one forest stand.

Reliability comparisons based upon the total residual variation are relevant only when the same measurements are applied to all trees of the population under survey, and the population is a random sample from the superpopulation. In a typical case, however, only a small fraction of the trees are measured as sample trees, and the population represents one part of the superpopulation. Then, the concepts residual variation within the populations and residual variation between the populations have to be employed in the reliability calculations.

Two formulae (5.3 and 5.10) for the mean-square error of the volume estimator and examples of the results derived by these formulae have been presented. The first formula is based upon the assumption that whenever a certain variable is measured from the sample trees, the estimates for this variable are derived from the measurements. In the second formula the optimally weighted averages of the a priori information and of the sample tree information are employed.

Efficiency of different sample tree measurement combinations has been studied by comparing the estimation errors with the sampling costs. The measurement of the crown height is usually included in the most efficient sampling procedures in the estimation of the stem volume of Scots pine. On the other hand, the value of the measurement of the upper diameter (d_6) from sample trees is not as self-evident as its common use might suggest.

The mixed utilization of the sample trees and a priori information clearly improved the efficiency of the measurements when the accuracy requirement for the volume estimate was relatively low.

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SELOSTE

KOEPUUT PUUSTON TILAVUUDEN ESTIMOINNISSA

Tutkimuksessa kehitetään menetelmä koepuumittausten optimoimiseksi. Optimaalisella koepuumittauksella tarkoitetaan mittaustapaa, jossa tietyllä mittauskustannuksella saadaan minimivirhe puujoukon tilavuuden estimaatille. Lähtökohtana on olettamus, että puujoukon rinnankorkeusläpimittajakauma tunnetaan tarkoin.

Runkokäyrämallit

Luvussa 2 esitellään simultaanisiin yhtälöryhmiin perustuvat runkokäyrämallit (taulukot 1 . . . 6), joita jatkossa käytetään rungon tilavuuden ja tilavuusfunktion osittaisderivaattojen estimointiin. Mallit ovat Kilkin ja Varmolan (1981) tutkimuksessa esitettyjen mallien parannettuja versioita. Parannusten ansioista on mallien kokoa ja samalla niiden vaatimaa laskenta-aikaa voitu supistaa. Lisäksi ovat mallin eksogeenisten muuttujien uudet transformaatiot poistaneet edellisissä malleissa lyhyissä puissa esiintyneen lievän harhaisuuden.

Puun tunnusten estimointi- ja mittausvirheet

Runkokäyräaineisto

Luvussa 3 on kolmen aineiston avulla tutkittu malleilla laskettujen puun tunnusten estimaattien virhevarianssin suuruutta ja varianssin jakautumista puujoukon sisäiseen ja puujoukkojen väliseen varianssiin.

Ensimmäisenä aineistona oli runkokäyrämallien perustana ollut 29 metsiköstä koottu 492 mäntykoepuun aineisto. Aineiston avulla tutkittiin suhteellisilta korkeuksilta mitattujen läpimittojen, kuuden metrin läpimitan, puun pituuden, latvusrajan korkeuden ja puun tilavuuden estimaattien virhevarianssia ja varianssin jakautumista metsiköiden sisäiseen ja metsiköiden väliseen varianssiin erilaisia koepuiden mittausyhdistelmiä käytettäessä.

Suhteellisilta korkeuksilta mitattujen läpimittojen, kuuden metrin läpimitan ja puun tilavuuden ennustamiseen käytettiin runkokäyrämallia. Puun pituuden ja latvusrajan korkeuden ennustamiseksi laadittiin erilliset regressioyhtälöt.

Runkokäyrää ja puun tilavuutta ennustettiin neljällä koepuutunnusyhdistelmällä: (1) d,h, (2) d,h,d_6, (3) d,h,d_6,h_c (4) ja d,h,h_c. Tulokset on esitetty taulukoissa 7 . . . 10. Kokonaisvarianssilla mitattu yhdistelmien paremmuusjärjestys runkokäyrää ja puun tilavuutta estimoitaessa oli (3), (2), (4) ja (1). Näillä mittausyhdistelmillä olivat kokonaisvirhevarianssia vastaavat keskivirheet yli 8.6 metrin puille 3.9, 4.3, 6.9 ja 7.5 prosenttia.

Valtaosa eri mittausyhdistelmillä saatujen runkokäyräestimaattien virhevarianssista ja vastaavasti tilavuusestimaattien virhevarianssista on metsiköiden sisäistä vaihtelua. Kuva 1 osoittaa että metsiköiden välisen varianssin suhde kokonaisvarianssiin on suurimmillaankin vain noin 0.25. Useimmiten suhde jää 0.15 alapuolelle. Tämä näkyy myös tilavuusestimaattien virhevarianssin jakautumisessa metsiköiden sisäiseen ja metsiköiden väliseen varianssiin. Eri mittausyhdistelmillä saadut tilavuusestimaattien virhevarianssit metsiköiden välillä ovat keskivirheinä ilmaistuina (3) 1.1, (2) 1.8, (4) 2.2 ja (1) 3.1 prosenttia. Tulos osoittaa myös sen, että ero menetelmien (2) ja (4) välillä on merkittävästi kaventunut. Täten on ilmeistä, että latvusrajan korkeuden mittauksella voidaan rinnankorkeusläpimitan ja pituuden avulla ennustetun puun tilavuusestimaatin virhettä metsiköiden välillä pienentää lähes yhtä paljon kuin kuuden metrin läpimitan mittauksella.

Kuuden metrin läpimitan virhevarianssin painottuminen metsiköiden sisäiseen varianssiin näkyy myös siitä, että kokonaisvirhevarianssi on keskivirheenä ilmaistuna 11.6 mm rinnankorkeusläpimitan ja pituuden ollessa ennustavina muuttujina ja 11.0 mm, jos näiden lisäksi ennustajana on latvusrajan korkeus. Metsiköiden välisen virhevarianssin vastaavat luvut ovat 3.6 ja 3.2 mm.

Runkokäyräaineistosta laskettiin lisäksi puun pituutta ja latvusrajan suhteellista korkeutta ennustavat regressioyhtälöt (3.5) . . . (.10) sekä (3.11) ja (3.12). Taulukoissa 11 ja 12 on esitetty yhtälöillä ennustettujen pituuksien ja latvusrajan suhteellisen korkeuden jäännösvaihtelun jakautuminen metsiköiden sisäiseen ja metsiköiden väliseen vaihteluun. Molempia tunnuksia estimoitaessa on virhe metsiköiden sisällä ja metsiköiden välillä likimain yhtä suuri. Aineiston keruutavasta johtuen ei pituuden estimoinnissa saaduille tuloksille ole syytä antaa kovin paljon painoa. Merkittävää on kuitenkin estimointivirheen voimakas pieneneminen lisättäessä pituuden selittäjiksi d_6 ja h_c rinnankorkeusläpimitan lisäksi.

VMI-aineisto

Toisena aineistona olivat valtakunnan metsien kuudennessa inventoinnissa Pohjois-Savon piirimetsälautakunnan alueella mitatut yli 7.5 metriä pitkät koepuut. Tämän aineiston avulla tutkittiin pituuden ja kuuden metrin läpimitan jäännösvaihtelua. Pituutta ennustettiin julkaisemattomilla valtakunnan metsien inventoinnin koepuuaineistoon perustuvilla pituusmalleilla, joissa selittäjinä olivat puulaji, rinnankorkeusläpimitta ja inventoinnissa mitatut metsikkötunnukset. Kuuden metrin läpimittaa ennustettiin Päivisen (1978) malleilla, joissa selittävinä muuttujina ovat puulaji, rinnankorkeusläpimitta, pituus ja kasvupaikkaa kuvaavat tunnukset. Laskelmien tulokset on esitetty taulukoissa 13 ja 14.

Pituusestimaatin virhevarianssi on jonkin verran pienempi koealojen välillä kuin koealojen sisällä. Kun kaikki puulajit yhdistetään, on pituuden jäännösvaihtelun koealojen välinen varianssi noin 40 prosenttia kokonaisvarianssista. Koealojen sisäisessä virhevarianssissa on mukana myös satunnainen mittausvirhe. Kaikille puulajeille laskettu koealojen välinen pituusestimaattien virhevarianssi on keskivirheenä ilmaistuna 1.3 metriä.

Kuuden metrin läpimitan estimointivirheestä valtaosa jää koealojen sisäiseksi vaihteluksi. Estimointivirheen vaihtelu metsiköiden sisällä on likimain saman suuruinen kuin ensimmäisessä aineistossa eli kaikilla puulajeilla keskihajontana ilmaistuna 11.0 mm. Männyllä vaihtelu on hieman pienempi, 9.8 mm. Valtakunnan metsien inventointien mittaukset ovat luokituksista ja mittausvirheistä johtuen kuitenkin epätarkempia kuin runkokäyräaineistossa. Tästä syystä on ilmeistä, että valtakunnan metsien inventointiaineistossa on kuuden metrin läpimitan todellinen vaihtelu koealojen sisällä jonkin verran pienempi kuin runkokäyräaineistossa. Tämä on ymmärrettävä, kun otetaan huomioon, että valtakunnan metsien inventoinnin relaskooppikoealat edustavat vain metsikön pientä osaa ja toisaalta runkokäyräaineiston keruussa pyrittiin metsikön puista saamaan mahdollisimman monipuolinen näyte. Lisäksi on huomattava, että läpimittajakaumat poikkeavat aineistoissa toisistaan. Tämä vaikuttaa tuloksiin, sillä virhevarianssi kasvaa jonkin verran rinnankorkeusläpimitan kasvaessa.

Keskihajontana ilmaistuna on d_6:n jäännösvaihtelu koealojen välillä 5.6 mm, mikä on 2 mm suurempi kuin runkokäyräaineiston männyillä. Koealojen välistä vaihtelua suurentaa ensisijaisesti koealojen pienuus. Toisena tekijänä saattavat olla mittaajista johtuvat systemaattiset erot. Näiden merkitystä ei kuitenkaan voitu osoittaa kovin suureksi luvussa 4 esitetyillä variaatiolaskelmilla. Ei kuitenkaan ollut mahdollista tutkia koealakohtaisia systemaattisia mittausvirheitä, joita on voitu tehdä sekä kuuden metrin läpimitan mittauksessa että sen ennustamisessa käytettävien rinnankorkeusläpimitan ja pituuden mittauksessa. Toisaalta olisi koealojen välinen virhevarianssi muodostunut hieman suuremmaksi, ellei kuuden metrin selitysmallissa olisi ollut mukana myös kasvupaikkatunnuksia.

Pystymitta-aineisto

Kolmantena aineistona oli 194 pystymittaleimikon 242 koepuualuetta. Yli 7.5 metrin pituisia koepuita oli yhteensä 58855 kappaletta. Pystymittakoepuuaineistolle tehtiin samat laskelmat kuin VMI-koepuuaineistolle. Laskelmien tulokset on esitetty taulukoissa 15 ja 16. Koska leimikoista ei ollut käytettävissä metsikkötunnuksia, jouduttiin näille tunnuksille antamaan vakioarvot sekä pituutta että yläläpimittaa ennustavissa malleissa. Ilmeisesti osittain tästä syystä olivat estimaatit harhaisempia kuin VMI-aineistossa.

Pituusestimaattien kokonaisvirhe kasvoi jonkin verran VMI-aineistoon verrattuna. Lisäys johtui pääosaksi leimikoiden sisäisen vaihtelun kasvusta. Täten ei ole todennäköistä, että metsikkötunnusten mukaanottamisella olisi virhettä voitu paljoakaan pienentää.

Yläläpimitan estimointivirhe oli leimikoiden sisällä samaa suuruusluokkaa kuin VMI-aineistossa ja runkokäyräaineistossa. Leimikoiden väliset erot olivat samaa luokkaa kuin metsiköiden väliset erot runkokäyräaineistossa ja huomattavasti pienemmät kuin koealojen väliset erot VMI-aineistossa.

Myös pystymitta-aineiston tuloksia tarkasteltaessa on otettava huomioon, että mahdolliset systemaattiset mittausvirheet ryhmien ja leimikoiden välillä kasautuvat leimikoiden väliseen virheeseen.

Mittausvirheet ja -kustannukset

Mittausvirheiden ja mittauskustannusten arvioinnin perustana käytettiin Hyppösen ja Roiko-Jokelan (1978) tuloksia. Puun pituuden mittaukseen oletettiin käytettävän käsivaraista Suunto-hypsometria ja yläläpimitan mittaukseen latvakaulainta. Mittauskustannuksiin otettiin mukaan vain muuttuvat kustannukset. Täten ei esimerkiksi yläläpimitan mittauksen edellyttämää ryhmän koon suurenemista ja tästä aiheutuvaa kiinteiden kustannusten nousua otettu huomioon.

Estimointimenetelmien luotettavuus

Luvussa 5 esitetään kaava (5.3) puujoukon tilavuusestimaatin keskineliövirheen laskemiseksi. Virhe koostuu tilavuusmallissa mukana olevien koepuutunnusten otanta-, mittaus- ja ennustevirheistä painotettuina niiden merkitystä osoittavilla osittaisderivaatoilla. Keskineliövirheeseen on vielä lisättävä tilavuusmallin harhan neliö.

Olettamalla että kaikista koepuista mitataan aina sama määrä koepuutunnuksia ja käyttämällä lähtökohtana taulukossa 18 annettuja vaihteluparametrien oletusarvoja, on kuvaan 2 laskettu esimerkkejä tilavuusestimaatin keskineliövirheistä erilaisia koepuumittausyhdistelmiä ja koepuumääriä käytettäessä.

Jos koepuita on 1–2, saadaan pienin keskineliövirhe käyttämällä koepuutunnusten h, d_6 ja h_c ennustamiseen olemassa olevia malleja. Jos koepuiden määrä on yli 2, mutta alle 16, saadaan pienin virhe yhdistelmällä d,h,h_c. Jos koepuita mitataan yli 16, antaa kaikkien tässä tutkimuksessa mukana olleiden koepuutunnusten d, h, d_6 ja h_c mittaus pienimmän keskineliövirheen.

Kuvassa 3 on verrattu mittausyhdistelmiä (d), (d, h) ja (d,h,h_c) mittausyhdistelmään (d,h,d_6) puujoukkojen välisen yläläpimitan eron ($s_{\hat{b}}d_6$) funktiona. Kuten oli odotettavissa, yläläpimitan mittauksen merkitys korostuu puujoukon puiden yläläpimitan poikkeaman kasvaessa. Virhe alkaa kasvaa jyrkästi poikkeaman ylittäessä 4 mm.

Luvussa 5.2 tarkastellaan Pekkosen (1982) esittämää yhdistettyyn estimointiin perustuvaa menetelmää. Siinä yhdistetään optimaalisesti olemassa olevien mallien ja koepuumittausten tieto. Kuva 4, joka vastaa vaihteluparametreiltaan kuvaa 2, osoittaa että pieniä koepuumääriä käytettäessä yhdistetty menetelmä antaa huomattavasti luotettavampia tuloksia kuin pelkkiin mitattuihin koepuihin perustuva estimointimenetelmä. Tulos osoittaa myös sen loogisesti pääteltävissä olevan tuloksen, että lisämittaukset oikein käytettyinä eivät koskaan suurenna estimaatin virhettä.

Otantamenetelmien tehokkuus

Luvussa 6 tarkastellaan eri otantamenetelmien tehokkuutta vertaamalla tilavuusestimaatin virhettä mittauskustannuksiin (kaava 6.1). Tiettyä mittauskustannusta vastaava optimaalinen mittaustapa saadaan ratkaisemalla epälineaarisen ohjelmoinnin tehtävä (6.2). Tehtävän yksinkertaistamiseksi on kuitenkin oletettu, että kaikista koepuista tehdään samat mittaukset. Tällöin optimi löydetään kokeilemalla kaikki koepuumittausvaihtoehdot.

Kuvassa 5 on esitetty kaavalla (5.3) saatu tilavuusestimaatin virhe koepuutunnusten mittausyhdistelmän ja mittauskustannuksen funktiona taulukossa 18 annettuja vaihteluparametrien arvoja käytettäessä. Mittauskustannukset on ilmaistu rungon tilavuusyksiköissä, ja niiden perustaso on saatu olettamalla, että pituuden mittauksen kustannus on 0.01 m^3. Tulos ei poikkea olennaisesti kuvan 2 tuloksista. Jos koepuiden mittauskustannus on alle 0.03 m^3, ei varsinaisia koepuumittauksia kannata tehdä lainkaan. Jos mittauskustannus on tätä suurempi mutta alle 0.25 m^3, antaa mittausyhdistelmä d,h,h_c parhaan tuloksen. Tätä suuremmilla mittauskustannuksilla saadaan paras tulos mittaamalla kaikki mukana olevat koepuutunnukset.

Kuvissa 6a ja 6b on tarkasteltu yläläpimitan poikkeaman vaikutusta otantamenetelmien tehokkuuteen. Jos puujoukon yläläpimitta ei poikkea lainkaan a priori mallilla saadusta (kuva 6a), on mittausyhdistelmä (d,h,h_c) tehokkain aivan alhaisia otantakustannuksia lukuunottamatta, jolloin koepuita ei kannata mitata. Jos yläläpimitan poikkeama on 6 mm (kuva 6b) on mittausyhdistelmä (d,h,h_c) edelleen tehokkain mittauskustannuksen ollessa alle 0.06 m^3; tätä suuremmilla mittauskustannuksilla yhdistelmä d,h,d_6,h_c antaa parhaan tuloksen.

Kuvissa 7a ja 7b on tarkasteltu puujoukkojen välisen latvusrajan korkeuden eron ($s_b\hat{h}_c$) vaikutusta otantamenetelmien tehokkuuteen. Mitä enemmän latvusrajojen keskimääräiset korkeudet poikkeavat puujoukkojen välillä sitä edullisemmaksi tulee latvusrajan mittaus. Erot ovat kuitenkin verraten pieniä odotettavissa olevalla latvusrajojen korkeuksien vaihtelualueella.

Kuvaan 8 on laskettu yhdistettyyn koepuiden käyttöön perustuvia tilavuusestimaatin virheen ja mittauskustannuksen riippuvuuksia. Samassa kuvassa on esitetty myös tehokkaimmat mittausmenetelmät kuvasta 5. Parametrien arvot ovat samat kuin kuvassa 5. Vain pienillä otantakustannuksilla tulokset poikkeavat olennaisesti pelkkiin koepuihin perustuvan estimoinnin tuloksista. Pelkän rinnankorkeusläpimitan mittaus ei odotetusti johda koskaan edullisimpaan tulokseen yhdistettyä estimointia käytettäessä.

Tuloksia tarkasteltaessa on merkille pantavaa, etteivät Suomessa yleisimmin käytetyt mittausyhdistelmät (d,h) ja (d,h,d_6) ole optimaalisia missään tilanteessa. Latvusrajan mittauksen edullisuus perustuu kahteen seikkaan. Ensiksi latvusrajan kokonaisvirhevarianssista noin puolet on metsiköiden välistä varianssia, kun välisen varianssin osuus kuuden metrin läpimitan kokonaisvirhevarianssista on vain noin kymmenesosa. Toiseksi latvusrajan mittaus on suhteellisesti luotettavampaa ja halvempaa kuin yläläpimitan mittaus.

Kaava (6.3) ilmaisee kokonaiskustannuksen odotusarvon mittauskustannusten ja estimointivirheen aiheuttaman kustannuksen summana. Lausekkeella (6.4) on ilmaistu epälineaarisen ohjelmoinnin tehtävä, joka ratkaisemalla voidaan löytää kokonaiskustannuksen odotusarvon minimoiva koepuumittausten yhdistelmä. Jos tehtävää yksinkertaistetaan ja oletetaan, että kaikista koepuista mitataan samat koepuutunnukset, tehtävä voidaan ratkaista yksinkertaisena rajoittamattomana ääriarvotehtävänä.

Estimointivirheen painokertoimen c_4 suuruutta pyrittiin arvioimaan olettamalla, että puuston tilavuuden mittausta käytetään puuerän hinnan määrittämiseen. Tällöin voidaan olettaa, että parametri c_4 saa sellaisen arvon, että optimiratkaisussa puuerän myyjän/ostajan lisäkustannus yhden koepuun mittauksesta on yhtä suuri kuin odotettavissa oleva estimointivirheen vähenemisen aiheuttama tappion pieneneminen.

Kuvassa 10 osoitetaan milloin koepuista kannattaa mitata kuuden metrin läpimitta, kun myyjä/ostaja arvottaa estimointivirheen kustannuksen yhtä suureksi kuin tappion odotusarvo on siinä tapauksessa, että puuerän tilavuus ali/yliarvioidaan. Jos myyjä/ostaja on halukas ottamaan riskin, hän soveltaa alhaista parametrin c_4 arvoa ja päinvastaisessa tilanteessa korkeaa parametrin c_4 arvoa. Vastaavasti tulee yläläpimitan mittaus joko edullisemmaksi tai epäedullisemmaksi.

Olennaista olisi tietää, miten puujoukkojen välinen kapenemisero pienenee puujoukon koon kasvaessa. Kun puujoukko kasvaa käsittämään mallien perustana olevan puujoukon, kapenemisero tulee nollaksi eikä kapenemisen mittaus luonnollisestikaan kannata. Hyvin pienen puujoukon kapenemista ei taasen kannata mitata puujoukon vähäisen merkityksen vuoksi. Lisätutkimuksin olisi selvitettävä, millaisissa tilanteissa kapenemisero on niin suuri, että puukaupan toisen osapuolen kannattaa mieluummin panostaa lisäkoepuiden mittaukseen kuin suostua puuerän yksikköhinnan muuttamiseen.

Tarkastelu

Tutkimuksen keskeinen tulos on huomion kiinnittäminen koepuumittauksissa puujoukkojen välisten erojen selvittämiseen. Koepuiden käyttöön perustuvassa puuston tilavuuden estimoinnissa on tilavuusmalleihin otettava selittäviksi muuttujiksi puujoukkojen eroja kuvaavia tunnuksia tähänastisten puiden tilavuuden kokonaisvaihtelua kuvaavien tunnusten lisäksi. Tämä tarkastelutapa muuttaa käsitystä esimerkiksi kuuden metrin läpimitan mittauksen kannattavuudesta.

Esitetyt numeeriset tulokset ovat vain esimerkkejä menetelmän soveltamisesta. Kussakin sovellutuksessa on sekä vaihtelu- että kustannusparametrien arvot estimoitava tilanteeseen sopiviksi. Kokonaan vaille tarkastelua on jätetty koepuiden valintatavan vaikutus tulosten luotettavuuteen.

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