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194

THE ECONOMIC BASIS OF FOREST POLICY

METSÄPOLITIIKAN TALOUDELLISET PERUSTEET

Seppo Vehkamäki

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ACTA FORESTALIA FENNICA 194

THE ECONOMIC BASIS OF FOREST POLICY

A study on the goals and means of forest policy

Seppo Vehkamäki

Seloste

METSÄPOLITIIKAN TALOUDELLISET PERUSTEET

Tutkimus metsäpolitiikan tavoitteista ja keinoista

To be presented, with the permission of the Faculty of Agriculture and Forestry of the University of Helsinki, for public criticism in Auditorium XII of the University, Aleksanterinkatu 5, Helsinki on 23rd May 1986, at 12 o'clock noon

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The goal-setting in forest policy and the means available for achieving the desired goal of forest policy are examined in this study. Examination of the goal is done using a macromodel of a closed economy. In the model GNP is assumed to be linearily dependent on the supply of raw wood. The model is used to derive the marginal conditions of the optimum equilibrium forestry with respect to the growing stock and the silviculture. The effect on the forest owners' behaviour of the following means are examined: the taxation of "pure income" from forestry, the taxation of income from selling raw wood, unit sales taxation and *ad valorem* sales taxation of forestry and corresponding sales subsidies, the support of silvicultural investment and the channelling of income from wood sales into silvicultural investments. The marginal conditions have been defined according to the maximum principle. An empirical study concerning the raw wood market in the case of softwood logs, and silvicultural investments in the case of young stand tending, has been carried out on the basis of the theoretical examination.

Tutkimuksessa tarkastellaan metsäpolitiikan tavoitteen asettamista ja erilaisten metsäpolitiikan keinojen vaikutusta metsänomistajien käyttäytymiseen. Tavoitteen asettamista on tutkittu teoreettisesti suljetun talouden makromallilla, jossa bkt:n oletetaan riippuvan lineaarisesti raakapuun tarjonnasta. Mallin avulla on johdettu metsätalouden optimitasapainon marginaaliehdot puuston ja metsänhoidon tason suhteen. Seuraavien metsäpolitiikan keinojen vaikutusta metsänomistajien käyttäytymiseen on tarkasteltu: metsätalouden "puhtaan tulon" verotus, raakapuun myyntitulojen erilaiset veromuodot, metsänhoitotöiden avustaminen ja lainoittaminen julkisista varoista ja raakapuun myyntitulojen ohjaaminen metsänhoitotöihin. Puuston ja metsänhoidon marginaaliehdot on määritelty maksimiperiaatteella. Empiirinen tarkastelu, joka koskee havutukkimarkkinoita ja taimikon hoitoa, on suoritettu teoreettisen tarkastelun pohjalta.

Key words: Forest ownership, wood supply, forest taxation, silvicultural investment, state subsidies
ODC 903+91+92

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Preface

This study was carried out in order to gather information of the economic basis of forest policy underlying the Finnish forestry. The topic of the study was initiated by Professor Päiviö Riihinen, whose contribution has been decisive to the completion of the study.

Valuable advice and statistical material was received from Dr. Veli-Pekka Järveläinen, Professor Matti Keltikangas, Messrs. Jari Kuuluvainen, Mikko Tervo and Ilpo Tikkanen. The original manuscript was translated by Messrs. John Derome and Vesa Kuittinen. In various stages of the manuscript assistance was received from Mrs. Raine Kamppinen, Mr. Kimmo Kekäläinen, Ms. Liisa Nikula, Mrs. Vappu-Tellervo Osmoviita and, in particular, Ms. Heli Suutarinen. The Academy of Finland offered financial support.

I express my gratitude to all those who were involved in and contributed to this study.

Helsinki, March 1986

Seppo Vehkamäki

List of symbols

The symbols used to depict the various variables, constants and functions are presented in the order:

1) Latin letters and

2) Greek letters.

Separately for the introduction and for the theoretical part of the study (Sections 1–4) and for the empirical part of the study (Section 5).

Sections 1–4

1) Latin letters

Symbol	*Explanation*
A	– coefficient vector of the first degree terms in the logistic growth function
a	– elements of the vector A
B	– coefficient matrix of the quadratic form in the logistic growth function
b	– elements of the matrix B
C	– consumption
D	– loan stock of forestry
e_s	– price elasticity of the raw wood supply
f	– multiplier of the financing support
G	– growth of the forests
$\mathcal{H}$	– Hessian matrix
$\mathscr{h}$	– Hamiltonian function
H	– cutting, supply of raw wood
I	– investment
K	– capital stocks or the silvicultural stock
k	– depreciation rates of the capital stocks
L	– Langrangean function
m	– shadow prices of stocks
p	– price of raw wood
Q	– gross national product, GNP
q	– price of forest
R	– taxable income of the forest owners
r	– rate of time preference of the forest owners
S	– area of the forests
s	– net sales of the forests
T	– rest of the generation time
t	– time
V	– growing stock
Y	– production function in the primary industry of the forest owners

2) Greek letters

α (alpha)	– raw wood input required per GNP unit
β (beta)	– rate of change in the price of raw wood
γ (gamma)	– silvicultural investment rate per cutting unit presribed by the government
ϑ (eta)	– *ad valorem* sales tax and *ad valorem* sales subsidy
θ (theta)	– coefficients of the linear tax function of the growing stock
ι (iota)	– interest rate of the loans of forestry
$\varkappa$ (kappa)	– share of self-financing in forestry
λ (lambda)	– shadow price of instrument variable constraints
ν (ny)	– characteristic root of the Hessian matrix
ξ (xi)	– mariginal rate of substitution between the growing stock and the silviculture
π (pi)	– amortization rate of the loans of forestry
ϱ (rho)	– rate of time preference of the government

τ (tau)	– taxable income from forestry
$\bar{\tau}$	– taxable mean growth of the forest
φ (phi)	– tax function of the income of the forest owners
χ (chi)	– unit sales tax and unit sales subsidy
ψ (psi)	– tax function of the stocks of the forestry

Section 5

1) Latin letters

a	– elasticities of the equilibrium stumpage price
a'	– elasticities of the stumpage price during the adjustment phase
b	– elasticities of the equilibrium raw wood supply
b'	– elasticities of the raw wood supply during the adjustment phase
c	– elasticities of the equilibrium silvicultural investment
c'	– elasticities of the silvicultural investment during the adjustment phase
d	– Durbin-Watson statistic
F	– F-statistic
G	– growth of the forests
H	– raw wood (saw timber) supply (log.)
h	– Durbin h-statistic
I	– silvicultural investment (the area of the young stand tending)
K	– silvicultural stock
p	– stumpage price (log)
N	– number of the periods to 95 % adjustment
n	– number of observations
R^2	– adjusted multiple correlation coefficient, adjusted coefficient of multiple determination
s	– time (year) as a variable
s^2	– residual variance
t	– time (year) as an index of the observation period
u	– rate of unemployment (log.)
v	– autocorrelated residual term
V	– growing stock
w	– end product price of the wood-working industry (log.)

2) Greek letters

δ_1 (delta)	– adjustment elasticity of the stumpage price
δ_2	– adjustment elasticity of the raw wood supply function
δ_3	– adjustment elasticity of the investment function
δ_i'	– elasticity of the lagged endogeneous variable
ε (epsilon)	– stochastic residual term
ϱ (rho)	– autoregressive coefficient of the first order

1. Introduction

1.1. Background to the study

Since the beginning of the 1960's, considerable attention has been paid in Finland to the drawing up and implemantation of forestry programmes. The basic concept used in drawing up such programmes has been that economic development is highly dependent on the amount of raw wood available, and that Finnish forestry has to be intensified if the maximum degree of well-being is to be achieved. The target of the programmes has been obtained from estimates based on the growth capacity of Finnish forests. The means which the government has used in putting the programmes into effect are associated with the arrangements used in financing investments in forestry. Investments in forestry have increased considerably in Finland during the 1960's and 1970's. The growth of forests has become capital-dominated and their character of a natural resource has decreased. By regulating investments in forestry, the public sector has attempted to strengthen the supply of raw wood. The programming of forestry in Finland, as well as the question which arise

- how a forestry goal, favourable from the point of view of the government, can be determined and
- by which economic policy means and in what way can the forestry behaviour of the forest owners be affected

have provided the motive of this study. The goal is the creation of instruments for assessing forest policy – instruments suited for examining the targets of forest policy and assessing the effectiveness of the various means available for practising forest policy.

1.2. The concept of forest policy

National economy can be defined as a group of people and organizations whose activity is aimed at satisfying the needs of the members of the group. The activity and organisation of such an economy can be illustrated by means of two element groups. The first group consists of elements which constitute the structure of the national economy, such as biological, technical, psychological, institutional and international elements which are taken as given in the activities of an economy. Biological and technical production functions, human preferences, restrictions set by economic competition, the taxation system, legislation, agreements etc. are examples of the first group of elements. The second group consists of elements describing the economic activity, in other words, economic variables, such as volume of production and consumption, prices, income and expenditure.

A national economy usually consists of public or private institutions which practise economic policy, in other words, try to make other institutions and members of the national economy act in accordance with their own aims. The most important public institution practising economic policy is the government. The economic policy of the government comprises the following stages:

1) setting of goals;
2) selection of the group of means required to attain the goals;
3) quantitative regulation of the means selected;
4) implementation and
5) follow-up.

The first three stages are called the planning of economic policy. In a democracy, the goals of the economic policy set by the government should represent the preferences of the members of society. The process by which a goal of the economic policy is shaped on the basis of individual preferences is called the rule of social choice. It has been proved, however, that there is no such goal of economic policy, based on the rule of social choice, which would necessarily reflect the preferences of all the members of a society

(HEAL 1974, pp. 29–59). Instead, either limits have to be set in the preferences of the members of society or else the members have to come to mutual agreement before a goal of the economic policy can be selected.

The means used to adapt the economy so as to conform with the goal can be divided into qualitative and quantitative means. The qualitative means of economic policy are used to affect the structure of the national economy. Selection of the group of means for quantitative economic policy is qualitative policy. Regulation of the means is quantitative economic policy.

The means of economic policy can also be classified on the basis of the way through which they affect the national economy. The means of fiscal policy exert their effect through the government budget, those of monetary policy through the money sypply and the interest rate in the national economy, those of income policy through prices and incomes etc. Depending on the intended effect of the economic policy on the national economy, economic policies can be divided into short-run and long-run economic policies.

The primary function of short-run economic policy is to regulate total demand within the framework of a given national economic performance. The primary function of long-run economic policy is, on the other hand, to improve the performance of the national economy: to aim at a long-run change, i.e. growth, which means the regulation of supply in the national economy.

The aim of the forest policy is to find those instruments of economic policy which affect the forestry behaviour (i.e. cuttings and investments) in such a way that the goals of economic policy can be attained. The means of forest policy may be those associated with fiscal policy, monetary policy or the direct regulation of the behaviour of forest policy. Forest policy is long-run economic policy in the sense that the stocks of forestry are always supposed to be variable.

1.3. Scope of the study

The starting point for this study is the national economy, which is divided into the following eight sectors

1) households
2) agriculture
3) wood-processing industry
4) other industry
5) forestry
6) public economy
7) financial institutions and
8) foreign countries.

The public economy, the government, acts as the controlling body which directs its economic policy with the aim of maximizing the utility of the entire national economy, its activities being restricted by budgets, balance of payments, appropriate distribution of income etc. The utility of a national economy is expressed as a function of the consumption of households and agriculture. Financial institutions channel accumulated private savings to the different sectors, and trade in foreign currency. Agriculture, the wood-processing and other industries as well as forestry, are responsible for the production of a national economy. Forestry production is biological production, whereas the harvesting of wood is regarded as part of the wood-processing industry. Forestry has been integrated horizontally with households, other sectors of production and the public economy and vertically with the wood-processing industry. There are no independent decision-making units in the forestry sector.

The following simplifications are made about the above described national economy for the purposes of assessing the goals of forest policy:

1) Agriculture is aggregated with the other industries, and the financial institutions are not taken into consideration.
2) The integration of forestry with the rest of the national economy is depicted by means of the linear dependence of GNP on the raw wood supply and by means of the silvicultural investment as an item in the total demand of the economy.
3) Industrial production in the national economy is homogeneous.
4) The economy has only one decision-maker, the government, which allocates the homogenous production to consumption and investments in the various sectors so that the utility of the government, expressed as a linear function of the consumption, is maximized.
5) A closed economy is examined.
6) The labour force is not included in the study.
7) The role of forests in man's physical environment is not taken into consideration.

The goal is to find the conditions of the growing stock and silviculture, which are optimum from the point of view of the government, to serve as the target of forest policy.

The applicability of certain means of forest policy for achieving the target of forest policy is then assessed using a model in which forestry has been integrated horizontally with one of the primary industries of the economic units. The optimum forestry behaviour of the economic units is first defined. The supply of raw wood in a situation of perfect competition are examined. The effect of certain governmental means of forest policy on the optimum equilibrium conditions in the forest-owners' economy and on the optimum forestry behaviour are then studied. The points under study are

- certain forms of income, sales and wealth tax of an economic unit practising forestry,
- subsidization of investments in forestry and
- channelling of income from the selling of raw wood directly to silvicultural investments.

When investigating the rules of optimum forestry behaviour and the effect of the means of forest policy, it is assumed that conditions of perfect competition hold as regards the supply of raw wood. In other words, the consideration is simplified by making the assumption that there are a great number of economic units practising forestry which act as decision-makers and which, because of the relative unimportance of each individual economic unit, have to take the government's economic policy measures and market impulses as given. The study is theoretical and deterministic. Biological, technical, psychological, institutional and economic hypotheses are made, and then presented as mathematical models.

Forestry behaviour, cuttings and silvicultural investments, is studied empirically on the basis of the theoretical examination. The study is limited by the scantiness of statistical material and the invariability of the Finnish forest policy instruments during the period for which there is statistical information available. The approach method has been selected so that it is from the forest owners' point of view possible consider cutting and silvicultural behaviour so separately as possible, not as institutionally or physically linked measures during the study period. The empirical study is, therefore, restricted to

- the examination of the market for softwood logs, especially their supply, and
- the examination of the effects of the tending of young stands and those of the financing conditions involved.

Finally the main results and methodological approach are summarized. In that connexion the literature of forest economics concerning the themes of this study is taken up for discussion. In the matter of the methodology special attention is paid to the tradition of optimum rotation and compared it with the approach of this study.

1.4. The biological production system of forestry

To start with, the biological production system of forestry is considered. The growth of forests constitutes a physical basis for the practice of forestry and forest policy. In this study, the growth of forests as an aggregate is assumed to be a strictly concave function of two stocks

- growing stock of the forests and
- the capital stock bound in the biological production of forests or silviculture.

The aggregate growth function [1] is written as

$$G(V(t), K(t)), \tag{1.1.}$$

---

[1] The aggregation of the growth function of forests is treated in Appendix.

where

$G(t)$ = growth (e.g. in cubic meters)

$V(t)$ = growing stock (e.g. in cubic meters) and

$K(t)$ = silviculture (expressed e.g. in monetary units).

(In order to simplify the notation the symbol of time t is omitted when that does not cause confusion.)

The growth function is a strictly concave function and has a maximum. The Hessian matrix formed from its second order derivatives is negative definite. The Hessian matrix can be presented in the form

$$(1.2)\quad \mathcal{H} = \begin{bmatrix} G_{vv} & G_{vk} \\ G_{kv} & G_{kk} \end{bmatrix}$$

the following assumptions about the elements are made

- tems G_{vv} and G_{kk} are always negative;
- the term representing the interactive effect of the growing stock and silviculture is positive, or it can be assumed to be zero in macroeconomic characterizations, which means that the growth function is separable with respect to the growth factors.

A growth function in which the interactive effect of the growing stock and silviculture is positive is called a general growth function in this study.

Growth constitutes the basis of the biological production system of forestry, which can be presented as a whole by means of the following equations:

$$(1.3)\quad \dot{V} = G(V, K) - H \text{ and}$$

$$(1.4)\quad \dot{K} = -kK + I,$$

where

$\dot{V}$ = time derivative of the growing stock,

H = cutting,

$\dot{K}$ = time derivative of the silviculture,

k = depreciation rate of the silviculture and

I = silvicultural investment.

Equations (1.3) and (1.4) depict changes in the stocks of foresty. Equation (1.3) is also called the forest balance.

The growth and depreciation rate of the silviculture are natural phenomena, endogeneous processes in the system. Cuttings and investments are economic events from the forest owners' point of view, and exogeneous events from the point of view of the biological production system. Forest owners remove wood for their own use through cuttings and invest their resources in forest growth.

Forestry can be said to be in equilibrium when

$$(1.5)\quad \dot{V} = G(V, K) - H = 0 \text{ and}$$

$$(1.6)\quad \dot{K} = -kK + I = 0,$$

i.e. when the cutting equal growth, and investment equal the depreciation of the silviculture. Forest growth can be controlled by regulating the stocks, growing stock and silviculture. The growing stock can be regulated by means of cuttings and silviculture by means of silvicultural investments.

The marginal rate of substitution between the growing stock and silviculture will be

$$(1.7)\quad \xi = \left(\frac{dV}{dK}\right)_{\substack{\text{constant}\\ \text{growth}}} = -\frac{G_k}{G_v},$$

i.e. the ratio which maintains the growth at the constant level. The entities G_v and G_k in ratio (1.7) represent the marginal growth in relation to the growing stock and silviculture.

Whenever the area of the forests is explicitly included in the consideration, the mean growth function calculated per unit area should be used

$$(1.8)\quad \bar{G} = \bar{G}(\bar{V}, \bar{K}) = \frac{G(V, K)}{S},$$

where

$\bar{G}(\cdot)$ = mean growth,

$\bar{V}$ = mean growing stock,

$\bar{K}$ = mean silviculture and

S = area of the forests.

A so-called logistic growth function is used as the macroeconomic specification of the growth of forests in this study. It is often written in the form (e.g. Braun 1978, p. 29)

$$(1.9)\quad G(V,K) = a_1V + a_2K + b_{11}V^2 + b_{22}K^2 + 2b_{12}VK.$$

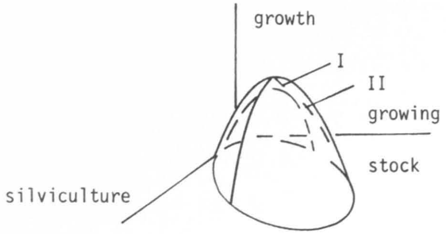

Fig. 1. Logistic growth function
I = growth curve of the constant growing stock
II = growth curve of the constant silviculture

If the above-mentioned system is in a natural state, it can be presented as a function of time in the form

$$(1.10)\quad G(V(t)) = a_1V + b_{11}V^2,$$

from which it can be concluded that when time increases unlimitedly, the forests will reach the steady state

$$(1.11)\quad \lim_{t \to \infty} V(t) = -\frac{a_1}{b_{11}} \text{ (Braun 1975, p. 30).}$$

The logistic growth function can be written in the vector form

$$(1.9')\quad G(V,K) = A' \begin{bmatrix} V \\ K \end{bmatrix} + [\,V\ K\,]\ B \begin{bmatrix} V \\ K \end{bmatrix}.$$

This is a quadratic function, about which we can make the following assumptions

- the quadratic coefficient matrix B (Hessian matrix of the growth function) is negative definite;
- vector A is positive;
- coefficients b_{11} and b_{22} are negative, but coefficient b_{12}, which represents the interactive effect of the growing stock and silviculture, is positive in the case of general growth function or zero in the case of separable growth function.

The logistic growth function is a geometrically downwards-opening paraboloid and its isoquants are ellipses. A schematic presentation of the logistic growth function is shown in Fig. 1. Figures 2 and 3 show the elliptic isoquants of the general and separable logistic growth function.

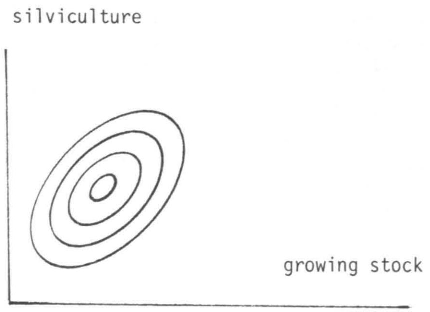

Fig. 2. Isoquants of the general logistic growth function.

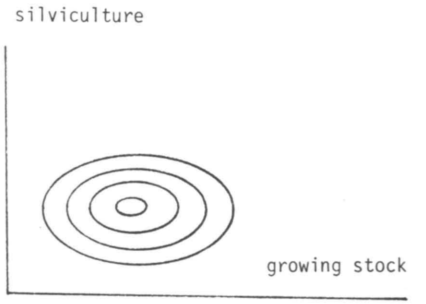

Fig. 3. Isoquants of the separable logistic growth function.

The marginal growths of the separable logistic growth function with respect to growing stock and silviculture are

$$(1.12)\quad G_v = a_1 + 2b_{11}V \text{ and}$$

$$(1.13)\quad G_k = a_2 + 2b_{22}K.$$

Empirically, the growing stock of the growth functions represents the wood of living trees. The silvicultural capital stock or silviculture is created as a result of various silvicultural measures. The capital stock of the biological production system of forestry is an index showing the silvicultural level. The silviculture is thus on aggregate entity consisting of measures like preparation of regeneration areas, seeding and planting, tending of young stand, forest fertilization, forest drainage and silvicultural cuttings. Cuttings, which are the means of regulating the growing stock and the removal of wood from the biological production system to the forest owners' economy, also have an investment aspect whenever they incur costs as a result of the development of the forests having been taken into account. The value of the silviculture can be calculated by means of labour, machine and material costs of the silvicultural measures. To be precise, the growth function should have the property that the soil alone, without the silviculture, would produce trees through natural regeneration. In this study, however, the natural regeneration is not assumed.

2. Determination of the government's target growing stock and silviculture

2.1. Model

A model has been drawn up in order to examine the optimum conditions of equilibrium forestry in order to determine the target growing stock and silviculture of the government. The biological production system of forestry is depicted in the model as a part of the national economy. The starting point of the model is the balance of resources in a closed economy.

(2.1) $Q = C + I_1 + I_2$,

where
Q = gross national product,
C = consumption,
I_1 = other than silvicultural investment,
I_2 = silvicultural investment.

The GNP is depicted using the production function

(2.2) $Q = Q(K_1)$,

where
K_1 = capital stock

which is a so-called well-behaved, strictly concave, production function, i.e.

$Q_k > 0$,
$Q_{kk} < 0$,
$Q_k \rightarrow \infty$, when $K_1 \rightarrow 0$ and
$Q_k \rightarrow 0$, when $K_1 \rightarrow \infty$.

A raw wood input, the magnitude of which is

α = raw wood input required per GNP unit,

is needed in the national economy to produce one GNP unit.

The dynamics of the model are depicted by the state equations of its stocks. The state equations for the capital stocks are

(2.3) $\dot{K} = - k_iK_i + I_i \ (i = 1,2)$,

where
k_i = depreciation rates of capital stocks
i = 1 capital stock of GNP
i = 2 silviculture.

The GNP affects the growing stock of the forests, the demand for raw wood generated by the GNP being realized as cuttings, and the growth of the forests. The state equation for the growing stock is then

(2.4) $\dot{V} = G(V, K_2) - \alpha Q(K_1)$.

The initial growing stock, silviculture and capital stock are given.

The object of the government is the maximization of exponentially discounted consumption

(2.5) $\max \int_{t=0}^{t=\infty} e^{-\varrho t} \ C(t)dt$

where
ϱ = positive rate of time preference.

The significance of the length of the time horizon and the relevance of the infinite time horizon will be discussed later in Section 6.

The government has two instrument variables available for solving the optimization problem

1) investment in the GNP of the national economy and
2) silvicultural investment,

in other words, the problem is the optimum allocation of GNP. The model is presented in diagrammatic form in Figure 4 on the next page.

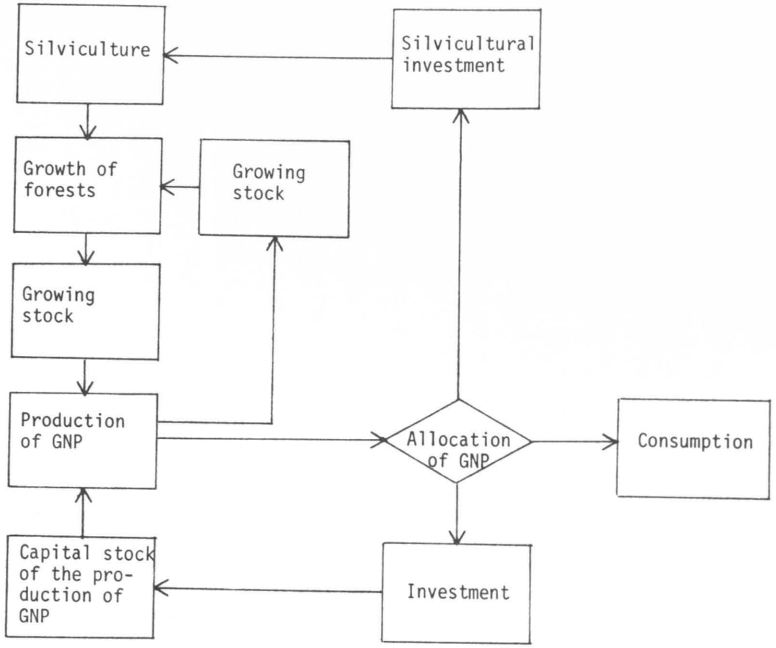

Fig. 4. Schematic presentation of the model for the allocation of GNP.

2.2. Derivation of the target stocks of the government's forest policy

Optimization

In order to set the target stocks of forest policy, the conditions of the optimum equilibrium of the national economy are solved from the above-mentioned problem. The problem is solved by means of the maximum principle (Intriligator 1971, pp. 344–369). According to this principle, the Hamiltonian function is formed from the object function and the state equations of the stocks

$$(2.6)\quad \mathcal{H} = e^{-\varrho t}C + \sum_{i=1}^{2} m_i\,(-k_iK_i + I_i) + m_3\,(G(V,K_2) - \alpha Q(K_i)),$$

where

$$(2.7)\quad C = Q(K_1) - I_1 - I_2.$$

The variables m_i of the Hamiltonian function indicate the margin effect of the corresponding stock variables on the object function (Dorfman 1969, p 821). The Hamiltonian function is the sum of two components:

1) direct consumption, which is expressed by means of the term $e^{-\varrho t}C$, and
2) consumption postponed to the future, expressed as a product of the variables m_i and the state equations of the corresponding stocks.

The variables m_i can be called the shadow prices of the stocks, due to the fact that they are derived from their future contribution to the object function.

In the economic literature it has often been paid attention to the current-value Hamiltonians

$$\tilde{\mathcal{H}} = e^{\varrho t}\,\mathcal{H}$$

and on the current-value shadow prices

$$\tilde{m}_i = e^{\varrho t}m_i$$

rather than on the discounted shadow prices m_i (Clark 1976, p. 105).

Solving the optimum conditions by applying the maximum principle results in the instrument variables I_i being chosen such that the sum of direct consumption and the consumption postponed to the future is maximized subject to the politically determined instrument variable boundary

$$(2.8)\quad \hat{I}_i \geqq I_i \geqq \check{I}_i.$$

The constraints are chosen so that minimum consumption and environmental requirements are taken into consideration.

The conditions of the concavity of the Hamiltonian function scan be examined by means of the characteristic equation of the Hessian matrix (the matrix of the second-order partial derivatives) derived from the Hamiltonian function (2.6) with respect to the stock and instrument variables. The roots of the characteristic equation are

$$(2.9)\quad v_1 = (e^{-\varrho t} - \alpha m_3)\,Q_{kk},$$

$$(2.10)\quad v_2 = m_3G_{kk},$$

$$(2.11)\quad v_3 = m_3G_{vv} \text{ and}$$

$$(2.12)\quad v_4 = v_5 = 0.$$

Because all roots are negative or zero in accordance with the assumptions made about the shape of the production and growth functions provided that

$$(2.13)\quad e^{-\varrho t} - \alpha m_3 \geqq 0 \text{ and}$$

$$(2.14)\quad m_3 \geqq 0,$$

the Hessian matrix is a negative semidefinite and the Hamiltonian function (2.6) concave (Intriligator 1971, p. 496). The conditions (2.13 and 2.14) mean that the concavity requirement of the Hamiltonian function (2.6) presupposes a non-negative shadow price of the GNP and growing stock[2].

Because the Hamiltonian function (2.6) is defined as a concave and differentiable function in instrument and stock variables and the instrument variable constraints (2.8) restrict a convex set, the necessary conditions of the maximum are also the sufficient conditions (Seierstad-Sydsaeter 1977, pp. 367–370, Theorem 2., and pp. 384–385, Theorem 10.)[3]. Because the Hamiltonian function is linear in the instrument variables the optimum solution is a combination of the singular and bang-bang solutions (Kamien-Schwartz 1981, pp. 186–198).

The Lagrangian function

$$(2.15)\quad L = \mathcal{H} + \sum_{i=1}^{2} \hat{\lambda}_i(\hat{I}_i - I_i) + \sum_{i=1}^{2} \hat{\lambda}_i(I_i - \check{I}_i)$$

is determined in order to carry out the optimization. In (2.15) the variables $\hat{\lambda}$ and $\check{\lambda}_i$ are the shadow prices of the investment constraints.

The optimum solution satisfies

a) the state equations (2.3) and (2.4),
b) the shadow price (costate, auxiliary, adjoint) equations

$$(2.16)\quad \dot{m}_1 = -L_{K_1} = m_1k_1 + m_3\alpha Q_k - e^{-\varrho t}Q_k,$$

$$(2.17)\quad \dot{m}_2 = -L_{K_2} = m_2k_2 - m_3G_k \text{ and}$$

$$(2.18)\quad \dot{m}_3 = -L_V = -m_3G_v,$$

c) the optimum conditions of investments

$$(2.19)\quad L_{I_i} = m_i - e^{-\varrho t} + \hat{\lambda}_i + \check{\lambda}_i = 0,$$

---

[2] It is to be noticed that linear functions are concave and convex, but not strictly, the negative of a strictly convex function is strictly concave, the non-negative weighted sum of concave functions is concave (e.g. Intriligator 1971, pp. 460–464 and Kamien-Schwartz 1981, pp. 260–264). Moreover, the shadow prices of stocks may assume any sign, when the state equations are linear (e.g. Kamien-Scwartz 1981, p 123 and p. 204).

[3] The sufficieney conditions are also considered in Kamien-Schwartz (1981), Sections 3 and 15.

(2.20) $\hat{\lambda}_i \geqq 0$ and $\check{\lambda}_i \geqq 0$,

(2.21) $\hat{\lambda}_i(\hat{I}_i - I_i) = 0$ and $\check{\lambda}_i(I_i - \check{I}_i) = 0$

or equivalently the optimum investments are

(2.22) $I_1 = \begin{Bmatrix} \hat{I}_1 \\ \dot{I}_1 \\ \check{I}_1 \end{Bmatrix}$ whenever (2.23) $m_1 \gtreqless e^{-\varrho t}$ and

(2.24) $I_2 = \begin{Bmatrix} \hat{I}_2 \\ \dot{I}_2 \\ \check{I}_2 \end{Bmatrix}$ whenever (2.25) $m_2 \gtreqless e^{-\varrho t}$,

where the conditions (2.23) and (2.25) are called switching functions, because according to them the investments are switched from an investment boundary to another in order to reach the equilibrium (steady, stationary) state. When the unequality signs of the switching functions are prevailing, some of the investment constrains $\hat{I}_i$ or $\check{I}_i$ are effective. The term blocked interval has been used to refer any time interval during which the equilibrium path of the instrument and stock variables cannot be followed. The design of the investment boundary determines how quickly the equilibrium state is reached or whether it is reached at all. If the investment boundary is set so that the singular solution lies outside of the boundary the equilibrium state is never reached (Clark 1976, pp. 56–57).

Conditions of the target stocks

The optimum equilibrium stocks, the target stocks of the government's policy, are solved as a singular solution from the above-mentioned conditions (e.g. Kamien-Schwartz 1981, pp. 193–198). The equilibrium stocks are supposed to be an interior solution with respect to the investment constraints. The shadow prices of the capital stock and silviculture in the optimum equilibrium are from (2.23) and (2.25)

(2.26) $m_i = e^{-\varrho t}$

The shadow price of the growing stock in the equilibrium, i.e.

$\dot{V} = 0$ and G_v = constant

can be determined by integrating (2.18), when

(2.27) $m_3 = \bar{m}_3 e^{-G_v t}$,

where
$\bar{m}_3 = m_3(0)$.

By inserting (2.27) into (2.16) and (2.17) and equating them with the time derivative of (2.26) the marginal conditions of the optimum equilibrium stocks in the economy are

(2.28) $Q_k = \dfrac{\varrho + k_1}{1 - \alpha \bar{m}_3}$,

(2.29) $G_k = \dfrac{\varrho + k_2}{\bar{m}_3}$ and

(2.30) $G_v = \varrho$,

where the equilibrium shadow price of the growing stock is determined as

(2.31) $\bar{m}_3 = \dfrac{\varrho + k_2}{G_k} = \dfrac{Q_k - (\varrho + k_1)}{\alpha Q_k}$.

By using the separable logistic growth function

(2.32) $\dot{K}_2 = \dfrac{1}{2b_{22}} \left(\dfrac{\varrho + k_2}{\bar{m}_3} - a_2\right)$ and

(2.33) $V^* = \dfrac{1}{2b_{11}} (\varrho - a_1)$

are the target stocks of the goverment's forest policy.

Graphical examination of the equilibrium price of the growing stock

The significance and the solution of the equilibrium (shadow) price of the growing stock and the significance of the fulfilment of the condition (2.31) can be examined graphically. The two equations (2.31) contains three unknowns G_k, Q_k and $\bar{m}_3$ that are to be solved from them. The graphs of the optimum equilibrium cutting are drawn in the upper right-hand quadrant of Figure 5 (next page) as a function of the price $\bar{m}_2$ ($\bar{m}_3 < \frac{1}{\alpha}$). The graphs are virtual supply curves (their derivation is presented in Section 3.2). The graph of the marginal GNP is presented as a function of the price of the growing stock in the upper left-hand quadrant. The correspondence between GNP and the marginal production is presented in the lower left-hand quadrant. The linear dependence of GNP on the raw wood supply is presented in the lower right-hand corner.

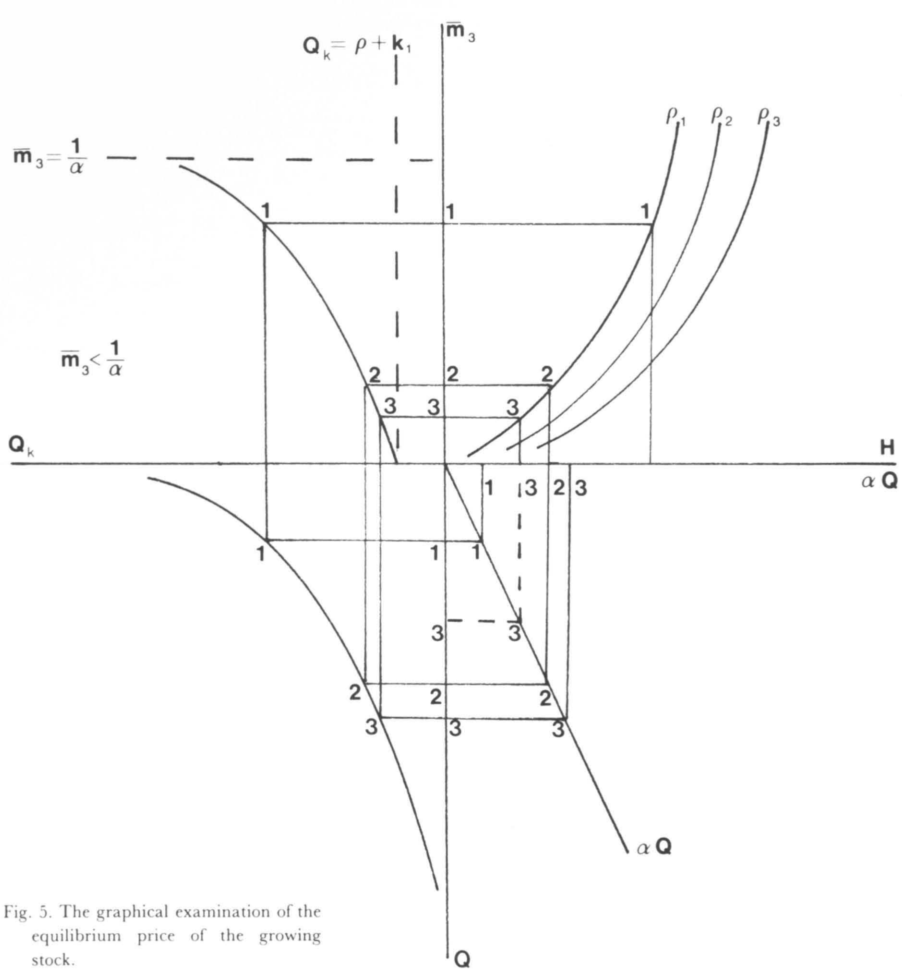

Fig. 5. The graphical examination of the equilibrium price of the growing stock.

If the price of the growing stock is at the level 1, then the optimum equilibrium cutting would be H_1 but the wood consumption in the production of GNP is only αQ_1. The price of the growing stock brings about a "state of oversupply" of raw wood in the economy and, owing to the cutting saving $H_1 - \alpha Q_1$, forestry moves into a state of imbalance. If the price of the growing stock is at level 2, then forestry is in the optimum equilibrium of the economy. Wood consumption and wood supply are equal and GNP is at its optimum. GNP has increased in comparison to the price at level 1 by $Q_2 - Q_1$. If, for instance, the price of the growing stock falls from level 2 to level 3, then the supply of raw wood falls to level H_3 and becomes a factor restricting GNP.

The marginal conditions of the optimum equilibrium forestry have been used to determine

- the target growing stock and
- the target silviculture

of forestry. The procedure has been based on the idea that the goal of forest policy should be set endogeneously at the same time as that of other economic policy. The GNP in model is linearly dependent on the supply of raw wood, while on the other hand the growth of the forests is dependent on the silvicultural investments which the government has allocated to forestry out of the GNP.

3. Optimum cutting and investment behaviour of forest owners

3.1. Definition

Model

The optimum forestry behaviour is defined subject to very general economic conditions. The definition is based on an aggregate of forest owners which is identical with respect to the household management conditions and which meets the conditions of perfect competition

- homogeneous product,
- perfect information,
- maximization of consumption,
- atomistic competition and
- free movement of resources.

Suppose that the object function of the forest owners is

$$\max \int_{t=0}^{t=T} e^{-rt} C dt, \tag{3.1}$$

where

C = consumption,

r = rate of time preference of the forest owners and

T = rest of the generation time of the forest owners.

The forest owners optimize their object function subject to the following income constraint

$$Y(K_1) + pH = C + I_1 + I_2 \tag{3.2}$$

where

$Y(\cdot)$ = well-behaved production function of the forest owners' primary industry (e.g. agriculture);

K_1 = capital stock in the forest owners' primary industry;

p = real price index of raw wood (price index of the primary industry = 1);

H = cutting, supply of raw wood;

C = consumption;

I_1 = investment in the primary industry and

I_2 = silvicultural investment

and subject to the following state equations of the stocks

$$\dot{K}_1 = -k_1K_1 + I_1, \tag{3.3}$$

$$\dot{K}_2 = -k_2K_2 + I_2 \text{ and} \tag{3.4}$$

$$\dot{V} = G(V, K_2) - H \tag{3.5}$$

where

k_1 = coefficient for the depreciation rate of the capital stock in the primary industry;

k_2 = coefficient for the depreciation rate of the silviculture

K_2 = silvicultural capital stock or silviculture

$G(\cdot)$ = growth function of the forest owners' forests and

V = growing stock in the forest owners' forests

and subject to the following initial and terminal conditions of the stocks:

$$K_1(0) = K_1', K_1(T) = K_1'', \tag{3.6}$$

$$K_2(0) = K_2', K_1(T) = K_2'' \text{ and} \tag{3.7}$$

$$V(0) = K', V(T) = V_1'', \tag{3.8}$$

The terminal stocks are bequests to the next generation, and are representing the intergenerational altruism in this consideration.

The real price of raw wood changes according to the equation

$$p = p(0)e^{\beta t}, \tag{3.9}$$

where $\beta \gtreqless 0$. In the model the treatment of the forests is dependent only on the current price of raw wood and current price change,

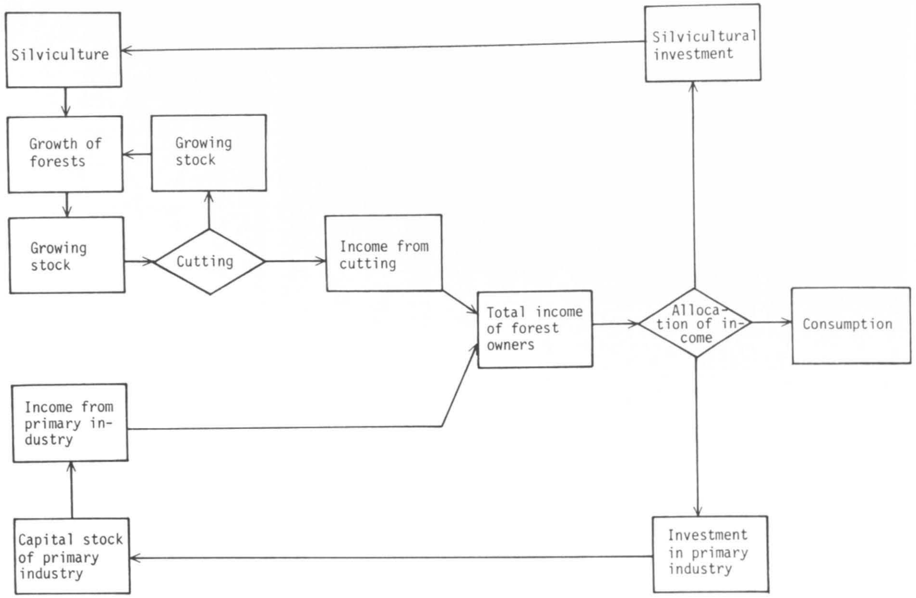

Fig. 6. Schematic presentation of the model for the allocation of the forest owners' income and growing stock.

not on predicted price changes. Future price changes are taken into account when they occur. Such a model is called myopic or it can be said that there are myopic price expectations in the model (Clark 1976, p. 75).

The following means are available to the forest owners at all times for achieving the goal

- investment and
- cutting

subject to the constraints

(3.10) $0 \leqq I_i \leqq \hat{I}_i$ and

(3.11) $0 \leqq H \leqq \hat{H}$.

The maximum investment is determined by the forest owners in accordance with their minimum concumption requirements. The maximum cutting obeys the institutionally determined environmental requirements.

The forest owners' problem is the optimum allocation of their income and growing stock. The problem is presented in diagrammatic form in Fig. 6 above.

Optimization

The Hamiltonian function of the above described problem is

(3.12) $$\mathcal{H} = e^{-rt}C + m_1(-k_1K_1+I_1) + m_2(-k_2K_2+I_2)+m_3(G(V,K_2) - H),$$

where
$C = Y(K) + pH-I_1-I_2.$

Because the Hamiltonian function is concave in instrument and stock variables and the instrument variable boundaries restrict a convex set, the necessary conditions of the maximum are also sufficient, and because it is linear in instrument variables the solution is a combination of the singular and bang-bang solutions as in the foregoing Section. Because of the finite time horizon the blocked intervals caused by the initial and terminal abjustment phases may overlap and the equilibrium state cannot be reached at all. However it is assumed that the equilibrium state can be reached. It must be emphasized that it is a numerical problem to determine when the equilibrium is reached or whether it is reached at all and when the equilibrium must be leaved in order to achieve the bequest stocks. Because there are one to one correspondence between the instrument and stock varibles in the problem it is possible to achieve the bequest stocks and to stay on them (Nyberg-Viotti 1978, pp. 73–81), provided that they are not in contradiction with the growth possibilities of the forests.

The optimum solution satisfies

a) the state equations (3.3)–(3.5),

b) the shadow price equations

(3.13) $\dot{m}_1 = m_1k_1 - e^{-rt}Y_k$,

(3.14) $\dot{m}_2 = m_2k_2 - m_3G_k$,

(3.15) $\dot{m}_3 = - m_3 G_v$ and

c) the optimum conditions of the instruments

(3.16) $I_i = \begin{Bmatrix} \hat{I}_i \\ I'_i \\ 0 \end{Bmatrix}$ whenever (3.17) $m_i \gtreqless e^{-rt}$, $i=1-2$,

and

(3.18) $H = \begin{Bmatrix} 0 \\ H^* \\ \hat{H} \end{Bmatrix}$ whenever (3.19) $m_3 \gtreqless pe^{-rt}$,

where the expressions (3.17) and (3.19) are called switching functions, and

d) the initial and terminal stock conditions (3.6) – (3.8).

Optimum equilibrium conditions

The equalities of the switching functions (3.17) and (3.19)

(3.17') $m_i = e^{-rt}$, $(i = 1-2)$ and

(3.19') $m_3 = pe^{-rt}$

are used in the derivation of the marginal conditions of the optimum equilibrium stocks. By equating (3.13)) with the time derivative of (3.17') it is got to the marginal condition of the optimum capital stock in the primary industry

(3.20) $Y_k = r + k_1$.

By inserting (3.19') into (3.14) and equating (3.14) with the time derivative of (3.17') it is obtained

(3.21) $G_k = \frac{r+k_2}{p}$ and

by equating (3.15) with the time derivative of (3.19')

(3.22) $G_v = r - \beta$.

The conditions (3.21) and 3.22) are interpreted as follows:

1) G_k, i.e. the marginal growth of the forest owners' forests which respect to the silviculture is, in an optimum equilibrium, equal to the ratio between the capital costs needed for accomplishing the marginal growth and the price of raw wood. Capital costs arise from

   r = rate of time preference and
   k_2 = depreciation rate of silviculture.

   Ceteris paribus it can be concluded that the higher the price of raw wood, the greater the capital stock because the growth function is concave. In an optimum equilibrium state, the investments are also the greater, the higher the price of raw wood. (3.21) can also be written in the form

   (3.23) $pG_k = r + k_2$,

   which indicates that, in an optimum equilibrium state, the value of the marginal growth of the forests is equal to the capital costs arising from the marginal growth.
2) G_v, i.e. the marginal growth of the forest owners' forests with respect to the growing stock is, in an optimum equilibrium, equal to their rate of time preference deducted by the relative change in the price of raw wood. The smaller

the marginal growth of forests as a function of the growing stock, the greater the optimum growing stock. If it is zero, the growing stock is the same as presupposed by the maximum growth. *Ceteris paribus* we can conclude that the smaller the absolute value of the marginal growth calculated with respect to the growing stock, the greater the cuttings equalling growth. The condition can also be interpreted to mean that, in an optimum equilibrium, the marginal growth with respect to the growing stock is equal to the rate of time preference, the effective value of which is the function of the forest owners' rate of time preference and the relative change in the price of raw wood.

Table 1. Optimum cutting and silvicultural investment combinations during the initial and equilibrium phase in linearized case.

Silvicultural investment condition	Cutting condition $\hat{G}_v > r-\beta$	 $\hat{G}_v = r-\beta$	 $\hat{G}_v < r-\beta$
$\hat{G}_k > \frac{\hat{G}_v+k_2}{pe^{-(\hat{G}_v-r)t}}$	$H = 0$ $I_2 = \hat{I}_2$	$H = H^*$ $I_2 = \hat{I}_2$	$H = \hat{H}$ $I_2 = \hat{I}_2$
$\hat{G}_k = \frac{\hat{G}_v+k_2}{pe^{-(\hat{G}_v-r)t}}$	$H = 0$ $I_2 = I_2^*$	$H = H^*$ $I_2 = I_2^*$	$H = \hat{H}$ $I_2 = I_2^*$
$\hat{G}_k < \frac{\hat{G}_v+k_2}{pe^{-(\hat{G}_v-r)t}}$	$H = 0$ $I_2 = 0$	$H = H^*$ $I_2 = 0$	$H = \hat{H}$ $I_2 = 0$

The condition (3.21) is a function of time. The optimum equilibrium of silviculture is dynamic because of the time-variant price of raw wood.

Solved from conditions (3.21) and (3.22), the optimum growing stock and silviculture become in the case of the separable logistic growth function

$$(3.24)\quad V^*(t) = V^* = \frac{b_{22}\,(r - \beta - a_1)}{2\det(B)}$$

$$(3.25)\quad K^*(t) = K^* = \frac{b_1\,(\frac{r + k_2}{p} - a_2)}{2\det(B)}$$

where symbol det (B) refers to the determinant of the coefficient matrix B of the logistic growth function.

Adjustment behaviour

The optimum adjustment behaviour from an arbitrary initial state of the stocks to the equilibrium state can be studied by using a linear approximation of the separable growth function in the vicinity of the equilibrium. In such a case the shadow price equations of the forestry are

$$(3.14')\quad \dot{m}_2 = m_2k_2 - m_3\hat{G}_k \text{ and}$$

$$(3.15')\quad \dot{m}_3 = -m_3\hat{G}_v,$$

where $\hat{G}_k$ and $\hat{G}_v$ are the coefficients of the silviculture and the growing stock in the linearized growth function. By solving the shadow prices from (3.14') and 3.15') it is obtained the functions of time only

$$(3.26)\quad m_2 = \bar{m}_2e^{k2t} + \bar{m}_3\frac{\hat{G}_k}{\hat{G}_v+k_2}(e^{-\hat{G}_vt} - e^{k2t}) \text{ and}$$

$$(3.27)\quad m_3 = \bar{m}_3e^{-\hat{G}_vt},$$

where $\bar{m}_2$ and $\bar{m}_3$ are the initial values of shadow prices which must be determined so that the equilibrium stocks are reached. By using the above expressions the switching functions (3.17) and (3.19) can be presented in the form

$$(3.28)\quad \bar{m}_2e^{k2t} + \bar{m}_3\frac{\hat{G}_k}{\hat{G}_v+k_2}(e^{-\hat{G}_vt} - e^{k2t}) - e^{-rt} \gtreqless 0 \text{ and}$$

$$(3.29)\quad \bar{m}_3e^{-\hat{G}_vt} - p\,e^{-(r-\beta)t} \gtreqless 0.$$

From the above switching functions can be concluded the optimum cutting and silvicultural investment combinations during the initial adjustment phase as listed in Table 1 above It is to be noticed that the conditions above in Table 1 differ slightly from the myopic conditions

$$(3.30)\quad G_v \gtreqless r - \beta \text{ and } G_k \gtreqless \frac{r + k_2}{p}\quad ,$$

which will be used later in this study. In the case $G_v = r$ they are indentical.

It should be remembered that the table covers the forest owners' forestry only. The growing stock and silviculture are directed towards an optimum equilibrium by regulating cutting and investment by means of the rules in Table 1.

3.2. Cutting and silvicultural investment behaviour as a function of constant price

In the following the price of raw wood is the given constant p and then

$$\beta = 0.$$

Suppose that in this static situation with respect to the price, growth and cutting match each other. The amount of wood supplies is then determined by the conditions

$$(3.31)\quad G_v = r,$$

$$(3.32)\quad G_k = \frac{r + k_k}{p},$$

$$(3.33)\quad \dot{V} = G(V, K_2) - H = 0 \text{ and}$$

$$(3.34)\quad \dot{K} = -k_2K_2 + I_2 = 0.$$

When the general logistic growth function is used as the specification of the growth of forests, conditions (3.31) and 3.32) can be written in the form

$$(3.35)\quad G_v = a_1 + 2b_{11}V + 2b_{12}K_2 = r \text{ and}$$

$$(3.36)\quad G_k = a_2 + 2b_{22}K_2 + 2b_{12}V = \frac{r + k_k}{p}$$

from which the growing stock and silviculture of an optimum equilibrium state can be solved with respect to the forest owners' rate of time preference and the price of raw wood. The optimum stocks become

$$(3.37)\quad V^* = \frac{b_{22}(r-a_1) - b_{12}\,(\frac{r+k_2}{p} - a_2)}{2\det(B)}$$

and

$$(3.38)\quad K_2^* = \frac{b_{11}(\frac{r+k_2}{p} - a_2) - b_{12}(r-a_1)}{2\det(B)}$$

Supply behaviour

When the optimum growing stock and silviculture are inserted into equation (3.33), the equation of supply becomes

$$(3.39)\quad H^* = G(V^*(r, p), K_2^*(r, p)).$$

The price elasticity of supply has been defined at each point of the supply curve by

$$(3.40)\quad e_s = \frac{dH^*p}{H^*dp}.$$

In the examined case the supply is

$$(3.41)\quad H^* = a_1V^* + a_2K_2^* + b_{11}\,V^{*2} + b_{22}K_2^{*2} + 2b_{12}V^*K_2^*$$

and its derivative with respect to the price

$$(3.42)\quad \frac{dH^*}{dp} = (a_1+2b_{11}V^*+2b_{12}K_2^*)\frac{dV^*}{dp} + (a_2+2b_{22}K_2^*+2b_{12}V^*)\frac{dK_2^*}{dp}$$

Since we can conclude from equations (3.37) and (3.38) that

$$(3.43)\quad \frac{dV^*}{dp} > 0 \text{ and } \frac{d^2V^*}{dp^2} < 0 \text{ and}$$

$$(3.44)\quad \frac{dK_2^*}{dp} > 0 \text{ and } \frac{d^2K_2^*}{dp^2} < 0 \text{ and}$$

$$(3.45)\quad \lim_{p\to\infty}\frac{dV^*}{dp} = 0 \quad\text{and}\quad \lim_{p\to\infty}\frac{dK_2^*}{dp^2} = 0$$

then the price elasticity of the raw wood supply decreases as the price increases. An increase in the price boosts the raw wood supply, but the higher the price level, the smaller the relative increase in supply resulting from an equal relative increase in the price. The above-mentioned supply curve is shown in Fig. 7 on the next page. The rate of time preference is given as the shift parameter of the supply curve, and the rate of time preferences, in order of magnitude, are:

$$r^1 > r^2 > r^3 = 0$$

Equations (3.37) and (3.38) indicate that

$$(3.46)\quad \lim_{p\to\infty,\, r\to 0} V^* = V^+ = \frac{(-b_{22}a_1 + b_{12}a_2)}{2\det(B)}$$

$$(3.47)\quad \lim_{p\to\infty,\, r\to 0} K^* = K^+ = \frac{(b_{12}a_1 - b_{11}a_2)}{2\det(B)}$$

which represent the growing stock and silviculture of maximum biological growth. A positive rate of time preference and limited

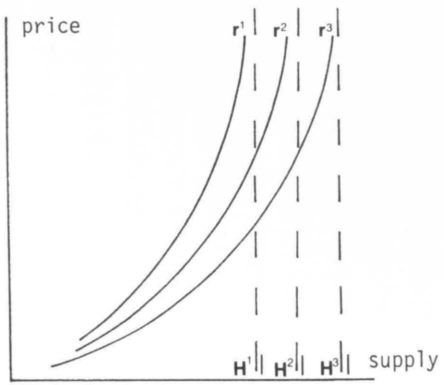

Fig. 7. The raw wood supply curve with rate of time preference r^1, r^2, r^3 ($r^1 > r^2 > r^3 = 0$). H^i represents the maximum supply at a given rate of time preference.

price decrease the supply from what it would be as defined by the biological production potential. Fig. 7 shows the supply resulting from maximum biological growth, H^3, which the supply curve at the rate of time preference $r = 0$ approaches asymptotically as the price increases. Other supply curves have their own maximum supply, based on their own rate of time preferences, which they approach asymtotically.

Silvicultural investment behaviour

Investment behaviour is investigated as a function of the constant price of raw wood by deriving an investment function which depicts the use of the forest owners' own money in financing investments in forestry. In a state of equilibrium, investments in silviculture equal the depreciation of the capital stock

(3.48) $I_2^\bullet = k_2 K_2^\bullet$.

When we suppose that the growth of forests has been specified with the general logistic growth function, then the silvicultural investments in the equiligrium state become on the basis of equations (3.38) and (3.48), as follows

(3.49) $$I_2^\bullet = k_2 \frac{b_{11}(\frac{r+k_2}{p} - a_2) - b_{12}(r - a_1)}{2\det(B)}$$

For the purposes of investigating the investment effect of the price of raw wood and the forest owners' rate of time preference in forestry, we define partial derivates

(3.50) $$\frac{dI_2^\bullet}{dp} = \frac{k_2^2 b_{11}}{2\det(B)\, p^2} \text{ and}$$

(3.51) $$\frac{dI_2^\bullet}{dr} = \frac{k_2}{2\det(B)} \left(\frac{b_{11}}{p} - b_{12}\right).$$

Equation (3.51) indicates, when

$b_{11} < 0$,

that the higher the price of raw wood, the more *ceteris paribus* forest owners invest in their silviculture. Equation (3.51) indicates that the greater the forest owners' rate of time preference, the less *ceteris paribus* they invest in their silviculture.

3.3. Cutting and silvicultural investment behaviour as a function of price changes

Let us now examine myopic economic behaviour in forestry in a dynamic adjustment situation when the price of raw wood is changing. For the sake of simplifying the examination suppose that the growth function of the forests is separable.

Suppose that the market has been in a static state of equilibrium, in other words, that

(3.52) $G_v = r$,

(3.53) $$G_k = \frac{r + k_2}{p}$$

(3.54) $\dot{V} = G(V^\bullet; K_2^\bullet) - H^\bullet = 0$ and

(3.55) $\dot{K} = -k_2 K_2^\bullet + I_2^\bullet = 0$.

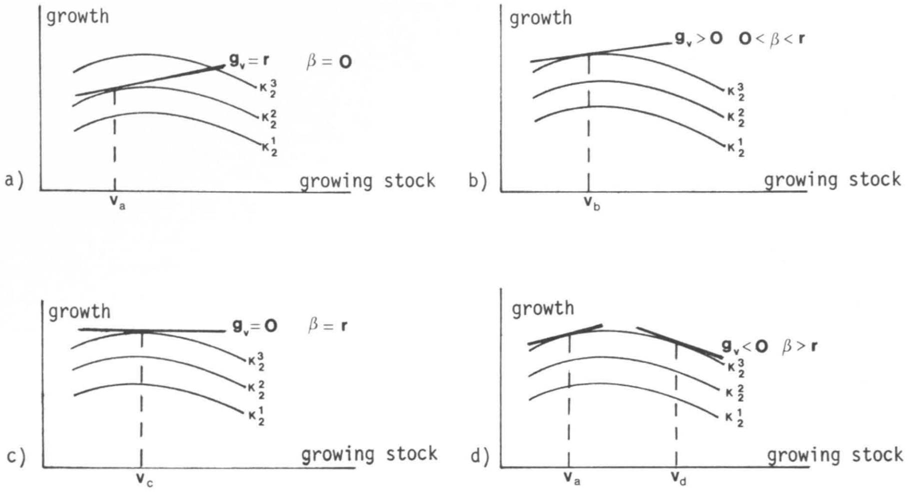

Fig. 8. Diagramme showing the effect of four different relative price increases on the growing stock and growth of forests. Movement from a lower growth curve to an upper one represents the silvicultural effect of a price increase. Curves K_2^1, K_2^2 and K_2^3 represent the growth of forests as a function of constant silviculture.

Price increase

The situation is as indicated in Fig. 8a. Suppose that the price of raw wood starts to increase at the rate β. The following holds true in the situation which prevails immediately after the price change

(3.56) $G_v > r - \beta$

(3.57) $$G_k > \frac{r + k_2}{p}.$$

The immediate reaction following the price change is that the growing stock is allowed to increase by omitting cutting and that maximum silvicultural investment is made. It is a question of a bang-bang situation. This immediately results in a decrease in the raw wood supply and an increase in the investments. As soon as the growing stock reaches the level conforming to the increase in the price, and the silviculture reaches the level conforming to the price at the given moment, we arrive at a situation where

(3.58) $G_v = r - \beta$,

(3.59) $$G_k = \frac{r + k_2}{p},$$

(3.60) $\dot{V} = G(V^\bullet; K_2^\bullet) - H^\bullet = 0$ and

(3.61) $\dot{K}_2 = -k_2 K_2^\bullet + I_2^\bullet = 0$,

and from this it can be concluded that the raw wood supply further increases through silvicultural condition (3.59), but, because of the shape of the growth function, the increase rate slows down as the price increases, and that the final effect of the increase through the growing stock condition depends on the magnitude of condition (3.58). If the relative increase in the price is such that

(3.62) $G_v = r - \beta > 0$, (Fig. 8.b),

then the cutting in conformity with the growth, i.e. the supply, increases with respect to the static state of equilibrium.

If, on the other hand, the price increase is such that

$$G_v = r - \beta = 0 \text{ (Fig. 8.c)}, \tag{3.63}$$

the growth and supply are at the level of the growing stock conforming to the maximum biological growth, and the cutting is at its maximum. If, again, the price increase is such that

$$G_v = r - \beta < 0 \text{ (Fig. 8.d)}, \tag{3.64}$$

then the optimum growing stock is greater than the growing stock conforming to the maximum biological growth and this results in cutting at a level below the maximum biological growth. Whether the supply will now increase in comparison to a static state of equilibrium or not, depends on the combined effect of the growing stock and silviculture on the growth.

What happens if the price increase stops, is presented in Figure 8.d. The cutting is then momentarily $V_d - V_a$ plus the growth and subsequently according to the growth factors K_2^3 and V_a.

Price decrease

Accordingly, if the price of raw wood starts to decrease at the rate β, then the result is a situation in which

$$G_v < r - \beta \text{ and} \tag{3.65}$$

$$G_k < \frac{r + k_2}{p}, \tag{3.66}$$

whic means that the optimum bang-bang reaction would be to decrease both the growing stock and the silvicultural level. This is accomplished by omitting investment and cutting. A decrease in the price, thus, immediately results in an increase in the supply. Again, when the growing stock reaches the level conforming to the price decrease, we come to a situation where

$$G_v = r - \beta, \tag{3.67}$$

$$G_k = \frac{r + k_2}{p}, \tag{3.68}$$

$$\dot{V} = G(V^*, K_2^*) - H^* = 0 \text{ and} \tag{3.69}$$

$$\dot{K}_2 = -k_2 K_2^* + I_2^* = 0. \tag{3.70}$$

It can be concluded from conditions (3.67) – (3.70) that cutting decreases, i.e. the supply continues to decrease since the optimum investment and silvicultural level decrease in conformity with silvicultural condition (3.68) when price is decreasing. The above can be summarized as follows:

a price increase

1) immediately during the bang-bang stage
   - decreases cutting and
   - increases silvicultural investment, and
2) in equilibrium
   - increases cutting and silvicultural investment

a price decrease

1) immediately during the bang-bang stage
   - increases cutting and
   - decreases silvicultural investment
2) in equilibrium
   - reduces cutting and silvicultural investment.

4. The effect of the means used in forest policy

4.1. The approach

In order to obtain some background for the use of the means of forest policy two cases are characterized. In the first case case it is question of

- *ex ante* planning of wood-processing copacity

in which the wood-processing industry and forest owners take part, through their organizations, together with the government. The role of the government in this case is to act as an auctioneer between wood-processing industry and forest owners. The aim of the goverment's forest policy is to affect the behaviour of forest owners in such a way that the cuttings consistent with the planned capacity of wood-processing industry are achieved.

In the second case the capacity of the wood-processing industry is given and in the use of the means of forest policy it is question of

- *ex post* stabilization of raw wood market.

The forest policy is aimed at influencing the raw wood supply and the selection of a short-run degree of capacity utilization of the wood-processing industry.

From the point of view of the wood-processing industry the first case is a long-run consideration and the second case a short-run consideration.

In the forestry the long-run behaviour cannot be distinguished from the shor-run behaviour, because forest owners can react only by regulating their stocks through cutting and silvicultural measures. It means that the same means of forest policy can be used in both cases mentioned above.

The purpose of the following study is to examine the effect of various means of forest policy on the behaviour of forest owners maximizing the present value of their consumption subject to the state equations of the stocks and the instrument variable constraints. The means of forest policy are taken as exogeneous variables or functions into account in the maximization task. Conclusions concerning the chosen means are drawn from their effects on the myopic marginal conditions of the optimum equilibrium stocks. The examination is a comparative statics analysis, although the price of raw wood is in the most cases used as a function of time. No reference is made to the time horizon and bequest stocks of the forest owners or to any blocked intervals. The means are examined one by one, and no combinations of means are included. The study covers two types of income tax: tax of a so-called "pure income" from forestry and sales income tax. The taxation of stocks in forestry is treated as a wealth taxation form. Two types of sales tax are included: ad valorem tax and unit tax. The financial support for silvicultural investments by the government is discussed, and the institutional channelling the sales income from forestry to silvicultural investments is treated.

The examination of the different means obey the following composition:

The definition of the means of forest policy
The determination of the consumption
The instrument variables
The stock variables and their state equations
The optimum equilibrium conditions
The conclusions

The optimum conditions of the myopic behaviour of forest owners are derived by aid of the maximum principle subject to the same mathematical conditions as in Section 3 on page 20. The results of the examination of the means are summarized in Table 3 on page 35 at the end of the Section 4.

4.2. Taxation

4.2.1. Income taxation

4.2.1.1. "Pure income" from forestry as the basis for determining income tax

Definition of the means

The taxable mean growth is priced using the current price of raw wood at a given moment. Let

p = constant price of raw wood
$\bar{t}$ = taxable mean growth and
S = area of the forest,

then the taxable income from forestry is

$$(4.1) \quad \tau = p\bar{t}S.$$

Taxes are based on the total income of forest owners, which is defined as

$$(4.2) \quad R = Y(K_1) + \tau$$

using a tax function

$$(4.3) \quad \varphi = \varphi(R)$$

with the property

$$\varphi_R > 0 \text{ and } \varphi_{RR} \geqq 0.$$

Determination of the consumption

The government is buying forests for the price

q = constant price of forest (forest land and growing stock),

and the net sales of forests to the government are

s = net sales of forests.

When the area of forest is used as an endogeneous variable and when the tax function and the income from the net sales of forests is inserted in the forest owners' income constraint, the consumption becomes

$$(4.4) \quad C = Y(K_1) + p\bar{H}S + qs - I_1 - \bar{I}_2S - \varphi(R).$$

The price of raw wood is taken as a constant.

Instrument variables

The forest owners' available instrument variables are

$0 \leqq I_1 \leqq \hat{I}$, investment in the primary industry,
$0 \leqq \bar{I}_2 \leqq \hat{I}$, mean silvicultural investment,
$0 \leqq \bar{H} \leqq \hat{H}$, mean cutting and
$\check{s} \leqq s \leqq \hat{s}$, net sales of forests.

Stock variables

There are four stocks in the forest owners' economy

K_1 = capital stock in the primary industry,
$\bar{K}_2$ = mean silviculture,
$\bar{V}$ = mean growing stock and
S = area of forests owned by forest owners

with the following state equations

$$(4.5) \quad \dot{K}_1 = -k_1K_1 + I_1,$$

$$(4.6) \quad \dot{\bar{K}}_2 = -k_2\bar{K}_2 + \bar{I}_2,$$

$$(4.7) \quad \dot{\bar{V}} = \bar{G}(\bar{V},\bar{K}_2) - \bar{H} \text{ and}$$

$$(4.8) \quad \dot{S} = s.$$

Optimum equilibrium conditions

The optimum equilibrium conditions in the forest owners' economy are

$$(4.9) \quad Y_k = \frac{r + k_1}{1 - \varphi_R R_Y}$$

for the primary industry provided that $1 - \varphi_R R_Y > 0$,

$$(4.10) \quad \bar{G}_k = \frac{r + Sk_2}{pS}$$

for the silviculture,

$$(4.11) \quad \bar{G}_v = \frac{r}{S}$$

for the growing stock and

$$(4.12) \quad \frac{p\,\bar{G}\,(\bar{V}^{*},\bar{K}_2^{*})}{q} = r + \frac{k_2\bar{K}_2^{*} + \varphi_R R_\tau \tau_s}{q}$$

for the forest ownership.

Conclusions

Conditions (4.9–4.12) indicate that this type of taxation affects the production, capital stock and investment of the primary industry and the area of forests owned by forest owners. If the "pure income" from forestry changes, then the taxable total income and marginal tax rate change. The only possibility for forest owners fo react to the new situation is to change the production and capital stock in their primary industry and the area of forests. The last of the conditions is of particular interest in this case. According to this condition, the ratio between the market value of the growth of the marginal area of forests and its selling price is equal to the forest owners' rate of time preference plus the ratio between the costs arising from taxes and the depreciation of silviculture of the marginal area and its selling price. The marginal income and marginal costs of forest ownership are equal. Suppose that *ceteris paribus* the bases of taxable income are raised, in other words, that the factor $\varphi_R R_\tau \tau_s$ or the marginal tax rate of forest ownership increases. The only possibility for forest owners to maintain their economy in an optimum state of equilibrium is to sell forest and thus reduce the effect of the marginal tax and bring the marginal conditions (4.12) into equilibrium.

If no sales of forests are allowed and the area of forest is exogeneous, the optimum equilibrium conditions become

$$(4.9') \quad Y_k = \frac{r + k_1}{1 - \varphi_R R_y},$$

$$(4.10') \quad G_k = \frac{r + k_2}{p} \text{ and}$$

$$(4.11') \quad G_v = r.$$

From (4.9'–4.11') it is very clear to see, that when the "pure income" from forestry, as defined in this study, is used as the basis for determining the taxable income, the resources of biological growth cannot be regulated. It is deceptive to use the term income tax when referring to the taxation of the "pure income" from forestry. In fact, it is a question of an area tax, a type of wealth tax.

4.2.1.2. Income from selling raw wood as the basis for determining tax

Definition of the means

The taxable income from forestry is determined as the sales income

$$(4.13) \quad \tau = p\,H$$

Taxes are based on the forest owners' total income

$$(4.14) \quad R = Y(K_1) + \tau$$

using a tax function

$$(4.15) \quad \varphi = \varphi(R)$$

with the property $\varphi_R > 0$ and $\varphi_{RR} \geqq 0$ [4].

Determination of the consumption

When the tax function is inserted in the forest owners' income constraint, the consumption becomes

$$(4.16) \quad C = Y(K_1) + p\,H - I_1 - I_2 - \varphi(R).$$

The price of raw wood is a function of time, $p = p(o)e^{\beta t}$.

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[4] In the case $\varphi_{RR} > 0$ the problem is non-linear with respect to cutting and the adjustment to the equilibrium state becomes gradual instead of the bang-bang adjustment.

Instrument variables

The forest owners' available instrument variables are

$0 \leqq I_1 \leqq \hat{I}_1$, investment in the primary industry
$0 \leqq I_2 \leqq \hat{I}_2$, silvicultural investment and
$0 \leqq H \leqq \hat{H}$, cutting

Stock variables

There are three stocks in the forest owners' economy

K_1 = capital stock in the primary industry,
K_2 = silviculture and
V = growing stock

with the usual state equations

$$(4.17)\quad \dot{K} = -k_1K_1 + I_1,$$

$$(4.18)\quad \dot{K}_2 = -k_2K_2 + I_2 \text{ and}$$

$$(4.19)\quad \dot{V} = G(V,K_2) - H.$$

Optimum equilibrium conditions

The optimum equilibrium conditions in the forest owners' economy are

$$(4.20)\quad Y_k = \frac{r + k_1}{1 - \varphi_R R_Y}$$

for the primary industry provided that $1 - \varphi_R R_Y > 0$,

$$(4.21)\quad G_k = \frac{r + k_2}{(1 - \varphi_R)p}$$

for the silviculture provided that $1 - \varphi_R > 0$ and

$$(4.22)\quad G_v = r - \beta$$

for the growing stock.

Conclusions

The use of sales income for determining the taxable income affects the forest owners' primary industry and silviculture. It can be concluded from conditions (4.20) and (4.21) that if the interaction between the growing stock and silviculture is positive, then the stocks, growth, cutting and investment in the optimum equilibrium state can be reduced by tightening taxation. Alleviations in taxation have the opposite effect. If the growing stock and silviculture have no interaction, this type of taxation does not affect the growing stock at all.

4.2.2. Taxation of stocks in forestry

Definition of the means

In this study we examine a case where tax is determined separately for the different types of wealth, in other words, forest owners are taxed separately for

- the silviculture and
- the growing stock.

The tax function of the silviculture is

$$(4.23)\quad \psi_2 = \psi_2(K_2)$$

with the property $\psi_k > 0$ and the tax function of the growing stock is

$$(4.24)\quad \psi_3 = \psi_3(V)$$

with the property $\psi_v > 0$.

Determination of the consumption

When the tax functions are inserted in the forest owners' income constraint, the consumption becomes

$$(4.25)\quad C = Y(K_1) + pH - I_1 - I_2 - \psi_2(K_2) - \psi_3(V).$$

Instrument and stock variables

The forest owners' instrument and stock variables are as in the preceding section, and the stocks obey the state equations (4.17–4.19).

Optimum equilibrium conditions

The optimum equilibrium conditions in the forest owners' economy are

$$(4.26)\quad Y_k = r + k_1$$

for the primary industry,

$$(4.27)\quad G_k = \frac{r + k_2 + \psi_k}{p}$$

for the silviculture and

$$(4.28)\quad G_v = r - \frac{\dot{p} - \psi_v}{p}$$

for the growing stock.

Conclusions

Because the marginal taxes are postive, a tightening of taxation results in a decrease in the silviculture, but the effect on the growing stock depends on price change.

The above-mentioned considerations can be examined by studying the logistic growth and linear tax function of the growing stock only. No tax is levied on other types of wealth. The tax function is

$$(4.29)\quad \psi_3 = \theta_0 + \theta_1 V$$

When we insert the partial derivative of the tax function in condition (4.28), the optimum growing stock and silviculture, in the case of the separable logistic growth function, become

$$(4.30)\quad V^{\bullet} = \frac{b_{22}\left(r - \frac{\dot{p} - \theta_1}{p} - a_1\right)}{2\det(B)} \text{ and}$$

$$(4.31)\quad K_2^{\bullet} = \frac{b_{11}\left(\frac{r + k_2}{p} - a_2\right)}{2\det(B)}$$

They indicate that the effect of a price change on the optimun condition of forests can be partly or wholly eliminated by means of the coefficient of the first begree term of the tax function. If the price is increasing, the coefficient must be positive, and if the price is increasing, the coefficient must be positive, and if the price is decreasing, the coefficient θ_1 must be negative so as to permit the elimination of the effect of price changes. In the last mentioned case the function (4.29) is a subsidy function. The constant term in the tax function can be used for regulating the level of taxation.

4.2.3. Sales tax

4.2.3.1. *Ad valorem* sales tax

Definition of the means

In the case a certain proportion of the price of each unit sold is paid as tax, the sales tax is determined on the ad valorem principle. The ad valorem tax imposed on the price of raw wood is denoted as

η = *ad valorem* tax rate.

Determination of the consumption

When the ad valorem tax imposed on the raw wood sales is inserted in the forest owners income constraint, the consumption becomes

$$(4.32)\quad C = Y(K_1) + (1-\eta)pH - I_1 - I_2.$$

Instrument and stock variables

The forest owners' instrument and stock variables are as in the section 4.2.1.2. and the stocks obey the state equations (4.17–4.19).

Optimum equilibrium conditions

The optimum equilibrium conditions in the forest owners' economy are

$$(4.33)\quad Y_k = r + k_1$$

for the primary industry,

$$(4.34)\quad G_k = \frac{r + k_2}{(1-\eta)p}$$

for the silviculture provided that $1 - \eta > 0$, and

$$(4.35)\quad G_v = r - \beta$$

for the growing stock.

Conclusions

Ad valorem sales tax does not affect the equilibrium in the primary industry. Conditions (4.34) and (4.35) indicate that a tightening of taxation reduces the silviculture, growth, cutting and investment of the target state in the case of a separable growth function. Alleviation of taxation has an adverse effect. In the case of the ad valorem sales subsidy the sign of the policy parameter is opposite in (4.32) and the conclusions are contrary to that of the ad valorem sales tax.

4.2.3.2. Unit sales tax

Determination of the consumption

In the case a tax is imposed per raw wood unit sold it is referred to as a unit tax. The unit tax is denoted as

χ = value of unit tax.

Determinatination of the consumption

When the unit tax imposed on the raw wood sales is inserted in the forest owners' income constraint, the consumption becomes

$$(4.36)\quad C = Y(K_1) + (p-\chi)H - I_1 - I_2.$$

Instrument and stock variables

The forest owners' instrument and stock variables are as in the section 4.2.1.2. and the stocks obey the state equations (4.17–4.19).

Optimum equilibrium conditions

The optimum equilibrium conditions in the forest owners' economy are

$$(4.37)\quad Y_k = r + k_1$$

for the primary industry,

$$(4.38)\quad G_k = \frac{r + k_2}{p - \chi}$$

for the silviculture provided that $p - \chi > 0$ and

$$(4.39)\quad G_v = r - \frac{\dot{p}}{p-\chi}$$

for the growing stock provided that $p - \chi > 0$.

Conclusions

The behaviour of the stocks according to conditions (4.38–4.39) in the different tax parameter and price change situations is shown in Table 2. on the next page.

From the table it is to be seen that in the situation of price increase the regulation of the unit sales tax have substitution effects between the growing stock and silviculture. The total effect of the forest policy on the growth and cutting depends on the growth function and its marginal rate of substitution (the equation (1.7)).

If the goverment pays forest owners

χ = subsidy for every raw wood unit sold

then the tax parameter has the opposite sign compared to the optimum conditions (4.38–4.39) and so the stock effects are opposite with the effects shown in Table 2.

Table 2. The effect of unit sales tax change on the optimum equilibrium growing stock and silviculture in the different price change situations. Symbol $\chi > \bar{\chi}$ represents a tightening in taxation and symbol $\chi < \bar{\chi}$ the opposite.

Tax parameter change	$\chi > \bar{\chi}$			$\chi < \bar{\chi}$		
Price change	$\beta>0$	$\beta=0$	$\beta<0$	$\beta>0$	$\beta=0$	$\beta<0$
Growing stock effect	>0	$=0$	<0	<0	$=0$	>0
Silvicultural effect	<0	<0	<0	>0	>0	>0

4.3. Support of silvicultural investments

Definition of the means

The government support silvicultural investments by financing them through the state budget on the following conditions (Forest Improvement Act):

$\varkappa$ = share of self-financing of silvicultural investments
ι = interest on loans
π = amortization rate of loans.

The budget constraint of the government has no effect on the behaviour of the forest owners. The use of governmental funds is regulated only by the financing conditions.

Determination of the consumption

When the financing conditions of the silvicultural investment are inserted in the forest owners' income constraint, the consumption becomes

$$(4.40)\quad C = Y(K_1) + pH - I_1 - \varkappa I_2 - (\iota + \pi)D,$$

where
D = loan stock.

Instrument and stock variables

The forest owners' instrument variables are as in the Section 4.2.1.2. The capital stock in the primary industry, the silviculture and the growing stock obey the state equations (4.17–4.19). The state equation of the loan stock is

$$(4.41)\quad \dot{D} = -\pi D + (1-\varkappa)I_2.$$

Optimum equilibrium conditions

The optimum equilibrium conditions in the forest owners' economy are

$$(4.42)\quad Y_k = r + k_1$$

for the primary industry,

$$(4.43)\quad G_k = f\,\frac{r + k_2}{p}$$

where

$$(4.44)\quad f = \varkappa + \frac{(1-\varkappa)\,(\pi+\iota)}{\pi + r}$$

the influence of the financing conditions on the forest owners' silvicultural behaviour, and

$$(4.45)\quad G_v = r - \beta$$

for the growing stock. It is provided that

$$0 \leqq f < 1.$$

Conclusions

An investment function which depicts silvicultural investment as a function of the financing conditions of the loans offered by the government is derived. The price of raw wood is constant and the growth of forests is specified with a separable logistic growth function, then, on the basis of equations (1.13) and (4.43), it is obtained

$$(4.46)\quad f\frac{r+k_2}{p} = a_2 + 2b_{22}K_2^{*}.$$

On the other hand, on the basis of equations (1.12) and (4.45) and above-mentioned facts, the following holds

$$(4.47)\quad r = a_1 + 2b_{11}V^{*}$$

When the optimum equilibrium states of the growing stock and the silviculture are solved from the last two equations, it is obtained

$$(4.48)\quad V^{*} = \frac{b_{22}(r-a_1)}{2\det(B)} \text{ and}$$

$$(4.49)\quad K_2^{*} = \frac{b_{11}\left(f\frac{r+k_2}{p} - a_2\right)}{2\det(B)}$$

In a state of optimum equilibrium, silvicultural investment equals the depreciation of the silviculture, i.e. the silvicultural investment function is

$$(4.50)\quad I_2^{*} = k_2\frac{b_{11}\left(f\frac{r+k_2}{p} - a_2\right)}{2\det(B)}.$$

The effect of the change in the self-financing share is considered as an example. From (4.44) and (4.50) it can be derived

$$(4.51)\quad \frac{dI_2^{*}}{d\varkappa} = k_2\frac{b_{11}\, f\frac{r+k_2}{p}}{2\det(B)}\left(1-\frac{\pi+\iota}{\pi+r}\right),$$

from which it can be concluded that

- if $\iota < r$, then the decrease of the self-financing share will raise the silvicultural investment and vice versa.
- if $\iota = r$, then the change of the self-financing share has no effect on the silvicultural investment,
- if $\iota > r$, then the decrese of the self-financing share will decrease the silvicultural investment,
- if $\pi = \iota = 0$, i.e. free financing of the government is available, then the silvicultural investment can be regulated by aid of the self-financing share only and the greater the self-financing share the smaller investment.

4.4. Channelling income from raw wood sales to silvicultural investment

Definition of the means

The government channels the income from raw wood sales to financing silvicultural investment (Act concerning Private Forests). The situation is depicted by the state equation of the silviculture

$$(4.52)\quad \dot{K}_2 = -k_2K_2 + I_2 + \gamma H,$$

where
γ = silvicultural investment rate per cutting unit presribed by the government.

Determination of the consumption

When the above defined silvicultural investment is inserted in the forest owners' income constraint, the consumption becomes

$$(4.53)\quad C = Y(K_1) + pH - I_1 - I_2 - \gamma H.$$

Instrument and stock variables

The forest owners' instrument variables are as in the Section 4.2.1.2. The transformation of the silviculture is depicted by the state equation (4.52). The capital stock in the primary industry and the growing stock obey the state equations (4.17) and (4.19).

Optimum equilibrium conditions

The optimun equilibrium conditions in the forest owners' economy are

$$(4.54)\quad Y_k = r + k_1$$

for the primary industry,

$$(4.55)\quad G_k = \frac{r+k_2}{p}$$

for the silviculture provided that

$$(4.56)\quad \gamma H^{*} \leqq k_2K_2^{*}$$

and

$$(4.57)\quad G_v = r - \beta$$

for the growing stock.

Conclusions

It can be concluded that the government's control activity has no effect on the forest owners' target state provided that the condition (4.56) is fulfilled. This can be interpreted to mean that it is quite the same as for as the forest owners are concerned whether the optimum silvicultural investment are based on their own or the government's target. What is essential is that the total silvicultural investment resulting from the requirements of these two bodies is optimum and that the forest owners can adapt their own investment and cutting to what the government wants. It is examined a situation in which the government has set so high an investment rate per cutting unit that the equilibrium in the forest owners' economy is upset, i.e. the condition (4.56) does not hold, and the forest owners' silvicultural behaviour becomes that of the bang-bang type. In such a case the forest

Table 3. Summary of the effects of the means of forest policy on marginal conditions of the optimum equilibrium stocks.

Means of forest policy	Marginal conditions		Remarks
	Silvicultural condition	Growing stock condition	
0. Unregulated forestry	$G_k = \frac{r+k_2}{p}$	$G_v = r - \beta$	r = rate of time preference, p = raw wood price β = proportional change in the raw wood price k_2 = depreciation rate of the capital
1. Taxation of "pure income" from forestry	$G_k = \frac{r+k_2}{p}$	$G_v = r - \beta$	Effect on forest owners' primary industry and forest ownership
2. Tax on sales income	$G_k = \frac{r+k_2}{(1-\varphi_R)p}$	$G_v = r - \beta$	φ_R = marginal income tax
3. Taxation of stocks in foresty	$G_k = \frac{r+k_2+\varphi_k}{p}$	$G_v = r - \frac{\dot{p}-\varphi_v}{p}$	φ_k = marginal tax on the silviculture φ_v = marginal tax on the growing stock
4. *Ad valorem* sales tax	$G_k = \frac{r+k_2}{(1-\eta)p}$	$G_v = r - \beta$	η = *ad valorem* sales tax
5. Unit sales tax	$G_k = \frac{r+k_2}{p-\chi}$	$G_v = r - \frac{\dot{p}}{p-\chi}$	χ = unit sales tax
6. Support of silvicultural investment	$G_k = \frac{r+k_2}{p}\left(\varkappa+\frac{\pi+\iota}{\pi+r}(1-\varkappa)\right)$	$G_v = r - \beta$	$\varkappa$ = share of self-financing of investments ι = interest on loans π = amortization rate of loans
7. Channelling income from raw wood sales to investments in forestry	$G_k = \frac{r+k_2}{p}$	$G_v = r - \beta$	No effect on the optimum equilibrium forestry

owners have to make blocked interval decisions. If the price is on the increase, then the capital condition (4.55) is reduced as a function of price, i.e. the optimum silviculture increases, and if the investment rate is supposed to be constant with respect to time, then equilibrium is reached after a short time. If the price is on the decrease, equilibrium cannot be reached unless the investment rate is decreased by the government.

The channelling of the income from raw wood sales into silvicultural investments is of great importance in a developing economy where the growth of forests in a natural state is not sufficient to satisfy the demand for raw wood and where the forest owners have no information about the economical possibilities afforded by silviculture. In such a case the government can force the forest owners to behave in accordance with the forest owners' and government's economical interests.

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5. Empirical study

5.1. Approach to the empirical study

Scope of the empirical study

The empirical study covers

- behaviour of the raw wood market, and supply in particular, during the period 1955–1970, and
- investment behaviour during 1960–1979.

Private and state forestry are dealt with separately. The behaviour of the raw wood market is assessed on the basis of

- the supply of and demand for softwood logs.

Investment behaviour is assessed on the basis of

- the area of young stand tending.

The study is based on annual time series and the consideration has been selected so as to

- make the raw wood market and investment examination physically and institutionally as independent as possible during the selected study period.

This means that it must be possible to explain the raw wood market and investment as independently as possible and not as a link in a chain of physical or institutional measures. Forest regeneration, for instance, is therefore not included in the investment study because it is institutionally (Act concerning Private Forests) and physically dependent on lagged cuttings. Thinnings have both an investment and a market aspect, the separation of which leads to difficulties. As far as technical and economical properties are concerned, the softwood log market and tending of young stands have been internally homogeneous during the entire study period.

Supply of softwood logs

The equilibrium supply of raw wood is depicted using the equation

(5.1) $H_t^* = b_o + b_1 p_{t-1} + b_2 s_t,$

where
H_t^* = the logarithm of the equiligrium supply of softwood logs in the year t
p_{t-1} = the logarithm of the softwood log stumpage price in the year t−1, and
s_t = the logarithm of time, the shift parameter of the supply curve.

The equilibrium supply is defined in the empirical study in the same sense as Marshall used the term "long-run normal" (Nerlove-Addison 1958, pp. 861–862). The model has been defined as logarithmic, the elasticity thus being constant, since this model gave the best explanation when the estimation was carried out. The supply equation is based on the supposition that forest owners make their selling decisions on the basis of the previous year's price. The variable s_t, the time, in the supply equation depicts the omitted variables and shows trend-like development, such as change of rate of time preference, bequest growing or structural change of forest, and acts as a shift parameter of the supply function. The supply function is a constant elasticity approximation of the supply functions presented in Fig. 7 on the page 24.

The supply, cutting and growth are equal in accordance with equation

(5.2) $H_t^* = b_o + b_1 p_{t-1} + b_2 s_t = G(V_t^*, K_t^*),$

where growth is given in a logarithmic form. In order to make equation (5.2.) unambiguous, it must be assumed that the growing stock V_t^* and capital stock K_t^* in the equilibrium do not exceed the growing stock and silviculture resulting from the maximum biological growth of the forests (i.e. the rate of time preference is positive).

There is not sufficient information available about the growth, growing stock and silviculture. On the other hand, there is information about the actual supply, H_t, which has the following relationship with the equilibrium supply

$$(5.3)\quad H_t \gtreqless H_t^{\bullet} = G(V_t^{\bullet}, K_t^{\bullet}).$$

The adjustment of the supply is approximated using Nerlove's partial adjustment equation (e.g. Nerlove-Addison 1958, pp. 863–865)

$$(5.4)\quad H_t - H_{t-1} = \delta_1(H_t^{\bullet} - H_{t-1}),$$

where

H_t = actual supply in the year t

H_{t-1} = actual supply in the year t−1

δ_1 = adjustment elasticity

$H_t^{\bullet}$ = equilibrium supply in the year t.

The magnitude of adjustment elasticity indicates how quickly measures carried out adjust the actual supply to the equilibrium supply. Since the growth and supply are a function of both the growing stock and silviculture, the reaction of the partial adjustment equation includes the adjustment effects through the growing stock and silviculture. It must be noticed that because of the lack of information about the actual lagged growing stock the adjustment is expressed in relation to the actual lagged raw wood supply, i.e. in the empirical study the lagged cutting substitutes for the growing stock of the theory. This controversial fact is based on the assumption that the forest owners are regarding the cutting of the previous year as normal because of the lack of information and the myopia in the forest owners' behaviour. According to the devised theory, the adjustment reaction of the growing stock has a sign opposite to that of the change in the equilibrium supply, in other words, the adjustment elasticity is negative.

By combining equations (5.1) and (5.4) and taking the above sign rule into account, it is obtained

$$(5.5)\quad H_t = \delta_1 b_o + \delta_1 b_1 p_{t-1} + \delta_1 b_2 s_t + (1+\delta_1)H_{t-1},$$

the variables of which are available in time series suitable for statistical treatment.

If the absolute value of adjustment elasticity is less than one, the state of the equilibrium behaviour is reached after infinite periods, but a 95 % adjustment is reached after N periods when N is defined on the basis of the formula (Nerlove-Addison 1958, p. 874).

$$(5.6)\quad (1 - |\,\delta\,|\,)^{N} \leqq 0{,}05.$$

If the absolute value of adjustment elasticity is one, complete adjustment is reached after one period. It the absolute value of adjustment elasticity is greater than one, a state of equilibrium cannot be reached. It should be noted that Nerlove's equation is used myopically in the sense that, on the basis of expression (5.3), decisions concerning adjustment towards equilibrium stocks are made at all times, the use of the absolute value of adjustment elasticity in the above expression (5.6) thus being justified.

In Nerlove's equation, adjustment occurs as a relative change. Use of a relative change in an empirical study can be justified by the heterogenity of the forest owners and by the aggregativity of the statistical material. The magnitude of the adjustment elasticity depicts also the extent of the instrument variable boundary. The greater the absolute value of the adjustment elasticity the broader is the instrument variable boundary.

Stumpage price of softwood logs

The equilibrium demand for raw wood is depicted with the stumpage price equation

$$(5.7)\quad p_t^{\bullet} = a_o + a_1 w_{t+1} + a_2 H_t,$$

where

$p_t^{\bullet}$ = the equilibrium stumpage price of the softwood logs in the year t,

w_{t+1} = the delivery price of sawn timber in the year t+1, and

H_t = the supply of softwood logs in the year t.

The equation is logarithmic and based on the hypothesis that the demand for softwood logs depends on the price of sawn timber processed from it and on the supply of softwood logs. The delivery price of sawn timber with one year's lead, in the year t+1, is assumed to be the same as its selling price in the previous year t. By assuming that the variation of stumpage price has a degree of slowness which can be described using the adjustment elasticity and by forming the partial adjustment equation

$$(5.8)\quad p - p_{t-1} = \delta_2(p_t^{\bullet} - p_{t-1})$$

and by inserting into it the equilibrium stumpage price equation, the following equation is obtained

$$(5.9)\quad p_t = \delta_2 a_0 + \delta_2 a_1 w_{t+1} + \delta_2 a_2 H_t + (1-\delta_2)p_{t-1}.$$

Area of young stand tending

Investmens are assessed in the empirical study on the basis of the area of young stand tending. Equilibrium investments are equal to the depreciation of the silviculture, i.e.

$$(5.10)\quad I_t^{\bullet} = kK_t^{\bullet}.$$

Equilibrium investments are described using the equation

$$(5.11)\quad I_t^{\bullet} = c_0 + c_1 p_{t-1} + c_2 u_t + c_3 s_t$$

where

$I_t^{\bullet}$ = the area of young stand tending in the year t,

p_{t-1} = the stumpage price of softwood logs in the year t−1,

u_t = the unemployment rate in the year t, and

s_t = time, the shift parameter.

The equation is logarithmic and based on the hypothesis that investments are dependent on lagged stumpage price, unemployment rate (which can be said to have been used as a pseudo-instrument in Finnish forestry investments, as is pointed out in the following) and time, which represents the omitted trend-like variables, such as change of rate of time preference or change of silvicultural know-how. By forming the partial adjustment equation

$$(5.12)\quad I_t - I_{t-1} = \delta_3(I_t^{\bullet} - I_{t-1})$$

and inserting it into the equilibrium investment equation, the equation to be estimated becomes

$$(5.13)\quad I_t = \delta_3 c_0 + \delta_3 c_1 p_{t-1} + \delta c_2 u_t + \delta_3 c_3 s_t + (1-\delta_3)I_{t-1}$$

Exogeneous variables

Exogeneous variables in the study are

- the price of sawn timber on the domestic market,
- the unemployment rate and
- time.

The price of sawn timber on the domestic market one year ahead is used to explain the price of softwood logs. This is based on the idea that when buyers purchase raw material, they either know, on the basis of their own sales of sawn timber, or can estimate the price of sawn timber the next year, which they then use to determine the purchase price of softwood logs.

The Finnish forest policy has remained unchanged ever since the 1920's. The regulation of the means of forest policy,

- forest administration,
- promotion of private forestry,
- public control of private forestry,
- forest improvement activities and
- forest taxation,

has remained practically the same throughout the whole period. A pseudo-instrument which has been incorporated into the system can be found in two of these items: The tax rate in forest taxation, which is based on forest area, varies as a function of variations in stumpage price. In accordance with theoretical consideration, a change in the tax rate in area taxation primarily affects the sale of forest land (p. 29). The buying and selling of land in Finland have such institutional restrictions, and the statistical data is so limited that any assessment in this connection is

Table 4. Maximum subsidy-% in the tending of young stands normally and when employing unemployed people (The Forest Improvement Act).

Area	Ordinary subsidy-%	Subsidy-% when employing unemployed
I	15	45
II	30	55
III	40	70
IV	65	80

impossible. The other pseudoinstrument is the fact that state finance for the tending of private young stands is linked with unemployment. If the tending of young stands provides unemployed people with work, the maximum state subsidy for the costs of work increases as shown in Table 4 above.

We can expect, on the basis of Table 4 and the equation (4.51), that unemployment increases the tending of young stands in the private forestry sector in Finland.

The use of time as an exogeneous variable is justified by the fact that it represents the trend-like variables which have been omitted. As a result of industrialization and urbanization during the study period, forest ownership, for instance, has undergone changes which, on their part, have changed the forest owners' forestry behaviour (e.g. Honkanen et al., 1975). These include changes in the forest owners' occupational and age structure and income level in their primary trade.

Derivation of the equations to be estimated

The combination of equilibrium and adjustment cutting and silvicultural behaviour in the theoretical study is described in the empirical study using Nerlove's partial adjustment equations.

The adjustment behaviour and stumpage price equations which are to be estimated,

(5.5) $H_t = \delta_1 b_0 + \delta_1 b_1 p_{t-1} + \delta_1 b_2 s_t + (1+\delta_1)H_{t-1}$,

(5.9) $p_t = \delta_2 a_0 + \delta_2 a_1 p_{t-1} + \delta_2 a_2 H_t + (1-\delta_2)p_{t-1}$, and

(5.13) $I_t = \delta_3 c_0 + \delta_3 c_1 p_{t-1} + \delta_3 c_2 u_t + \delta_3 c_3 s_t + (1-\delta_3)I_{t-1}$

can be written for the estimation in the form

(5.14) $p_t = a_0' + a_1' w_{t+1} + a_2' H_t + \delta_2' p_{t-1} + \varepsilon_{2_t}$,

(5.15) $H_t = b_0' + b_1' p_{t+1} + b_2' s_t + \delta_2' H_{t-1} + \varepsilon_{1_t}$ and

(5.16) $I_t = c_0' + c_1' p_{t+1} + c_2' u_t + \delta_3' I_{t-1} + \varepsilon_{3_t}$,

where

(5.17) $a_i' = \delta_2 a_i$, $i = 0-2$,

(5.18) $b_i' = \delta_1 b_i$, $i = 0-2$,

(5.19) $c_i' = \delta_3 c_i$, $i = 0-2$, and

(5.20) $\delta_i' = 1-\delta_i$, $i = 1$ and 3, and $\delta_2' = 1+\delta_2$

and where the residual term ε_{i_t} are random variables independent of each other.

Estimation methods

Equations (5.14) – (5.16) form a recursive equation system, the equations of which can be estimated separately using the least square method (Koutsoyiannis 1981, pp. 340–342). All the equations have the lagged endogeneous variable as one explaining variable, which makes the resulting estimates biased. If the residual terms ε_{it} are not autocorrelated, the estimates of the ordinary least square (OLS) are consistent and asymptiotically effective in the case of large samples (Koutsoyiannis 1981, pp. 319–320). To provide against inconsistency caused by the autocorrelatedness of the residual terms, the equations have also been estimated using the generalized least square method (GLS) which takes into account the autoregressive process of the first order of the residual terms

(5.21) $v_t = \varrho v_{t-1} + \varepsilon_t$,

where

v_t = autocorrelated residual term,
ϱ = autoregression coefficient and
ε_t = stochastic residual term

and

$|\varrho| < 1$.

In order to take the autocorrelation of the residual terms into account,

- two-step procedures and
- search procedures

can be used. In this study the two-step Aitken's Generalized Least Squares procedure has been used owing to the availability of suitable computer programmes[5]. The first step in this procedure involves obtaining and estimate of the autoregression coefficient. The second step involves estimating the regression parameters. The steps can be integrated (Maddala 1977, p. 279).

The autoregression coefficient has been calculated using the formula (Theil 1971, p. 254)

(5.22) $$\varrho = \frac{\sum_{t-1}^{n-1} v_t v_{t+1}}{(n-1)s^2},$$

where

- v_t and v_{t+1} are the residual terms in the OLS estimation,
- n the number of observations, and
- s^2 the residual variance in the OLS estimation

In the GLS estimation used, the OLS estimation is used twice in succession:

- first with the original data data so as to determine the autoregression coefficient of the first order and
- then with the data from which the autoregressive process has been removed (Theil 1971, p. 253).

However, it should be noted that the autoregression coefficients which are obtained are biased in the sense that the absolute value of the autocorrelation indicated by them is smaller than in reality, when a lagged endogeneous variable is used as an explaining variable in the equation being estimated. The Durbin-Watson-statistic, which is frequently used for testing autocorrelation, are equally biased. A test for autocorrelation when a lagged endogeneous variable is present has been developed by Durbin (Johnston 1972, pp. 312–313). From (5.22) the statistic

(5.23) $$h = \varrho\sqrt{\frac{n}{1-n\,\mathrm{Var}(b)}}$$

is computed. In (5.23)

Var(b) = variance of the coefficient of the lagged endogeneous variable in the OLS estimation.

The statistic h is tested as a standard normal deviate. This is a large sample test ($n > 30$). If $n\,\mathrm{Var}(b) \geqq 1$, the test breaks down. Although the sample periods of this study ($n=20$ or $n=25$) don't fulfil the requirement of the large sample test, the statistic h is computed when then lagged endogeneous variable is present. In the other case the conventional Durbin-Watson statistic is computed in spite of its shortcomings (Koutsoyiannis 1981, pp. 214–216).

The GLS estimation is used if the autoregression coefficient $\varrho > 0{,}2$ or if Durbin-Watson-statistic or Durbin-h-statistic is significant in the OLS estimation.

Statistical material

Quantity and stumpage price statistics concerning the sales of softwood logs by the private forest owners were compiled from the basic forest taxation material by picking out a sample of 100 municipalities at the Department of Forest Economy, the Finnish Forest Research Institute⁶⁾. (There were 461 municipalities in Finland in 1982.) The sales figures of the state forestry sector were based

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5) The estimation has been carried out using econometric library programmes (IBM 1977) which are written in APL language and based on Theil's OLS (Theil 1971, p. 101) and GLS (Theil 1971, p. 236) estimation.

6) The same statistics have been used as a semiannual series in the publication: Jari Kuuluvainen: Sawtimber markets and business cycles in the Finnish sawmilling industry. Finnish Forest Research Institute, publication 63. The semiannual consideration has been rejected in this study because of the high seasonal variation in the timber sales.

Table 5. The time series of the empirical study.

Year	Price index of sawn timber	Rate of unemployment, in percent	Private forestry Stumpage price of softwood logs, in FIM	Private forestry Sales of softwood logs in 10^6 cubic metres	Private forestry Tending of young in 1000 hectares	State Forestry Stumpage price of softwood logs, in FIM	State Forestry Sales of softwood logs in 10^6 cubic metres	State Forestry Tending of young stands in 1000 hectares
1955	85.6		72.05	4.752	53.3	45.02	1.598	32.4
1956	81.0		61.99	2.674	98.9	32.84	1.826	53.9
1957	74.9		62.43	5.525	119.5	24.18	1.673	62.1
1958	73.2		55.20	4.705	56.2	36.82	1.976	61.7
1959	71.4		61.02	8.547	68.3	34.71	1.590	43.7
1960	74.5	1.5	69.32	8.680	52.6	47.48	1.797	36.4
1961	79.0	1.2	71.28	6.753	34.6	53.56	1.647	30.4
1962	78.5	1.2	72.91	6.408	39.6	50.83	1.531	33.1
1963	79.6	1.5	82.64	7.815	73.9	48.00	1.422	25.6
1964	86.5	1.5	92.11	8.015	88.0	54.02	1.270	31.4
1965	89.8	1.4	88.28	7,004	90.0	64.41	1.426	29.6
1966	86.4	1.6	79.90	7.363	118.8	62.40	1.323	39.6
1967	82.5	2.8	65.25	6.384	151.5	54.37	1.359	59.0
1968	78.1	1.0	63.55	9.954	155.1	47.64	1.431	58.5
1969	79.8	2.8	69.92	12.471	113.1	57.97	1.275	52.1
1970	83,9	1.9	78.15	13.128	92.1	60.30	1.181	33.7
1971	87.2	2.3	89.59	9.053	141.7	60.48	1.118	56.3
1972	83.0	2.5	73.01	12.157	155.3	59.48	1.197	69.4
1973	97.4	2.3	132.40	16.495	168.3	56.09	1.082	65.7
1974	107.6	1.7	126.71	10.968	175.3	118.32	1.075	70.5
1975	100.0	2.2	92.74	3.848	296.4	93.80	1.148	116.3
1976	95.3	4.0	95.07	9.896	287.0	73.37	1.218	136.1
1977	92.4	6.1	80.02	9.399	293.2	77.97	1.452	160.3
1978	86.3	7.5	79.56	15.854	280.4	69.59	1.718	151.7
1979	85.8	6.1	82.02	17.152	191.9	69.21	1.705	119.8

on the quantity and stumpage price statistics concerning softwood logs supplied by the state. These statistics were provided by the National Board of Forestry. The domestic basic price index of timber goods and articles (Statistical Yearbook of Finland, various years) was used as the price series for sawn timber. It should be noted that the price of sawn timber is an index serie.

The figures indicating the areas of young stand tending in the private and state forestry sectors are based on the 1982 Forest Statistical Yearbook (SVT XVII A:14, p. 100). The unemployment rate is based on official statistics (Statistical Yearbook of Finland, various years) from 1960 onwards.

All the time series are annual series. The stumpage price and sawn timber index series have been deflated using the total index for the wholesale price index so as to make them correspond to the price level for 1975 (Statistical Yearbook of Finland 1982, p. 264).

The time series are shown in Table 5. The series have been converted into a logarithmic form for computing.

5.2. Results of the raw wood market study

The results of the raw wood market are shown in Tables 6 and 7 on the pages 43 and 44. The results are discussed and interpreted in the following. Because the results of OLS and GLS estimation are very close, only the OLS estimates are reported in the text.

Table 6. Estimation of stumpage price functions (The figures are truncated without rounding.)

Sector of forestry	Method	Specification	a_0'	a_1'	a_2'	δ_2'	δ_2	a_0	a_1	a_2	R^2	F	ϱ	d/h
Private forestry	OLS	1	–3.57 (1.25)	1.64 (0.42)	0.00 (0.07)	0.13 (0.17)	0.86	–4.14	1.91	0.00	0.76	24.85*	–0.31	h=2.99
"	GLS	1	3.92 (1.39)	1.75 (0.38)	–0.04 (0.07)	0.13 (0.15)	0.87	–4.53	2.02	–0.04	0.86	45.30*	–0.22	h=–1.78
"	OLS	2	–3.65 (0.97)	1.68 (0.28)	–	0.12 (0.13)	0.87	–4.19	1.92	–	0.77	39.21*	–0.34	h=2.30
"	GLS	2	–3.54 (1.01)	1.59 (0.23)	–	0.18 (0.11)	0.81	–4.37	1.96	–	0.87	73.37*	–0.14	h=0.86
"	OLS	3	–3.89 (0.91)	1.86 (0.20)	–	0	1.00	–3.89	1.86	–	0.77	82.19*	–0.26	d=2.55**
"	GLS	3	–4.02 (0.92)	1.88 (0.16)	–	0	1.00	–4.02	1.88	–	0.85	131.48*	–	d=2.16
State forestry	OLS	1	–3.62 (3.29)	1.31 (0.74)	–0.09 (0.42)	0.46 (0.16)	0.53	–6.73	2.43	–0.16	0.62	13.84*	–0.15	h=–1.41
"	OLS	2	–4.17 (2.01)	1.42 (0.53)		0.47 (0.15)	0.52	–7.88	2.68	–	0.66	21.73*	–0.16	h=–1.33

Methods:
OLS = Ordinary Least Square
GLS = Generalized Least Square

Specification:
1. $p_t = a_0' + a_1' w_{t+1} + a_2' H_t + \delta_2' p_{t-1}$
2. $p_t = a_0' + a_1' w_{t+1} + \delta_2' p_{t-1}$
3. $p_t = a_0' + a_1' w_{t+1}$

Variables:
p_t = logarithm of softwood log price
w_t = logarithm of sawn timber price
h_t = logarithm of softwood log sales

Results:
a_1', a_2', δ_2' = elasticities of adjustment equations of stumpage price (standard errors in parentheses)
δ_2 = adjustment elasticity of stumpage price
a_1, a_2 = elasticities of equilibrium equations of stumpage price
ϱ = autoregressive coefficient of first order
R^2 = multiple correlation coefficient
F = F-statistic, * = significance more than 1 %
d = Durbin-Watson statistic, when $\delta_2' = 0$, ** = autocorrelation at 1 % level
h = Durbin h-statistic, when $\delta_2' \neq 0$
levels of significance of h-statistic
$h_{0,01} = 2,326$
$h_{0,05} = 1,645$

Stumpage price of the softwood logs

The estimation gave the following stumpage price function in the private forestry

(5.24) $p_t^* = -4.418 + 1.913 w_{t+1} + 0.009 H_t$

and the following adjustment elasticity

(5.25) $\delta = 0.862.$

Since the standard errors in the coefficients of cutting in the adjustment equation of the stumpage price, and the lagged stumpage price are great in comparison to the absolute values of the coefficients, the stumpage price equation was estimated both without the cutting variable only and without the cutting and lagged price variable. In the first case the equilibrium stumpage price equation becomes

(5.26) $p_t^* = -4.192 + 1.928 w_{t+1}$

and the adjustment elasticity

(5.27) $\delta_2 = 0.872$

and in the latter case

Table 7. Estimation of raw-wood supply function (The figures are truncated without rounding.)

Sector of forestry	Method	Specification	b_0'	b_1'	b_2'	δ_1'	δ_1	b_0	b_1	b_2	R^2	F	ϱ	d/h
Private	OLS	1	6.13	−1.34	0.63	0.14	−0.85	−7.20	1.57	−0.74	0.72	20.95*	−0.20	h=−1.56
forestry			(1.13)	(0.28)	(0.10)	(0.15)								
"	OLS	2	6.21	−1.32	0.69	0	−1.00	−6.21	1.32	−0.69	0.72	31.05*	−0.07	d= 2.12
			(1.13)	(0.28)	(0.08)									
State	OLS	1	0.07	0.05	0.07	0.68	−0.31	−0.23	−0.16	0.23	0.62	14.44*	−0.01	h=−0.12
forestry			(0.41)	(0.10)	(0.04)	(0.16)								
"	OLS	3	0.26	–	0.05	0.65	−0.34	−0.76	–	0.17	0.64	22.33*	−0.06	h=−0.43
			(0.10)		(0.03)	(0.14)								

Methods:
OLS = Ordinary Least Square
GLS = Generalized Least Square

Specification:
1. $H_t = b_o' + b_1' p_{t+1} + b_2' s_t + \delta_1' H_{t-1}$
2. $H_t = b_o' + b_1' p_{t+1} + b_2' s_t$
3. $H_t = b_o' + b_2' s_t + \delta_1' H_{t-1}$

Variables:
H_t = logarithm of softwood log sales
p_t = logarithm of softwood log price
s_t = logarithm of the year of selling

Results:
b_1', b_2', δ_1' = elasticities of adjustment equations of raw wood aupply (standard errors in parentheses)
δ_1 = adjustment elasticity of raw wood supply
b_1, b_2 = elasticities of equilibrium equations of raw wood supply
ϱ = autoregressive coefficient of first order
R^2 = multiple correlation coefficient
F = F-statistic, * = significance more than 1 %
d = Durbin-Watson-statistic, when $\delta_1' = 0$
h = Durbin h-statistic, when $\delta_1' \neq 0$
levels of significance of h-statistic
$h_{0,01} = 2{,}326$
$h_{0,05} = 1{,}645$

(5.28) $p_t^* = -3.895 + 1.861 w_{t-1}$

and

(5.29) $\delta_2 = 1.000.$

The coefficient of determination of all the equations is by and large the same. They indicate that estimated changes or changes which have already occurred in the price of sawn timber are almost immediately reflected in the stumpage price of softwood logs – the adjustment elasticity is almost one – and that the elasticity of the equilibrium softwood log price in relation to the price of sawn timber is nearly two. This means that changes in the price of the end product have an overproportinonal effect on the stumpage price of softwood logs. On the other hand, the price of softwood logs is practically independent of the supply, i.e. the elasticity of the stumpage price in relation to the quantity sold is zero. This should be interpreted to mean that, for institutional reasons, the price development of other factors of production in the woodworking industry, such as capital and the labor, is less flexible than that of softwood logs, and that as a result of competition between the buyers the variations in the price of sawn timber are reflected overproportionally in the price of softwood logs.

The equilibrium stumpage price equation for the state forestry becomes

(5.30) $p_t^* = -6.738 + 2.439 w_{t+1} - 0.169 H_t$

with the adjustment elasticity

(5.31) $\delta_2 = 0.538.$

Since the coefficient for the offered quantity is statistically very non-significant, the stumpage price equation for the state forestry was estimated without the quantity variable, giving

(5.32) $p_t^* = -7.885 + 2.684 w_{t+1}$

and

(5.33) $\delta_2 = 0.529.$

Table 8. Estimation of silvicultural investment function (The figures are truncated without rounding.)

Sector of forestry	Method	Specification	c_0'	c_1'	c_2'	c_3'	δ_3'	δ_3	c_0	c_1	c_2	c_3	R^2	F	ϱ	d/h
Private	OLS	1	−1.12	0.88	0.36	0.23	0.26	0.73	−1.53	1.20	0.49	0.32	0.89	41.60*	−0.12	h=−1.25
forestry			(1.28)	(0.33)	(0.21)	(0.11)	(0.20)									
"	OLS	2	−0.93	1.06	0.57	0.28	0	1.00	−0.93	1.06	0.57	0.28	0.89	52.55*	−0.04	d= 1.80
			(1.30)	(0.31)	(0.14)	(0.11)										
State	OLS	1	−1.16	0.68	0.25	0.12	0.46	0.53	−2.17	1.27	0.47	0.23	0.89	41.36*	−0.12	h=−0.88
forestry			(0.87)	(0.25)	(0.17)	(0.21)	(0.17)									
"	OLS	3	−1.32	0.77	0.28	–	0.49	0.50	−2.60	1.52	0.56	–	0.89	57.39*	−0.13	h=−0.86
			(0.81)	(0.12)	(0.16)		(0.16)									

Methods:
OLS = Ordinary Least Square
GLS = Generalized Least Square

Specification:
1. $I_t = c_o' + c_1' p_{t+1} + c_2' u_t + c_3' s_t + \delta_3' I_{t-1}$
2. $I_t = c_o' + c_1' p_{t+1} + c_2' u_t + c_3' s_t$
3. $I_t = c_o' + c_1' p_{tt-1} + c_2' u_t + \delta_3' I_{t-1}$

Variables:
I_t = logarithm of young stand tending area
p_t = logarithm of softwood log price
s_t = logarithm of the year of investment (1960: Ln 1, 1961:ln 2 etc.)
u_t = logarithm of unemployment rate

Results:
c_1', c_2', δ_1' = elasticities of adjustment equations of silvicultural investment (standard errors in parentheses)
δ_1 = adjustment elasticities of silvicultural investment
c_1, c_2 = elasticities of equilibrium equations of silvicultural investment
ϱ = autoregressive coefficient of first order
R^2 = multiple correlation coefficient
F = F-statistic, * = significance more than 1 %
d = Durbin-Watson-statistic, when $\delta_3' = 0$
h = Durbin h-statistic, when $\delta_3' \neq 0$
levels of significance of h-statistic
$h_{0,01} = 2{,}326$
$h_{0,05} = 1{,}645$

The coefficient of determination of both stumpage price equations in the state forestry is equally good. The elasticity of the softwood log price in relation to the price of sawn timber is 30 to 40 % greater in the state forestry than in the private forestry. The adjustment elasticity in the state forestry, on the other hand, is about half of that for private forestry. The great elasticity in relation to the price of sawn timber is due to the fact that the state forests are situated in the northern and eastern parts of the country, where the economic development has been the slowest and where changes in the demand for sawn timber, as a result of less flexible development in in the prices of other factors of production and high costs caused by the remote location, are reflected in the price of raw wood to a more extreme extent than in other parts of the country. The great elasticity of the stumpage price of softwood logs in relation to the price of sawn timber in the state forestry is, within a short period, compensated by the adjustment elasticity which is clearly smaller than in the private forestry. The time it takes to reach a 95 % adjustment of the stumpage price caused by a change in the price of sawn timber is 4 to 5 times longer in the state forestry than in the private forestry. This phenomenon can be interpreted to mean that the competition for softwood logs in the state forestry is subject to an administratively set minimum price constraint (cf. Tervo 1978, p. 10).

Supply of softwood logs

The equilibrium supply equation in the private forestry becomes

(5.34) $H_t^* = -7.206 + 1.576 p_{t-1} - 0.743 s_t$

with the adjustment elasticity

(5.35) $\delta_1 = -0.851.$

Since the standard error in the coefficient of the lagged endogeneous variable is relatively large, an alternative was tried in which

(5.36) $\delta_1 = -1.000.$

Then the equilibrium supply equation becomes

(5.37) $H_t^* = -6.210 + 1.322p_{t-1} - 0.693s_t.$

The coefficient of determination of both the equations is almost equally good. The equations indicate that the stumpage price elasticity of the softwood log supply in the private forestry sector is about 1.3, and the elasticity in relation to time, or rather to those omitted trend variables, is negative and its absolute value less than one. The adjustment elasticity of supply in the private forestry sector is almost minus one, which means that a 95 % adjustment is reached during a 1–2 year period.

The negativity of the coefficient of the trend variable can be interpreted to mean that private forests are grown with the aim of leaving them as bequests for the next generation or that willingness to grow denser and older stands is increasing. That is in accordance with the results of the investment study below and with the empirical observation that the growing stock has increased during the period (Yearbook of forest statistics 1982). The positive adjustment elasticity with respect to time variable (see Table 7) indicates together with the facts above that either blocked interval supply behaviour or negative effective rate of time preference or both have been present during the sample period.

An essential factor leading to the above interpretation is the fact that the average age of farmer forest owners is increasing. Farmers account for more than one half of all forest owners and they own two thirds of the forests (Järveläinen 1983, p. 8). The proportion of farmers over 55 years had increased from 35 % in the farmer census of 1959 to 43 % in the farmer census of 1969. More recent comparable statistics are not available, but we can assume that the trend has remained similar, although the development has been slower. The aging of forest owners affects their rate of time preference and willingness to grow forest for the next generation in the following ways:

- The rate of time preference of old people can be assumed to be smaller than that of young people. The physiological and psychological development of old people encourages saving.
- Elderly farmer forest owners prepare for the coming generation change by withholding cuttings. Forests are used for financing the generation change at a farm in two ways:

1) the forest is left to all the heirs and one of them continues working the farm (Honkanen et. al. 1975, p. 42, Table 9.), and
2) timber is sold for financing the division of inheritance (Honkanen et al. 1975, p. 53).

- If there is no one to carry on after an elderly farmer forest owner, omission of cuttings is one way of safeguarding or improving his income in his later years. In national economies where industrialization and urbanization have been rapid, as in Finland, the proportion of the population engaged in primary production has dropped rapidly and at the same time the proportion of those farms which have no one to take over has become very high (Honkanen et al. 1975, p. 61).

When the older generation is getting economically prepared for the coming generation change, their behaviour can be called altruistic towards the younger generation (in fact the rate of the intergenerational time preference can be considered to be negative). The other two interpretations above cannot actually be called altruistic, but in reality they will most likely become such indirectly. The above interpretations are equally applicable to elderly forest owners other than farmers. The average age is relatively high also in this group of Finnish forest owners (Järveläinen 1983, p. 9). The decrease in the rate of time preference and weakening of supply is definitely associated with forest owners' expectations that the stumpage price level will remain the same or rise over a long period.

In addition to, or together with, the above factors which affect the raw wood supply in the private forestry sector, the supply has been diminishing due to the fact that in their primary industry forest owners are less and less dependent on forestry as a source of income. Other sources of income are also being used to a greater and greater extent in financing the operations on a farm (e.g. Riihinen 1982, p. 337). As no statistical data are available, it is impossible to draw any conclusions about whether some changes in the age or dimension structure or species of tree structure of the forest, or in the dimension or timber assortment structure in the wood working industry, have been a trendlike diminishing factor in the sales of softwood logs.

The equation which best explained the equilibrium supply in the state forestry is

(5.38) $H_t^* = -0.235 - 0.169p_{t-1} + 0.230s_t$

with the adjustment elasticity

(5.39) $\delta_1 = -0.312.$

The stumpage price elasticity of the raw wood supply in the state forestry is negative, but not statistically significant. The negative stumpage price elasticity indicates a backward bending supply curve for the reason that the state has been compelled to sell timber in order to achieve the surplus target set by the government.

The stumpage price elasticity of the raw wood supply in the state forestry can be considered to be zero due to the fact that cuttings in the state forestry are very closely tied to planning. The time variable in the supply equation can also be interpreted as a policy variable resulting from the planning of the the allowable cut. Supply over a long period has been strengthened, but this has been possible only by weakening it over a short period. This situation has developed from the cuttings carried out in the first half of the century and mainly from the effect of postwar settlement activities on the structure of forests (National Board of Forestry, various years).

5.3 Results of the investment study

The results of the estimation of the investment study are shown in Table 8 on the page 45. The OLS estimation gives the following equilibrium investment function for the private forestry

(5.40) $I_t^* = -1.533 + 1.200p_{t-1} + 0.495w_t + 0.320s_t$

with the following adjustment elasticity

(5.41) $\delta_3 = 0.735.$

As the standard error of the adjustment elasticity is great, it is set

(5.42) $\delta_3 = 1.000,$

when the investment function becomes

(5.43) $I_t^* = -0.934 + 1.061p_{t-1} + 0.570u_t + 0.287s_t.$

The coefficient of determination of both the investment functions is almost equally good. The elasticity of the instrument variable in the estimation function corresponding to the investment function (5.40) is not statistically significant, wheras all the elasticities in the estimation of the investment function (5.43) are significant. The latter investment function can, therefore, be considered better to explain the investments made by the private forest owners than function (5.40).

On the basis of investment function (5.43) we can conclude that

- investment made by the private forestry is unit elastic in relation to the price of raw wood,
- elasticity of investment in relation to unemployment is considerable, and
- investment by private forest owners has increased along with time during the study period.

The last-mentioned factor may be caused by the bequest growing, as was postulated in the raw wood market study. As it is a question of the tending of young stands, the changes in the deterioration of capital can be omitted.

The equilibrium investment function in the state forestry becomes

(5.44) $I_t^* = -2.279 + 1.278p_{t-1} + 0.477u_t + 0.239s_t$

and the adjustment elasticity

(5.45) $\delta_3 = 0.532.$

As the elasticity with respect to time is not significant, it was left out of the examination. The equilibrium investment function in the state forestry subsequently becomes

(5.46) $I_t^* = -2.609 + 1.522p_{t-1} + 0.561u_t$

and adjustment elasticity

(5.47) $\delta_3 = 0.506$.

The functions indicate that the equilibrium elasticity of investment in the state forestry in relation to the price of raw wood is great which is in accordance with the slightly negative stumpage price elasticity of the supply. On the other hand, there are no trend-like factors which would affect the investment in the state forestry. Forestry investments are used to a great extent by the government as a means of employment policy.

6. Discussion

The main results

In this study the following items have been examined theoretically:

1) the determination of the target silviculture and the target growing stock, the targets of forest policy, from the government's point of view,
2) the optimun cutting and silvicultural behaviour of forest owners and
3) the effect of the various means of forest policy exercised by the government on the cutting and silvicultural behaviour.

Empirically has been studied

4) the forest owners' cutting behaviour as part of the raw wood market, and their investment behaviour.

The thought underlying the determination of the target stocks in the government's forest policy has been that by inserting the biological production of forestry into a simple macroeconomic model, the optimum equilibrium silviculture and growing stock are solved together with the optimum equilibrium condition of the GNP. The GNP has been assumed to be linearly dependent on the raw wood grown in the forestry.

From the result it can be seen that the optimum synchronizing of the GNP and the biological production of the forestry can be considered as a shadow pricing problem of raw wood. If the price is too low, raw wood will become a factor restricting the rest of the economy. If, on the other hand, the price is too high, the rest of the economy cannot sufficiently utilize, the raw wood offered by forestry. If a suitable price is chosen for raw wood, then an optimum equilibrium production capacity of GNP in accordance with the optimum equilibrium growth of forests is arrived at.

Rules for forest owners' optimum equilibrium cutting and silvicultural behaviour have been defined to serve as the basis for the examination of the means of forest policy and to derive the supply function of raw wood and the silvicultural investment function. The forestry has been included in the forest owners' income constraint so that they have two decision tasks:

1) to allocate their income and
2) to allocate their growing stock

In the first case they have as the alternatives

- to consume
- to invest in their primary industry and
- to invest in their silviculture

In the second the alternatives are

- to grow (to wait) and
- to cut.

In a myopic economy, where the raw wood price is changing with time, the marginal conditions of the optimum equilibrium state of the forest owners' economy are

1) marginal production in the forest owners primary industry is equal to its capital costs,
2) marginal growth in the forest owners' forests with respect to the silviculture is equal to the ratio between the capital costs due to the silviculture and the price of raw wood, and
3) marginal growth in the forest owners' forests with respect to the growing stock is equal to the difference between their rate of time preference and the proportional change in the raw wood price.

From the raw wood supply function and the investment function can be concluded that a price change will cause, during and adjustment phase of the stocks, a cutting change of opposite sign and an investment change of same sign.

The purpose of the examination of the means of forest policy is to find out how the means affect the marginal conditions of the optimum equilibrium state of the forest owners' economy. They have been examined separately, not in combinations. The means covered in this study are

1) taxation of the "pure income" from forestry,
2) taxation of sales income,
3) taxation of stocks,
4) sales taxes,
5) financial support for silvicultural investment, and
6) institutional channelling of sales income to silvicultural investment.

The effects of these means on the marginal conditions of the optimum equilibrium stocks are summarized shortly in the following. (See also Table 3, on the page 35.) Taxation of the "pure income" from forestry, which is used in Finland, cannot be used for affecting the stocks in forestry but instead the forest owners' primary industry and ownership of forests. Taxation of sales income and ad valorem sales tax affect the forestry through silviculture. In the case of linear income tax function the above-mentioned types of taxes are similar. Taxation of stocks is the most flexible and versatile of the means of forest policy examined. The taxation of the growing stock can be used for eliminating the effects of price changes on forestry. By the means of unit sales tax substitution effects between the growing stock and the silviculture can be created. By supporting the silvicultural investment, the government can affect the forestry through the silvicultural condition. During the past few decades Finnish forest policy has mainly meant the regulation of the terms of financing by the government. Channelling of income from sales of raw wood to silvicultural investment by the government is important when forest owners are ignorant of the economic possibilities offered by the silviculture. In Finland, the public authorities direct, by means of the Privatte Forest Law, a considerable proportion of the forest owners' cutting incomes into silvicultural investment.

The empirical study covers the cutting and investment behaviour in private and state foresty. The cutting behaviour has been studied as part of the softwood log market. The results concerning cutting behaviour in private forestry can be summarized as follows:

1) Over a short adjustment period, the change in the raw wood price causes a change in the raw wood supply of opposite sign. A rise in the stumpage price discourages the supply over an adjustment period, and vice versa, in accordance with the theory.
2) The elasticity of equilibrium raw wood supply in relation to price has been greater than one during the sample period 1955–1979.
3) The raw wood supply has decreased in Finland during the sample period. Since silvicultural investment has increased during the same period, the decrease in supply has been assumed to have been caused by the following factors

- the average age of forest owners has increased and forests have been grown to the bequests for the next generation,
- cuttings carreid out to provide income have become less frequent as forest owners have become less dependent on income from forests, and
- possible structural changes in forests or in the raw wood requirements of wood working industry.

The raw wood supply in state forestry is not affected by changes in the stumpage price. None of the forest policy means has been used to regulate the raw wood supply during the study period and it has not been possible to examine effects of such means.

Silvicultural investment behaviour has been studied by means of the tending of young stands only. The most important results are as follows:

1) The elasticity of the young stand tending in relation to the raw wood price is high in both private and state forestry.
2) In private forestry the silvicultural investment adjusts rapidly to price changes, where as adjustment in state forestry is more slowly.
3) In Finland the rate of unemployment has worked as a means of forest policy. The government finances the young stand tending carried out by private forest owners, the proportion of state finance being dependent on the employment creation effect of the young stand tending. The elasticity of silvicultural investment is high with respect to the unemployment rate, i.e. to the state support.
4) Silvicultural investment has a raising trend during the study period 1960–1979.

Remarks

In the theoretical part attention has been paid mainly on the optimum equilibrium states (steady states) in the national economy and the forest owners' economy. On the basis of the optimum equilibrium results a comparative statics analysis can be made (Kamien-Schwartz 1981, p. 155). At the chosen level of generalization the steady state has the important property that it is defined independently on the initial state, the time horizon and the terminal state. It is also possible to draw conclusions on the initial adjustment of the stocks from an arbitrary state to an optimum equilibrium state and from the last-mentioned state to a terminal state in the finite horizon case. In order to draw conclusions on the timing of adjustments numerical methods should be applied. Although the time horizon has no bearing on the properties of the optimum equilibrium state, two different time horizons have been used. In the case of the derivation of the government's target stocks the infinite time horizon has been used. By aid of that it has been avoided to define any other terminal state than the optimum equilibrium state. It has not been assumed that the government's time horizon in the economic planning or decision making should be infinite; in the reality it may be very short, only a few months, even in the connexion of forest policy. Because there is no essential information concerning the choice of the government's time horizon, it is a highly arbitrary value judgement. The date of the retirement or the division of inheritance has provided with the way of choosing the time horizon in the case of the derivation of the forest owners' optimum behaviour rules. By aid of the bequest stocks, the forest owners are taking into consideration all the forthcoming generations.

In the examination of the government's target stocks constant prices have been used. In fact the treatment includes the determination of the optimum raw wood price as a shadow price by means of the optimum conditions of the economy. In the examination of forest owners' behaviour the raw wood price is a constant or a variable as a function of time, and taking the raw wood price into account is myopic, i.e. the behaviour depends only on the current price and the current price change and not on predicted price changes.

The growth of forests has been presented as a function of two input factors: the growing stock and the silvicultural capital stock or the silviculture. By using that kind of aggregate or uneven-aged growth function it is possible to determine cutting and silvicultural investment endogenously in connexion with other instrument variables of the economy. (At the stand level the uneven-aged growth function and the determination of the optimum growing stock has been treated by Duerr and Bond (1953).) The thinking in the study is macroeconomic. The growth function is aggregative over two dimensions. First it is aggregated over forest stands within wood lots, and second it is aggregated over wood lots (see Appendix). A wood lot is the microeconomic decision making unit in the forestry. There is no qualitative difference between a microeconomic and macroeconomic growth function of forests, when the growing stock and the silviculture are used as input factors. The applied growth function does nos imply any special silvicultural technique, both even-aged and uneven-aged growing are applicable. If the even-aged growing or stand method is applied, as usual in Finland, the stand patterns and rotation periods are determined as endogenous data. No requirements are set on the age structure of the stands.

A very common approach in the forest economics and forest management is to apply a growth function with stand age as an input factor. This tradition is called maximum soil rent school or optimum rotation school, and its modern version goes back to König (1835), whose forest economic thinking has been reported by Hagfors (1936). König depicted the economy of an even-aged stand as a formula, that was improved by Faustmann (1849), and at the present it is called the Faustmannian formula. Pressler (1858–1885) optimized the formula with respect to the stand age, i.e. he solved the optimum rotation period of a stand, and by letting the time horizon go to the infinity he defined the rotation period of the maximum soil rent. The German forest economists Heyer (1871), Judeich (1879) and Endres (1895) have presented very comprehensive results concerning the choice of optimum rotation period based on the maximum soil rent in different situations and especially the case of the normal forest as optimum rotating forest. The maximum soil rent school has undergone a renascence after the mid-1960's. The optimum rotation is treated for example by Clark (1976), Heaps (1981), Comolli (1981), Dasgupta (1982) and Rotschild and Kovenock (1983). (See also Riihinen 1978). Nyyssönen (1958) has investigated the determination of the rotation based on the marginal rate of return of the growing stock. Nyyssönen and Ojansuu (1982) have developed criteria for decision making concerning the priority of stands for regeneration. These criteria are based on detailed stand information: site class, species of tree, age, basal area, mean diameter, mean height and stumpage prices of logs and pulp wood. Kilkki and Väisänen (1969) have investigated the determination of the optimum cutting policy for the forest stand numerically using a growth function specification for a stand (Kuusela and Kilkki 1963), which is often cited in the literature for example by Clark (1976) and Heaps (1981).

In determining the maximum soil rent and the optimum rotation period the forestry and every forest stand is treated as a separate capital asset in relation to the other economy, i.e. the forestry is an exogeneous phenomenon in the economy. At the aggregate level no general characteristics of an optimum cutting and silvicultural behaviour are known (Dasgupta 1981, p. 191). The normal forest, or the synchronized stationary forest, as it is also called, has been shown to be the optimum target state only, if the rate of time preference is zero and silvicultural investment is costless, i.e. there are no adjustment costs from an arbitrary age structure of forest stands to a normal forest (Dasgupta 1982, p. 191). This means that the age structure of stands determines the supply of raw wood and silvicultural investment. Theoretically and in many applications a weakness in the maximum soil rent thinking is the infinite time horizon; it does not take into consideration the behaviour of the forest owners as beings with finite life time. The maximum soil rent thinking implies the stand method as the only silvicultural technique. In the reduced form of the method the optimum rotation determines the time point for the clearcutting and regeneration of a stand.

Although the age distribution of forests has not been taken into account in this study, its importance is not denied in the Finnish forestry (see also Appendix). Biological production models have been developed (Vehkamäki 1976, Lyon and Sedjo 1983), where the forests are disaggregated with respect to stand properties like age, site, species of tree and stand type. The growth function for different stratum and transition function between the strata must be defined. The last mentioned functions depict the transition of land, growing stock andsilvicultural capital stock from a stratum to another. The disaggregation of the forests means also the disaggregation of the cuttings and silvicultural measures. Lyon and Sedjo (1983) have made computation with a relatively simple model and investigated, by aid of it, the development of the supply in different demand situations.

The explicit consideration of the stocks and the dynamic features caused by the stocks bring about difficulties in the empirical studies of the raw wood supply and silvicultural investment, because there are statistical data about growing stock and silvicultural-level only at about ten years' intervals. Specially in the raw wood supply studies, the stock aspect is seldom treated at all or is frequently abstracted away (Tervo 1984, Kuuluvainen 1984).

The importance of the stocks (or the allowable cutting) in the forest owners' decision making is taken into account in many cases by the interview or survey technique (e.g. Järveläinen 1983 and Riihinen 1963). The cross-section data collected by Järveläinen (1983) have been used in an attempt to construct raw wood supply function without any regard to the price (Kuuluvainen et al., 1984). Kasanen (1982) has done a very interesting attempt to estimate the forest owners' rate of time preference by means of the growing stock and annual growth of the Finnish forests as results of the Finnish forest inventories. Unfortunately it has not been possible

to take, for example, the silviculture into consideration.

In the empirical part of this study the stocks have been taken into consideration through the theoretically derived raw wood supply and silvicultural investment functions. The dynamics of the forestry behaviour has been approximated by means of the Nerlove's partial adjustment model. Because the silvicultural investment function has been estimated only with regard to the yound stand tending, the silvicultural stock accumulated by the tending measures bears a minor importance. Tikkanen (1981 and 1983) has constructed investment functions for different silvicultural measures, and there is no essential difference between results of his examination (e.g. Tikkanen 1981, p. 48) and the results of the investment examination of this study.

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Seloste

METSÄPOLITIIKAN TALOUDELLISET PERUSTEET

Tutkimus metsäpolitiikan tavoitteista ja keinoista

Tutkimuksen tarkoituksena on selvittää hallituksen metsäpolitiikan tavoitteen asettelun ja keinojen valinnan kansantaloudellisia perusteita. Tutkimus jakaantuu

- johdantoon
- tavoitetarkasteluun
- metsänomistajien optimaalisen metsätaloudellisen käyttäytymisen tarkasteluun
- tarkasteluun metsäpolitiikan keinojen vaikutuksesta metsänomistajien taloudelliseen käyttäytymiseen
- empiiriseen tarkasteluun ja
- yhteenvetoon.

Johdannossa tarkastellaan metsätalouden asemaa kansantaloudessa ja esitellään metsien makrotaloudellinen kasvufunktio. Metsätalouden oletetaan aina sekä horisontaalisesti että vertikaalisesti integroituneen johonkin muuhun toimialaan sekä metsien kasvu puuston ja metsänhoidon konkaaviksi funktioksi.

Tavoitteen asettelua tarkastellaan huoltotaseyhtälöön sijoitetun mallin avulla, jossa tuotanto on jaettu kahteen osaan: metsätalouden biologiseen tuotantoon, metsien kasvuun, ja muuhun tuotantoon, bruttokansantuotteen tuottamiseen. Malli perustuu varantoajatteluun, ja sen dynamiikka on kuvattu varantojen muutosyhtälöillä. Bkt:n oletetaan olevan lineaarisesti riippuvainen raakapuusta. Kansantalouden tavoitteeksi valitun kulutuksen maksimointi suoritetaan maksimiperiaattella kansantalouden investointien suhteen. Mallin optimitasapainotilaa voidaan pitää talouspolitiikan pitkän aikavälin tavoitteena. Metsäpolitiikan kannalta ovat merkittäviä seuraavat tasapainotarkastelun johtopäätökset:

- Optimitasapainon saavuttamiseksi on puuston hinnoittelu tärkeää: liian korkea hinta saattaa talouden epätasapainotilaan ja pienentää bkt:ta ja kulutusta. Liian alhainen hinta johtaa tasapainoon, jossa metsätalouden kaikkia mahdollisuuksia ei käytetä hyväksi.

- Tavoitetilassa on metsien kasvuun sidotun pääomakannan, metsänhoidon, rajakulumisen arvo yhtä suuri kuin sen tuloksena syntyvän puuston arvo.

- Jos puuston varjohinta on nolla, eivät metsätalouden investoinnit ole aiheellisia.

- Pienen aikapreferenssin käyttö johtaa suureen tavoitepuustoon ja korkeaan metsänhoidon tasoon.

Tutkimuksen metsäpoliittisten keinojen tarkastelun tarkoituksena on selvittää, miten yksittäiset metsäpolitiikan keinot vaikuttavat metsänomistajien taloudelliseen tavoitteen asetteluun ja sen välitykellä heidän metsätaloudelliseen käyttäytymiseensä. Yhtä metsäpolitiikan keinoa on tarkasteltu kerrallaan, keinoyhdistelmiä ei ole tutkittu. Raakapuun hinnan on oletettu olevan ajan funktio. Keinotarkastelun pohjaksi on määritelty optimaalisen metsätaloudellisen käyttäytymisen säännöt, jotka perustuvat metsänomistajien talouden optimitilan määritelmään, jonka mukaan heidän taloutensa tavoitetilana olevan optimitasapainon ehdot ovat

- metsänomistajien pääelinkeinon rajatuotanto on yhtä suuri kuin sen pääomakustannukset

- metsänomistajien metsien rajakasvu metsänhoidon suhteen on yhtä suuri kuin pääomakustannusten suhde raakapuun hintaan ja

- metsänomistajien metsien rajakasvu puuston suhteen on yhtä suuri kuin heidän aikapreferenssinsä ja raakapuun hinnan suhteellisen muutoksen erotus.

Edellä mainittujen ehtojen erisuuruustapauksissa metsänomistajat pyrkivät mahdollisimman nopeasti kehittämään varantonsa optimitilaan säätelemällä hakkuita ja investointeja.

Tutkitut metsäpolitiikan keinot ovat

- metsätalouden "puhtaan tulon" verotus
- myyntitulon verotus
- metsätalouden varantojen verotus
- myyntiverotus
- metsänhoitoinvestointien rahoituksen tukeminen ja
- myyntitulojen ohjaaminen investointeihin.

Tutkituista keinoista puuston verotus on teoreettisesti joustavin ja monipuolisin metsätalouden säätelyväline. Sillä voidaan esimerkiksi eliminoida raakapuun hinnan muutosten metsätaloudellisia vaikutuksia. Voimakas hinnan nousunopeus voi johtaa niin suureen tavoitepuustoon, että metsien kasvu on jopa maksimaalista pienempää.

Yksikkömyyntiverolla ja -tukipalkkiolla voidaan ohjata metsätalouden tavoitetilan puuston ja metsänhoidon suhdetta eli saada aikaan substituutiovaikutus puuston ja metsänhoidon välille eräissä tilanteissa. Metsätalouden investointeja tukemalla voidaan kohottaa metsänomistajien tavoitepuustoa ja -metsänhoitoa. Suomalainen metsäpolitiikka on suurelta osalta ollut valtion rahoitusehtojen säätelyä. Rahoitusehtojen säätelyn vaikutuksia investointikäyttäytymiseen on tarkasteltu investointifunktion avulla.

Raakapuun myyntitulojen hallinnollisella ohjaamisella metsänhoitoon on merkitystä silloin, kun metsänomistajat eivät tunne niitä taloudellisia mahdollisuuksia, joita metsät tarjoavat. Raakapuun myynnin tuloja *ad valorem*-verotuksella sekä *ad valorem*-tukipalkkioilla voidaan selkeästi vaikuttaa metsänomistajien metsätaloudelliseen käyttäytymiseen. Sen sijaan Suomessa sovellettavalla metsätalouden "puhtaan tulon" verotuksella ei voida vaikuttaa metsätalouden tuotannollisiin varantoihin, mutta kylläkin metsänomistajien pääelinkeinoon ja metsien omistukseen.

Empiirisesti on tarkasteltu yksityis- ja valtion metsätalouden hakkuu ja investointikäyttäytymistä sopeutusmallin avulla. Hakkuukäyttäytymisen tutkiminen on rajattu havutukkimarkkinoihin. Hakkuukäyttäytymistä koskevat tulokset voidaan tiivistää seuraavasti:

- Raakapuun tarjonta reagoi lyhyellä aikavälillä vastakkaismerkkisesti argumenttiensa muutoksiin esitetyn teorian mukaisesti. Sopeutus puuston tasapainoa kohti on havaittavissa empiirisesti. Esimerkiksi kantohinnan nousu aiheuttaa raakapuun tarjontaa heikentävän sopeutusreaktion.

- Tasapainotilan raakapuun tarjonnan jousto hinnan suhteen on ollut positiivinen ja suurempi kuin yksi.

- Tasapainotilan raakapuun tarjonta on heikentynyt Suomessa kaudella 1955–1979. Koska samanaikaisesti metsätalouden investoinnit ovat voimistuneet, on tarjonnan heikkenemisen oletettu johtuvan

  - metsänomistajien keski-iän kohoamisesta, minkä takia metsiä on kasvatettu perinnöksi
  - tulotavoitteisten hakkuiden vähenemisestä metsänomistajien tultua entistä riippumattomimmiksi metsätuloista ja
  - mahdollisista metsien tai metsäteollisuuden raaka-ainetarpeen rakennemuutoksista.

- Valtion metsätalouden raakapuun tarjontaan ei hinnan muutoksilla ole ollut vaikutusta. Tarjonta määräytyy kvantitatiivisen metsätaloussuunnittelun perusteella.

- Metsäpolitiikan keinojen vaikutusta raakapuun tarjontaan ei ole voitu tutkia, koska suoraan raakapuun tarjontaan vaikuttavia keinoja ei ole käytetty.

Investointikäyttäytymisen tutkiminen on rajattu taimikonhoitoon. Tuloksista voidaan mainita seuraavaa:

- Taimikonhoidon tasapainojousto raakapuun hinnan suhteen on korkea sekä yksityis- että valtion metsätaloudessa.

- Yksityismetsätalouden investointien sopeutuminen hinnanmuutoksiin on nopeaa, sen sijaan valtion metsätalous on hidasliikkeisempää.

- Valtio rahoittaa yksityismetsätalouden taimikonhoitoinvestointeja ja rahoituksen yksityistaloudellinen edullisuus on tehty riippuvaiseksi töiden työllistävästä vaikutuksesta. Työttömyysastetta on tarkastelussa käytetty sen takia eräänlaisena metsäpolitiikan instrumenttina josta taimikon hoitoinvestointien on todettu olevan riippuvainen.

- Taimikonhoitoinvestoinnit ovat voimistuneet autonomisesti tarkastelukaudella.

Appendix

On the aggregation of the separable logistic growth functions

An important feature of the separable logistic growth function is that it is easy to see, how the micro and macro-economic forest growth function can be derived by aggregation from the underlying forest stand growth functions.

Basic assumptions

Every forest stand comprises 1-n management units with uniform area. How the area of the management units is determined, is not treated in this study. For illustrative purposes, a homogeneous site quality is assumed. The forest stands and management units differ as concerns growing stock and silviculture. The aggregation is carried out by means of the management units; no reference is made to the stands.

The growth function of the management unit

The growth function of the management unit is

$$g_{il} = x + y_1 v_{il} + y_2 k_{il} + z_{11} v_{il}{}^2 + z_{22} k_{il}{}^2, \tag{A1.1.}$$

where

g = growth

v = growing stock

k = silviculture

i = subscript of the i th wood lot, $i = 1-N$

l = subscript of the l th management unit, $l = 1-L_i$

$x, y_1, y_2, z_{11}, z_{22}$ = constants of the growth function.

The aggregation of the micro-economic forest growth function

In order to derive the micro economic forest growth function, the growth functions of the management units (A1.1.) are summed up to [1].

$$g_{i.} = \sum_{l=1}^{L_i} g_{il} = xL_i + y_1 \sum_{l=1}^{L_i} v_{il} + y_2 \sum_{l=1}^{L_i} k_{il} + z_{11} \sum_{l=1}^{L_i} v_{il}^2 + z_{22} \sum_{l=1}^{L_i} k_{il}^2 + z_{22} \sum_{l=1}^{L_i} k_{il}^2. \tag{A1.2.}$$

From the formula of the population variance (e.g. Koutsoyiannis 1981, p. 534) it can derived

$$\sum_{l=1}^{L_i} v_{il}^2 = L_i\,(\bar{v}_{i.}^2 + S^2(\bar{v}_{i.})) \text{ and} \tag{A1.3.}$$

$$\sum_{l=1}^{L_i} k_{il}^2 = L_i(\bar{k}_{i.}^2 + S^2(\bar{k}_{i.})), \tag{A1.4.}$$

where

$\bar{v}_{i.}$ = mean growing stock of the management units

$\bar{k}_{i.}$ = mean silviculture of the management units

$S^2(.)$ = variance of the management units with respect to the mean of the growing stock and silviculture.

When it is assumed that the variances are depending linearly on the respective means, i.e.

$$S^2(\bar{v}_{i.}) = c_1\bar{v}_{i.} \text{ and} \tag{A1.5.}$$

$$S^2(\bar{k}_{i.}) = c_2\bar{k}_{i.} \tag{A1.6.}$$

we get

$$g_{i.} = xL_i + y_1 v_{i.} + y_2 k_{i.} + z_{11} L_i(\bar{v}_{i.}^2 + c_1\bar{v}_{i.}) + z_{22} L_1(\bar{k}_{i.}^2 + c_2\bar{k}_{i.}). \tag{A1.7.}$$

Because

$$L_i\bar{v}_{i.} = v_i\,, \tag{A1.8.}$$

$$L_i\bar{v}_{i.} = k_i\,, \tag{A1.9.}$$

$$L_i\bar{v}_{i.}^2 = L_i(\frac{v_{i.}}{L_i})^2 = \frac{1}{L_i}\,v_{i.}^2 \text{ and} \tag{A1.10.}$$

$$L_i\bar{v}_{i.}^2 = L_i(\frac{k_{i.}}{L_i})^2 = \frac{1}{L_i}\,k_{i.}^2\,, \tag{A1.11.}$$

the micro-economic forest growth function can be expressed as

$$g_{i.} = xL_i + (y_1+c_1z_{11})\,v_{i.} + (y_2+c_2z_{22})\,k_{i.} + \frac{z_{11}}{L_1}\,v_{i.}^2 + \frac{z_{22}}{L_i}\,k_{i.}^2 \tag{A1.12.}$$

The aggregation of the macro-economic forest growth function

In order to derive the macro-economic forest growth function, the micro-economic forest growth functions (A1.12.) are summed up to

$$g_{..} = \sum_{i=1}^{N} g_{i.} = xL_{.} + (y_1+c_1z_{11})\,v_{..} + (y_2+c_2z_{22})\,k_{..} + \sum_{i=1}^{N} \frac{z_{11}}{L_i}\,v_{i.}^2 + \sum_{i=1}^{N} \frac{z_{11}}{L_i}\,k_{i.}^2\,. \tag{A1.13.}$$

By denoting

$$\frac{1}{\sqrt{L_i}} = d_i \tag{A1.14.}$$

we get

$$\frac{1}{L_i}\,v_{i.}^2 = (d_i v_{i.})^2 \text{ and} \tag{A1.15.}$$

$$\frac{1}{L_i}\,k_{i.}^2 = (d_i k_{i.})^2\,, \tag{A1.16.}$$

when (A1.13.) can be expressed as

$$g_{..} = xL_{.} + (y_1+c_1z_{11})\,v_{..} + (y_2+c_2z_{22})\,k_{..} + z_{11} \sum_{i=1}^{N} (d_i v_{i.})^2 + z_{22} \sum_{i=1}^{N} (d_i k_{i.})^2\,. \tag{A1.17.}$$

By applying the population variance formulas to the quadratic terms of (A1.17.) and by assuming the linear dependencies

$$S^2(\overline{d_i v_{i.}}) = f_1\overline{d_i v_{i.}} \text{ and} \tag{A1.18.}$$

$$S^2(\overline{d_i k_{i.}}) = f_2\overline{d_i k_{i.}}\,, \tag{A1.19.}$$

where

$$\overline{d_i v_{i.}} = \frac{\sum_{i=1}^{N} d_i v_{i.}}{N} \text{ and} \tag{A1.20.}$$

$$\overline{d_i v_{i.}} = \frac{\sum_{i=1}^{N} d_i k_{i.}}{N} \text{ and} \tag{A1.21.}$$

we arrive at the macro-economic forest growth function

$$g_{..} = xL + (y_1+c_1z_{11})\,v_{..} + (y_2+c_2z_{22})\,k_{..} + f_1 z_{11} \sum_{i=1}^{N} d_i v_{i.} + f_2 z_{22} \sum_{i=1}^{N} d_i k_{i.} + \frac{z_{11}}{N}(\sum_{i=1}^{N} d_i v_{i.})^2 + \frac{z_{22}}{N}(\sum_{i=1}^{N} d_i k_{i.})^2\,. \tag{A1.22.}$$

When it is assumed that the micro-economic forestry units, wood lots are of the same size, i.e.

$$L_1 = L_2 = \cdots = L_i = \cdots = L_N = \bar{L} \tag{A1.23.}$$

and when

$$d_1 = d_2 = \cdots = d_i = \cdots = \bar{d} = \frac{1}{\sqrt{\bar{L}}} \tag{A1.24.}$$

the macro-economic forest growth function can be written

$$g_{..} = xN\bar{L} + (y_1+(c_1+\frac{f_1}{\sqrt{\bar{L}}})z_{11})\,v_{..} + (y_2+(c_2+\frac{f_2}{\sqrt{\bar{L}}})z_{22})\,k_{..} + \frac{z_{11}}{N\bar{L}}\,v_{..}^2 + \frac{z_{22}}{N\bar{L}}\,k_{..}^2. \tag{A1.25}$$

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[1] The dot in place of an subscript means summation over that subscript.

Summary of the results

The main results are expressed in Table A1. The consideration can be extended to include also other variables as the growing stock and silviculture as growth factors. For example the age of the management unit can be taken into consideration. At the aggregate level the separable growth function has the total of the management unit ages as a variable with analogous coefficients to the growing stock and silviculture. The non-homogeneous site quality can be taken under examination. The aggregation over the site types leads to the weighted sums. The variance functions can be assumed to be dependent on the total stocks or their shape can be assumed concave quadratic without any difficulties in the derivation of the aggregate growth function. Because the management unit growth functions are strictly concave, so the aggregate growth functions are also concave as weighted sums.

Table A1. Coefficients of the separable logistic growth function $g = a_0 + a_1v + a_2k + b_{11}v^2 + b_{22}k^2$ expressed by means of the coefficients of management unit growth function, numbers of the management units in a wood lot L_i and mean size of the wood lots L. Constants c_1, c_2, f_1 and f_2 are the coefficients of the variance functions of the growing stock and silviculture within wood lots (c_1 and f_1) and between various wood lots (c_2 and f_2).

coefficient of the growth function	constant	first-order terms		second-order terms	
		growing stock	silviculture	growing stock	silviculture
level of the consideration	a_0	a_1	a_2	b_{11}	b_{22}
management unit	x	y_1	y_2	z_{11}	z_{22}
micro-economic, wood lot	xL_1	$y_1+c_1z_{11}$	$y_2+c_2z_{22}$	$\frac{z_{11}}{L_i}$	$\frac{z_{22}}{L_i}$
macro-economic, wood lots of the same size	$xN\bar{L}$	$y_1+(c_1+\frac{f_1}{\sqrt{\bar{L}}})z_{11}$	$y_2+(c_2+\frac{f_1}{\sqrt{\bar{L}}})z_{22}$	$\frac{z_{11}}{N\bar{L}}$	$\frac{z_{22}}{N\bar{L}}$

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