ACTA FORESTALIA FENNICA

204

AN ECONOMIC MODEL UNDERLYING THE CHOICE OF CAPITAL INTENSITY IN TIMBER PRODUCTION

PUUNTUOTANNON PÄÄOMAINTENSITEETIN VALINNAN PERUSTANA OLEVA TALOUDELLINEN MALLI

Raymond K. Omwami

SUOMEN METSÄTIETEELLINEN SEURA 1988

Suomen Metsätieteellisen Seuran julkaisusarjat

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Publications of the Society of Forestry in Finland

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Raymond K. Omwami

Tiivistelmä

PUUNTUOTANNON PÄÄOMAINTENSITEETIN VALINNAN
PERUSTANA OLEVA TALOUDELLINEN MALLI

To be presented, with the permission of the Faculty of Agriculture and Forestry of the University of Helsinki,
for public criticism in Auditorium II of Metsätalo, Unioninkatu 40B,
Helsinki, on 26th of August 1988 at 12 o'clock noon

HELSINKI 1988

OMWAMI, R. K. 1988. An economic model underlying the choice of capital intensity in timber production. Tiivistelmä: Puuntuotannon pääomaintensiteetin valinnan perustana oleva taloudellinen malli. Acta Forestalia Fennica 204. 42 p.

The process of capital accumulation in timber production has been examined in this study. A detailed explanation of forest industry new investment in terms of productive capacity as the determinant of national forest policy target growing stock and silviculture is presented. The basis of the explanation of forest industry productive capacity was a linear vertically integrated input-output production model. The model was used to derive a macroeconomic equilibrium condition specifying forest sector aggregate demand as an integral part of the national economy. Timber production has been construed as a state variable system and the Maximum Principle used to derive silvicultural investment criterion. The derivation of the investment criterion was formulated as a dynamic problem in a labour surplus economy with linkage between savings and choice of silvicultural technology defined via income distribution between wages and profit. Maximization of aggregate consumption was specified as the goal of timber production.

By assuming a state of sub-optimal savings rate, it is shown that the real cost of labour is not zero in a labour surplus economy. Because unemployed labour is not a free commodity, it is concluded that capital-intensive silvicultural technology represents an optimal means of maximizing aggregate consumption in a labour surplus economy, contrary to the recommendation of social marginal productivity theory.

Tutkimus käsittelee pääoman akkumuloitumisprosessia puuntuotannossa. Tehdyssä tarkastelussa selitetään yksityiskohtaisesti, kuinka metsäpolitiikan tavoitteenasettelussa metsäteollisuuden uusinvestoinnit määräävät tavoitepuuston ja metsänhoidon tason. Uusinvestoinnit ymmärretään muutoksiksi tuotannollisessa kapasiteetissa. Metsäteollisuuden tuotannollinen kapasiteetti perustuu lineaariseen, vertikaalisesti integroituun panos-tuotosmalliin. Tätä mallia, jossa metsä- ja puutalouden kokonaiskysyntä on osa kansantalouden kokonaiskysyntää, käytetään makrotaloudellisen tasapainoehdon johtamisessa. Puuntuotantoa kuvataan tilamuuttujasysteemillä ja puuntuotannon investointikriteerin johtamisessa käytetään maksimointiperiaatetta. Investointikriteerin johtaminen muotoillaan dynaamiseksi ongelmaksi taloudessa, jossa on ylimääräistä työvoimaa. Yhteys säästöjen ja metsätaloudessa käytettävän teknologian valinnan välillä määritetään palkkojen ja voiton välisen tulonjaon avulla. Puuntuotannon tavoitteeksi määritetään kokonaiskulutuksen maksimointi. Olettamalla, että säästöaste on sub-optimaalinen, osoitetaan, että työvoiman todellinen kustannus ei ole nolla, vaikka taloudessa on ylimääräistä työvoimaa. Koska ylimääräinen työvoima ei ole vapaa hyödyke, päätellään, että pääomaintensiivisen teknologian käyttö metsätaloudessa on optimaalinen keino maksimoida kokonaiskulutusta tällaisessa taloudessa. Tämä on sosiaalisen rajatuottavuusteorian mukaiseen suositukseen nähden vastakkainen päätelmä.

Key words: vertical integration production, population, technology, consumers' preferences, intensive silviculture
ODC 903 + 91 + 92

Author's address: University of Helsinki, Department of Social Economics of Forestry, Unioninkatu 40B, 00170 Helsinki, Finland

ISBN 951-651-082-5
ISSN 0001-5636
Karisto Oy:n kirjapaino
Hämeenlinna 1988

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List of symbols

LATIN LETTERS

A	– matrix of fixed coefficient technology
a_n	– a vector of coefficients of labour inputs
a_{n-1}	– a vector of roundwood supplied by timber production to the forest industry
B	– roundwood cuttings measured in cubic meters
b	– a coefficient of capital input required in the production of unit of forest sector final commodity
G	– forest growth in cubic meters
g	– population growth rate
H	– Hessian matrix
$\mathscr{H}$	– current value Hamiltonian function
$\mathscr{h}$	– discounted value Hamiltonian function
I	– investment
K	– capital stock
k	– rate of depreciation of capital stock
L	– labour services
M	– imports
m	– premium on savings
N	– population
n	– total number of sectors in the national economy
p	– welfare (shadow) prices
Q	– national total supply comprised of domestic production and imports
q	– forest sector total supply
r	– social rate of time preference
S	– savings
t	– time
U	– measure of standard of living
V	– growing stock measured in cubic metres
W	– real cost of labour
w	– nominal wage rate
X	– exports
x	– forest sector per capita demand for investment
Y	– national aggregate demand
y	– forest sector aggregate demand
Z	– national production

GREEK LETTERS

α (alpha)	– proportion of labour force employed in the forest sector
β (beta)	– a coefficient of labour input required in the production of a unit of the forest sector final commodity
Γ (capital gamma)	– forest growth function
γ (gamma)	– propensity to consume
δ (delta)	– market rate of interest
ζ (zeta)	– forest sector exports
η (eta)	– rate of change in disembodied technological advance
λ (lambda)	– welfare (shadow) prices
μ (mu)	– imports
ξ (xi)	– per capita consumption of forest sector final commodity
Π (capital pi)	– real domestic absorption
π (pi)	– the value of forest sector imports measured in export-equivalent
ρ (rho)	– rate of change in per capita demand for forest sector final commodity
υ (upsilon)	– labour employment per unit of capital
ψ (psi)	– consumers' preference function
ω (omega)	– coefficient of roundwood input required per unit of forest sector final commodity

PREFACE

The aim of this study was to apply concepts of economic theory relevant to the formulation of a long-term timber production model as a basis of forest policy. Professor Päiviö Riihinen has been a perpetual source of inspiration and guidance through discussions and encouraging support that have surpassed formal academic relationships.

I have also had an opportunity to study under the supervision of Professor Matti Keltikangas and Professor Hans Gregersen. Dr. Markku Simula and Dr. Seppo Vehkamäki have read the manuscript several times and made valuable comments. Professor Seppo Honkapohja and Mr. Jari Kuuluvainen read earlier versions of the manuscript and made valuable comments.

My son Päiviö and daughter Irene have been very cooperative during the course of this study. Maija-Liisa has made it possible to pursue this study and have a family without the two goals becoming a zero sum game.

Wendy Vardaman proofread the manuscript. The study has been supported financially by the Government of Kenya and Suomen Kehitysyhteistyö.

I am deeply grateful to my teacher Professor Riihinen and express my sincere thanks to the above mentioned persons and institutions whose contributions have been decisive to the completion of this study.

Helsinki, May 1988

Raymond K. Omwami

1. INTRODUCTION

1.1. Timber production goal

Economic agents' (households, business firms, and government agencies) behaviour is assumed to be governed by the Economic Principle (Smith 1776). According to the Economic Principle, the purposeful behaviour of economic agents is to move economic resources from less to more advantageous lines of production whenever an opportunity to do so arises.

In forest management (planning and decision making for use of economic resources in forestry production) forest owners, like other agents, set goals for themselves whenever they have a desire to accomplish something. For example, goals are set for timber production as a source of raw material and energy in the economy. Goals may also be set in order to meet other benefits of forestry production, e.g., range forage, environmental conservation, recreation, etc. All these examples show that the study of the purposeful behaviour of forest owners is a study of forest production goals.

Given the wide range of benefits of forestry production, it seems plausible to assume that goal-setting in forest management is a combination of conscious and unconscious forest owners' purposeful behaviour. The combination of conscious and unconscious goals in forestry production raises the issue of economic relationships between them. For example, it is important to know whether the relationship between timber production and environmental conservation is competitive, neutral, or complementary in cost and output range for a given forest area.

Goal-setting in practice should take into account all benefits accruing from the forest resource (Row 1977). At the analytical stage of goal-setting, it is desirable to discern forestry production goals in order to determine how they may be accomplished. This study is restricted to the goal of timber production. Timber production is comprised of all forestry activities (excluding logging) responsible for the supply of roundwood as a raw material to the wood processing firms in a developing economy. A developing economy is defined as an economy experiencing changes in population, technology, and consumers' preferences. In this economy, the timber production goal model is the basis of national forest policy whose aim is to ensure the supply of roundwood that is consistent with wood processing firms' productive capacity.We shall assume that timber growing stock is a product of silvicultural investment. It is an investment in the sense that it represents a net real increase in the nation's real capital.

1.2. The need for forest policy

In reality, the Economic Principle by itself gives no guarantee that economic resources in a developing economy will be fully utilized (employed). When a nation's resources are not fully employed, its economy is said to be in a state of disequilibrium. When economic agents' purposeful behaviour as guided by the Economic Principle as an invisible planner fails to attain equilibrium in a national economy, full employment of economic resources has to be actively pursued as an explicit target of economic policy by a visible central planner.

A nation in which the Economic Principle as an invisible planner and government as a visible planner co-exist is said to have a mixed economy. The aim of government is to ensure an efficient national economy in which the balance of demand and supply equilibrium in the labour, commodity, and money markets is attained. The goals of economic policy are (Musgrave 1959):

1) equity in the distribution of income
2) efficiency in the allocation of resources, and
3) stabilization of aggregate demand.

We may add a fourth goal of economic policy to Musgrave's list:

4) growth of the national economy.

Efficiency and growth are considered to be goals of economic policy consistent with the purpose of forest policy. Links between equity and stability as goals of economic policy and the purpose of forest policy may exist, but they are tenuous.

Efficiency as a goal of economic policy seeks to eliminate inefficiency in resource allocation which may arise as a result of market failure and/or missing markets (Pigou 1952). Policy instruments and tools of policy analysis are well documented in this area of economic policy; see, e.g., (Just et al 1982, Friedman 1984, Johansson 1987). The aim of government intervention is to ensure that marginal social benefits and costs are in equilibrium. Growth constitutes the goal of development strategy.

It is growth policy that is consistent with the aim of this study. Therefore, the causes of dynamic changes in silvicultural investment in timber production constitute the basis of the study. These causes are the same as the factors responsible for dynamic changes in a developing economy (population, technology, and consumers' preferences).

1.3. The paradigm

Various paradigms contend for supremacy in the area of economic policy (Hicks 1965). On the basis of these paradigms, refutable hypotheses are formulated (Hicks 1979). The purpose of policy analysis, which links an economic model (like the one to be derived in this study) and a policy model (like the one used in forest policy analysis), is to ascertain the effectiveness of a paradigm's specified causal factors.

The nature of the problem posed by timber production constitutes the paradigm criterion. In the short run the quantity of growing stock sets limits upon roundwood cutting possibilities, although new plantings may be undertaken during this period. The feasible cutting programmes during the period must be consistent with the current amount of growing stock. Thus, from that point in time when feasible cutting possibilities are constrained by the existing growing stock until this stock ceases to be a constraint, timber is a scarce commodity and it presents an allocation problem to policymakers over a finite time horizon.

The terminal growing stock, unlike the current (initial) growing stock, does not limit roundwood cutting possibilities. It can be changed by current silvicultural investment behaviour; it is nevertheless of great importance just how it is changed. It presents to policymakers the problem of ascertaining a dynamic silvicultural investment programme that will guarantee a maximum roundwood cutting programme from a set of feasible silvicultural investment programmes. Growing stock, unlike merchantable timber, is a nonscarce commodity in the sense that it can be varied through time because of advances in technology.

Based on the preceding concise account of the nature of the problem of timber production, we conclude that timber is a stock and a flow variable commodity. This conclusion is indeed a common feature in studies of demand for and supply of merchantable timber, e.g., Duerr (1960), Zivnuska (1975), and Omwami (1986); however, its implication has not been considered seriously in traditional forest production goal studies. To show how the dual nature of timber production has been modelled inconsistently, it will suffice to point out the theory underlying traditional forest production goal studies.

Meaningful theoretical research work in the field of forest production goal studies was pioneered by Vaux and Zivnuska (1952), Gregory (1955), and Riihinen (1963). The theory used in these studies is of Marshall (1920). Since then, we have voluminous literature on this subject employing the same theory as that adopted in the pioneer research just mentioned.

Marshallian theory is essentially a static model, suitable for price determination by equilibrium of demand and supply. No distinction exists between stocks and flows in this model; the two variables enter into the determination of equilibrium price in exactly the same way. Applied literally to timber production, it means that the balance of supply and demand equilibrium in a single period contains stock as well as flow elements.

The lack of distinction between stock and flow aspect in timber production is a major constraint to equilibrium studies in this area. Silvicultural investment and roundwood cuttings in the current period determine the amount of growing stock in the following period. An increase in demand for roundwood gives a signal to increase future growing stock; the converse holds with respect to a decrease in demand.

If, instead of assuming a single stock - flow equilibrium in a given period, we consider stock equilibrium separate from flow equilibrium, we immediately notice that it is the absence of stock equilibrium that is the cause of disequilibrium in timber production. Specifically, the stock aspect of timber production makes it possible for mistakes in the current investment and cutting behaviour to be carried forward.

Optimal timber production behaviour in the short run (static equilibrium) does not imply optimality in the long run (dynamic equilibrium). When the stock aspect of timber production is explicitly taken into account, we need a dynamic theory in order to derive dynamic equilibrium conditions for growing stock.

By applying Marshallian theory in the study of timber production, all traditional forest production goal studies inherited the shortcomings of investigating a dynamic problem on the basis of a static theory (Duerr 1977).

We consider growth theory (a theory of long term economic growth) as a particular method of dynamic economics (Hicks 1965, 1985; Baumol 1970) to be propitious to the study of long-term timber production. The classical models of steady state growth that imply economic development has a natural tendency to a stationary state (Hicks 1985, pp. 29–43) are inconsistent with a developing economy as defined in this study. The temporary equilibrium methods of Marshall (1920) – short run and long run analysis – and Lindahl (1939) – period analysis – are also inappropriate because of their lack of distinction between stock equilibrium and flow equilibrium concepts (Hicks 1985, pp. 44–51; 62–80).

The concepts and tools of analysis required for this study may be found in growth models that are a derivative of Keynes' short run macroeconomic analysis (Keynes 1936). These models are distinguished by the following equilibrium concepts:

- the von Neumann Maximal rate of growth (von Neumann 1937)
- the turnpike theorem (Dorfman et al. 1958)
- the Golden Rule (Phelps 1961).

The focus of attention in these models is on the time path of an evolving economy always in momentary full employment equilibrium. The analysis is based on restrictive assumptions that consider consumers' preferences and technology to be unchanging as the economy expands.

Pasinetti's growth model (Pasinetti 1981), investigates dynamic changes in production, consumption, investment and employment by relaxing the restrictive assumptions that keep consumers' preferences and technology constant in the previous growth models. Because of the way a developing economy has been defined in this study, the basis of the timber production model we shall formulate is Pasinetti's growth theory (Pasinetti 1981).

1.4. Study outline

Timber production has been totally integrated in an open national economy. This means that a state of equilibrium in the national economy implies a state of equilibrium in timber production and vice versa. The inherent link of a state of equilibrium between the national economy and timber production has been defined by Keynes' macroeconomic equilibrium condition. Based on this concept, the influence of changes in population, consumers' preferences, and technology on growth and structural changes in the wood processing firms linked to timber production by the supply of roundwood has been investigated from a national economy point of view. The concept of forest industry has been used to define all wood processing firms in the national economy.

Changes in the national economy caused by business cycle phenomena are transmitted to timber production and vice versa, through the household sector which

- receives forest final products from wood processing firms and roundwood as a pure consumption good directly from timber production
- supplies labour services and investment goods to wood processing firms and timber production.

This approach to the description of the national economy as a circular process subsumes all aspects of a modern economy – production, demand, employment, investment, and trade – in the household sector. The simple description of the circular process made it possible to study timber production from a national economy point of view without subdividing it into various industries characteristic of Leontief input-output analysis (Leontief 1951).

In a Leontief input-output model, the subdivision criterion is the industry producing a certain commodity, intermediate or final, and the problem is to reckon where its inputs come from and where its outputs go to. In the vertically integrated model formulated, the criterion was the process of production of forest final commodities (final demand), and the problem was to build conceptually behind these commodities a vertically integrated sector which, by passing through all the intermediate commodities, went back to the original inputs: a vector of labour coefficients and a vector of the stocks of physical capital goods. Labour has been treated as the only primary factor of production and the vector of the stocks of physical capital goods regarded as a particular composite commodity representing a unit of vertically integrated productive capacity for the forest final commodities. The concept of vertical integration implied that no intermediate stage, and consequently no intermediate forest product, has been represented explicitly.

The analysis was independent of any kind of institutional set-up, i.e., neither a market nor a socialist economy was considered since the purpose of the study was to derive a planning model. In reality, the means adopted in order to achieve the optimum timber production goal would be specific to an institutional set-up. Since a study of the application of means of forest policy for achieving the target of timber production has not been done, it is unnecessary to restrict the model formulation to a particular institutional set-up.

From the concise account of the nature of the problem of timber production, we learn that policymakers are concerned with two interrelated but distinct problems:

- allocation of roundwood over a finite time horizon
- changing the level of the growing stock.

Allocation of roundwood over a finite time horizon constitutes a trade problem. A solution to it will entail the regulation of the roundwood market under a given forest industry (wood processing firms) capacity. The logical starting point of analysing this problem is the roundwood market.

Changes in the level of the growing stock constitute a production problem. In the long run, the level of the growing stock is variable; hence, it becomes possible to eliminate the constraint the current level of growing stock imposes on the feasible roundwood cutting programmes consistent with the existing or planned forest industry capacity. The logical starting point of analysing this problem is the context of production and consumption of forest products. In this respect, the concept of forest sector has been used to define all activities of timber production, forest logging, and forest industry that are responsible for the supply of forest final products.

The main objective of this study has been to derive a planning model of choice of capital intensity in timber production. Forest growth has been specified as a function of biological growing stock and inanimate capital stock. The latter has been referred to as silviculture. The equilibrium conditions for forest industry productive capacity derived determine the level of growing stock and silviculture that constitutes the target of long-run national forest policy. Determination of optimum capital intensity, i.e., the ratio of silvicultural inputs to a unit land area or simply the level of silvicultural intensity – the amount of investment per unit area – has been studied as an optimization problem concerned with the choice of a particular technologically feasible path that maximizes aggregate consumption. Silvicultural technology was defined as an input package comprised of forestry machinery and equipment, intermediate commodities, and quantities of labour made up of different skills. In the optimization problem, only uncertainty attributed to the pure role of time, i.e, the existence of a future that will generally differ from the present, was taken into account.

The discussion paid special attention to the policy implications of the analytical investigation accomplished. Finally, a summary of the main results of what is considered to be a new approach to the study of long-term timber production has been presented.

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2. A VERTICALLY INTEGRATED FOREST SECTOR PRODUCTION MODEL

2.1. An open national economy

2.1.1. Integration of exports and imports in an input-output analysis

To show how export and import considerations are made an inherent part of the subsequent input-output analysis, we assume that they are perfect substitutes in all uses. This simplifying assumption facilitates the integration of trade in the analysis without taking into account explicitly the reality of product differentiation characterized by different prices and degrees of substitutability.

The following non-negative vectors and matrix are defined:

a production vector

$$\mathbf{Z} = (Z_1 . . . Z_n)$$

an import vector

$$\mathbf{M} = (M_1 . . . M_n)$$

a domestic aggregate (final) demand vector

$$\mathbf{Y} = (Y_1 . . . Y_n)$$

an export vector

$$\mathbf{X} = (X_1 . . . X_n)$$

a matrix of technical coefficients

$$\mathbf{A} = \begin{pmatrix} a_{11} & \cdot & \cdot & \cdot & a_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{n1} & \cdot & \cdot & \cdot & a_{nn} \end{pmatrix}$$

A Leontief (1951) material balance equation for an open national economy may be specified as follows:

$$(2\text{–}1) \quad \mathbf{Z} + \mathbf{M} = \mathbf{AZ} + \mathbf{Y} + \mathbf{X}$$

Total production required to satisfy domestic demand and trade is given by

$$(2\text{–}2) \quad \mathbf{Z} = (\mathbf{I} - \mathbf{A})^{-1}(\mathbf{Y} + \mathbf{X} - \mathbf{M})$$

If in equation (2–2) **M** is sufficiently large, aggregate demand will be negative. Such an undesirable outcome may be avoided by assuming that the ratio of imports to domestic production is fixed by industries (Chenery and Clark 1959):

$$(2\text{–}3) \quad \mu_i = M_i / Z_i \geq 0$$

where industries are indexed by i and μ is the import ratio.

If we define a diagonal matrix of import ratios,

$$\hat{\mu} = \mathrm{diag}(\mu_1 . . . \mu_n)$$

and denote total supply by Q, then

$$Q = \mathbf{Z} + \mathbf{M} = \mathbf{Z} + \hat{\mu}\mathbf{Z} = (\mathbf{I} + \hat{\mu})\,\mathbf{Z}$$

and therefore,

$$(2\text{–}4) \quad \mathbf{Z} = (\mathbf{I} + \hat{\mu})^{-1} Q$$

Now,

$$(2\text{–}5) \quad Q = \mathbf{AZ} + (\mathbf{Y} + \mathbf{X})$$

and substituting equation (2–4) in (2–5) and solving for Q we obtain:

$$(2\text{–}6) \quad Q = (\mathbf{I} + \mathbf{A^*})^{-1}\mathbf{Y^*}$$

where

$$\mathbf{A^*} = \mathbf{A}\,(\mathbf{I} + \hat{\mu})^{-1}$$

$$\mathbf{Y^*} = \mathbf{Y} + \mathbf{X}$$

Note that

$$(\mathbf{I} + \hat{\mu}) = \mathrm{diag}(1/1+\mu_1 \quad . . . \quad 1+\mu_n)$$

hence the technical coefficients of **A** are the the same as those of **A*** except for the latter matrix diagonal coefficients which become $a_{ij}{}^* = a_{ij} /(I + \mu_j)$. The coefficient $a_{ij}{}^*$ defines the intermediate inputs from industry i required per unit of total supply of industry j.

Aggregate demand in equation (2–6), unlike that in equation (2–2), will never be negative.

2.1.2. Derivation of coefficients of labour and capital inputs

The starting point of a vertically integrated analysis is the traditional Leontief input-output analysis. Using Leontief concept of an industry, the physical quantities of a forest sector may be defined as follows:

(2–7) $(\mathbf{I} - \ddot{\mathbf{A}})q_t = y_t^*$

(2–8) $\mathbf{a}_{(n)}q_t = L_t$

(2–9) $\mathbf{A}^*q_t = K_t$

where

$\mathbf{A}^*$ = a non-negative square matrix of capital goods in national economy in which the jth column represents fixed capital goods required for the production of a unit of forest sector final commodity

$\ddot{\mathbf{A}}$ = a non-negative square matrix of capital goods in national economy in which the jth column represents circulating capital goods used up annually in the production of a unit of forest sector final commodity

$\mathbf{a}_{(n)}$ = a non-negative row vector whose jth component a_{nj} denotes the annual input of labour required by a unit of forest sector final commodity

L_t = labour services in man years required in the forest sector in year t

K_t = capital goods required in the forest sector at beginning of year t, in order to obtain total supply of commodities from the forest sector at the end of year t

q_t = total supply of commodities produced in the forest sector in year t

y_t^* = forest industry aggregate (final) demand comprised of consumption and new investment in year t

Equation (2–7) and (2–8) specify the flow of forest sector production and labour services required annually to obtain a unit of the final commodity y*. Equation (2–9) specifies the amount of capital stock required at the beginning of the year for production to be effected.

The industry classification defined by equations (2–7) – (2–9) has the advantage of being immediately observable. However, note that to define equations (2–7) – (2–9), we implicitly assumed that timber production, logging, and forest industry (which we have defined in this study as the forest sector) make up a single industry in the national economy and temporarily referred to it as the 'forest sector.' According to the traditional input-output analysis, in defining equations (2–7) – (2–9), we must clearly specify timber production, logging, wood processing firms as distinct industries in the national economy. The painstaking approach of defining the activities of the forest sector is appropriate for empirical analysis but does not lend itself easily to a dynamic analysis of production and consumption of forest products from a macroeconomic point of view.

The production of final commodities in a vertically integrated national economy is defined by the matrix

(2–10) $$\begin{pmatrix} y_{1t}^* & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & y_{2t}^* & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & y_{mt}^* \end{pmatrix} = Y_t^*$$

where

y_t^* = the proportion of the national economy final demand coming from sector j (j = 1, . . . , m) in year t

Y_t^* = national economy final demand in year t

Let the jth component of matrix (2–10) represent the forest sector in a vertically integrated input-output model of the national economy. The relationship between 'forest sector' in the national Leontief model and forest sector in a vertically integrated model is obtained from equations (2–7) – (2–9) as follows (Pasinetti 1973) :

(2–11) $q_t = (\mathbf{I} - \ddot{\mathbf{A}})^{-1} y_t^*$

(2–12) $L_t = \mathbf{a}(\mathbf{I} - \ddot{\mathbf{A}})^{-1} y_t^*$

(2–13) $K_t = \mathbf{A}^*(\mathbf{I} - \ddot{\mathbf{A}})^{-1} y_t^*$

The coefficient in equation (2–11) is the familiar Leontief inverse matrix (Leontief 1951), whose jth column contains the series of heterogeneous commodities that are directly and indirectly required in the national economy to obtain a unit of forest sector final commodity. To interpret the coefficients in equations (2–12) – (2–13), we define

(2–14) $\mathbf{a}(\mathbf{I} - \ddot{\mathbf{A}})^{-1} \equiv \beta$

(2–15) $\mathbf{A}^*(\mathbf{I} - \ddot{\mathbf{A}})^{-1} \equiv b$

and rewrite equations (2–12) – (2–13) as follows:

(2–12') $L_t = \beta y^*$

(2–13') $K_t = b y^*$

β and b express in a consolidated way the quantity of labour and capital stock required directly and indirectly in the national economy to obtain a unit of forest sector final commodity whether for consumption or investment.

The derivation of equations (2–12') – (2–13') from (2–7) – (2–9) shows that in a vertically integrated model, all inputs that enter the production of a unit of a final commodity are resolved into labour and capital stock. Capital stock in this sense is a composite commodity made up of different physical goods in different proportions.

Neither β nor b are directly observable, but they can be obtained through post-multiplication by $(\mathbf{I} - \ddot{\mathbf{A}})^{-1}$ from quantities **a** and **A*** that are directly observable.

Let us exclude the vector of circulating capital goods in which the jth component represents roundwood assortments supplied by timber production to the forest industry from the interindustry matrix; post-multiplying this vector by the Leontief inverse matrix, the proportion of roundwood content in the composite capital stock of the forest sector required to produce a unit of final commodity is derived:

(2–16) $\mathbf{a}_j(\mathbf{I} - \ddot{\mathbf{A}}) = \omega$

where

$\mathbf{a}_j$ = forest industry roundwood requirements

ω = vertically integrated roundwood coefficient, specifying the amount of roundwood required per unit of forest sector final demand (commodity)

Given the coefficient ω, the link between timber production and forest sector aggregate demand (i.e., final demand) is defined by

(2–17) $B = \omega y^*$

where

B = roundwood cuttings measured in cubic meters.

2.2. Model

The starting point of investigating production, consumption, investment, and employment in the forest sector is a national economy in which changes in population, technology, and consumers' preferences are assumed to be constant through time.

The entire household sector (population) contributes to forest sector aggregate demand. However, only a certain proportion of the population is employed in the forest sector:

(2–18) $L = \alpha N$

where

L = the amount of labour services employed in the forest sector

N = national population

α = the proportion of national population employed in the forest sector

Since there is no change in the state of technology, the technology employed in the forest sector is constant. To simplify the analysis, we shall assume that labour is the only primary factor of production. The coefficient of production of the forest sector final commodity and capital stock are denoted, β and b, respectively. The production of the final commodity is by means of labour supplemented by the capital stock. Because of the simplifying assumption made, production of capital stock is by means of labour alone. We also assume that capital stock is subject to an exogenously determined geometric rate of depreciation denoted by k. Since the forest sector technology is a subset of the national technology A* equation (2–6), which was assumed to be viable, it must be true that the technology we are now describing is equally viable.

Forest sector final commodity is partitioned into

(2–19) $y^* = (y_1^*, y_2^*)$

where
y^* = final commodity
y_1^* = final commodity allocated to consumption
y_2^* = final commodity allocated to investment

Per capita demand for consumption and investment are denoted by, ξ and x, respectively.

Recalling that the forest sector is a sub-system of an open economy, allocation to consumption of the final commodity is comprised of domestic consumption and exports. External consumption eliminates domestic consumption as a constraint to full employment. It may also, depending on the state of development of the national economy, serve as a source of financing domestic production by means of foreign technology. Moreover, the existence of foreign trade frees the level of investment from the constraint of domestic saving. There are now two ways of securing resources for investment: through use of domestic savings and by borrowing from the rest of the world. Nevertheless, all the benefits accruing from the existence of foreign trade must explicitly take into account the external and internal balance of payments equilibrium constraint (David 1985, p. 35):

(2–20) $X - M = S - I$

where
X = exports
M = imports
S = domestic saving
I = domestic investment

The balance of payments equilibrium constraint (2–20) is used in the discussion of self-contained development strategy in Appendix B. The description of the production process of forest sector final commodity and its allocation to consumption and investment is depicted in Figure 1. Figure 1 does not explicitly show the rest of domestic and world economy because they are subsumed in the household sector.

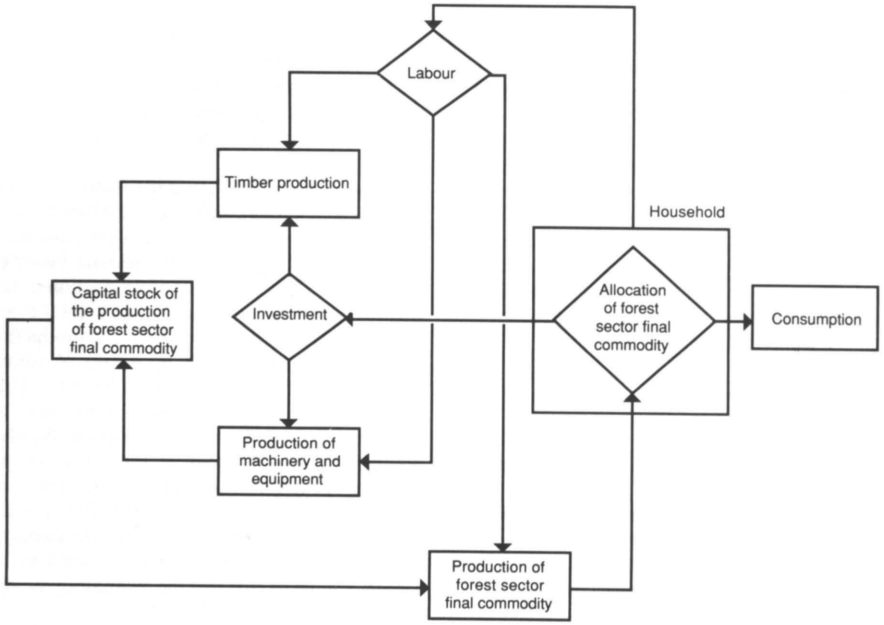

Fig. 1. A diagrammatic representation of the process of the production and the allocation of forest sector final commodity.

The description of the forest sector given on pages 13–14 is now specified in the form of a linear and homogeneous system of equations as follows:

$$(2\text{–}21)\quad \begin{pmatrix} -1 & 0 & \xi \\ k & -1 & x \\ \beta & b & -1 \end{pmatrix} \begin{pmatrix} y_1^* \\ y_1^* \\ N \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

All coefficients are restricted to be non-negative. By developing the determinant of the coefficient matrix in (2–21), Keynes' effective demand condition is obtained (Pasinetti 1981):

(2–22) $\beta\xi + kb\xi + bx - 1 = 0$

Equation (2–22) is a sum of binomials made up of production coefficients (labour β and capital b)[1] and demand coefficients (consumption ξ and investment x). Production coefficients are determined by the state of technology. Demand coefficients are determined by the preference of economic agents; however, the magnitude of the consumption coefficient is bound by technology, i.e.,

(2–23) $\xi = (1 - bx) \div (\beta + kb)$

and the amount of investment necessary to maintain consumption level specified by equation (2–23) is

(2–24) $x = (1 - \beta\xi - kb\xi) \div b$

Thus, when the preferred per capita consumption of the forest sector final commodity is fixed at the level defined by equation (2–23), an amount of investment defined by equation (2–24) must be undertaken in order to maintain a balance of demand and supply equilibrium specified by condition (2–22); otherwise there would be either unemployment or inflationary pressure.

When the condition for balance of demand and supply equilibrium is satisfied, the forest sector final commodity is allocated to consumption and investment as follows:

(2–25) $y^*_1 = \xi N$

(2–26) $y^*_2 = (x + k)N.$

Equation (2–25) says that the amount of the final commodity allocated to consumption is determined by per capita demand for forest products. Equation (2–26) says that demand for new investment and replacement necessary for the production of the forest sector final commodity determines the amount of investment allocated to the production of capital stock.

Since only a proportion of the population is employed in the forest sector, equation (2–18), production and demand coefficients in the system of equations (2–21) are made comparable by dividing each production coefficient by α. Hence, the equilibrium condition (2–22) is modified as follows:

(2–22') $(\beta/\alpha)\xi + k(b/\alpha)\xi + (b/\alpha)x - 1 = 0$

Total employment is:

(2–27) $L_1 = \beta y_1^*$

(2–28) $L_2 = b y_2^*$

where L_1 and L_2 denote the quantity of labour services used in the production of the final commodity and capital stock respectively, see Figure 1. The flow of labour services specified adds up to the proportion of labour force to the total population employed in the forest sector; hence, equation (2–18) may be rewritten as follows:

(2–18') $\beta y_1^* + b y_2^* = \alpha N$

Because of the stationary nature of the national economy, only the flow aspect of the technology of production has been considered. In a developing economy characterized by changes in technology, capital accumulation as a means of production becomes indispensable. This means that in a developing economy, we must consider the stock aspect of the technology of production and the need to investigate the direct relation between the requirement of the production process and capital accumulation. The stock of capital is quantitatively determined by the requirement of the national economy given the state of technology, the level of the labour force, and preferred final demand.

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[1] In equation (2–21), k is the coefficient of depreciation of the forest sector of the forest sector capital stock.

3. A FOREST SECTOR PRODUCTION MODEL IN A DEVELOPING ECONOMY

3.1. State equations

Changes in the

- national population,
- state of technology,
- consumers' preferences,

constitute the laws of motion that govern production and consumption and the context of production and consumption of the forest sector final commodity in the national economy.

3.1.1. Population

It would be interesting and meaningful to an economic model to ascertain how decisions regarding procreation are based upon economic factors. However, in this study, population growth is assumed to be given exogenously by demographic forces.

Based on this assumption, we further assume that the population grows at a constant percentage rate g per time period:

$$(3\text{–}1)\quad N(t) = N(0)\exp(gt)$$

The growing population leads to an increase in the labour force and potential demand for the forest sector final commodity. In (3–1), we do not specify the effect of attrition due to retirement, disability, and death for the sake of simplifying the analysis.

3.1.2. State of technology

The process by which the national economy changes over time in respect to the products it produces and the process used to produce them constitutes what is meant by changes in the state of technology (Burmeister and Dobell 1970). This change is responsible for:

a) The increase in the productivity of the existing capital goods and skills, i.e., the existing production structure; so that greater output is obtained from the same inputs as time proceeds. In the literature, it is referred to as disembodied technological advance (Burmeister and Dobell 1970, p. 66).

b) The increase in the set of technology at the disposal of the national economy. It is comprised of improvements that can be introduced only by investment in new capital goods and skills (newly trained and retrained labour). In the literature, it is referred to as embodied technological change (Burmeister and Dobell 1970, p. 66).

In this sub-section, the change in the state of technology we are modelling is what has been called disembodied technological advance. Based on empirical studies on the economy at large and forestry in particular (Fabricant 1940, 1942; Barger and Landsberg 1942; Barger and Schurr 1944, Gould 1946; Barger 1951; Manning and Thorburn 1971; Greber and White 1982; Simula 1983; Borger and Buongiorno 1985), it is hypothesized that coefficients of the existing production structure decrease at a constant rate η, through time:

$$(3\text{–}2)\quad \begin{cases} \beta(t) = \beta(0)\exp(-\eta t) \\ b(t) = b(0)\exp(-\eta t) \end{cases}$$

The decrease in production coefficients is assumed to be an exogenous phenomenon in this study although it seems plausible to treat change in the state of technology as a result of the conscious decision to invest in research (Stoneman 1983).

Disembodied technological advance has many effects in the national economy, some desirable and others undesirable (Kendrick 1977). The influence of the following are explicitly taken into account:

- increase in output,
- dismissal of workers on grounds of improved efficiency in the production process,
- increase in per capita real income at the disposal of consumers.

3.1.3. Consumers' preferences

Frontier studies in consumer theory (see, e.g., Richter 1966; Takayama 1985, pp. 169–294; Afriat 1987) seek to establish laws of rational behaviour at an instant of time. The core of these studies is the decision problem of how to choose an optimal consumption vector at given levels of per capita income, and the examination of the speed of variation of marginal utilities (possibilities of substitution) due to variations in commodity prices. The dynamic problem of investigating the influence of changes in consumers' preferences is simply assumed away in this more or less indiscernable literature.

An empirical approach to the study of consumer behaviour demonstrates that there is some regularity in the patterns of what people consume as they move from lower to higher incomes (Deaton and Muellbauer 1980). According to this approach Engel curves may be used to show the structural dynamics of per capita demand for forest products as a result of changes in consumers' preferences. The basic shape of the Engel curve is determined by the relationship between variations in income structure and expenditure. Variations in the price structure only influence the slope of the Engel curve. Therefore, it seems plausible to deduce that in the long-term, it is the level of real income that governs variations over time in per capita demand for forest products. Although actual consumption need not bear any simple relation to desired consumption at a given point in time, it also seems plausible to deduce that price changes can only postpone or anticipate a time path for forest products which in the long-term, if real income increases, is bound to take place anyhow.

Since increase in real incomes can be traced back to increase in productivity, Figure 2 is used to illustrate the dynamic interrelations between technical change and the evolution of consumers' preferences.

On the basis of an empirical approach to the study of consumer behaviour that demonstrates a non-proportional expansion in demand for various commodities in the national economy, changes in per capita demand for the forest sector final commodity are specified as follows:

$$(3\text{–}3)\quad \xi(t) = \xi(0)\exp(\rho(t))$$

where $\rho(t) \gtreqqless 0$, is the rate of change of per capita demand.

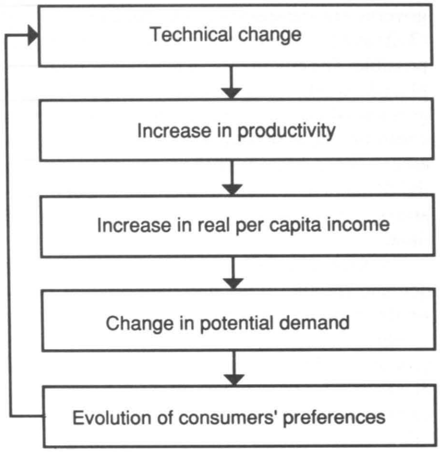

Fig. 2. A diagrammatic explanation of the long-term relationship between technical change and consumers' preferences.

The influence of changes in the state of technology on the rate of change of per capita demand, will be different depending on whether we are considering the short-term or long-term period. In the short-term, technical change will affect it through variations in the structure of prices. In the long-term, technical change will affect it through the level and rate of change of real per capita incomes. Thus,

$$(3\text{–}4)\quad \rho(t) = \psi[\beta, b; (\dot{\beta}, \dot{b})]$$

where ψ is a function determined by consumers' preferences and their evolution through time; the dot $(\cdot)$ indicates the time derivative of the respective coefficients.

Equation (3–3) is now rewritten as follows:

$$(3\text{–}3')\quad \xi(t) = \xi(0)\exp(\rho(t,\psi))$$

where the dependence of ρ on t and ψ is taken into account.

3.2. Derivation of equilibrium investment allocation

The shift from a stationary economy to a developing economy whose behaviour is governed by the specified state equations (3–1) – (3–2) and (3–3') makes a state of disequilibrium possible. The emergence of new workers and displaced workers (previously employed) on grounds of efficiency in the production process confronts the economy with the problem of unemployment. Moreover, the emergence of potential demand as a result of increase in population and per capita incomes must be satisfied through time.

To maintain full employment and satisfy demand through time, a certain amount of the final commodity must be allocated to investment in order to expand capital stock as a means of production. Only changes in the demand coefficient for new investment affect the size of the capital stock. Therefore, structural changes in this coefficient must be equal to the expansion of potential demand in order to obtain the level of capital stock consistent with equilibrium conditions in the economy, i.e.:

$$\text{(3–5)} \quad x(t) = (g + \rho)\xi$$

When condition (3–5) is satisfied, the system of equations (2–22) becomes;

$$\text{(3–6)} \quad \begin{pmatrix} -1 & 0 & \xi(t) \\ k & -1 & x(t) \\ \beta(t) & b(t) & -1 \end{pmatrix} \begin{pmatrix} y_1^{*}(t) \\ y_2^{*}(t) \\ N(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

All dated coefficients are assumed to be non-negative.

By inserting condition (3–5) in (2–23'), the dynamic macroeconomic effective demand condition is obtained:

$$\text{(3–7)} \quad (\beta(t)/\alpha(t))\xi(t)\exp(\rho(t,\psi)-\eta t) + (g + \rho + k)(b(t)/\alpha(t))\exp(\rho(t,\psi) - \eta t) - 1 = 0$$

When the dynamic condition (3–7) is satisfied, the forest sector final commodity is allocated to consumption and investment as follows:

$$\text{(3–8)} \quad y_1{}^{*}(t) = \xi(t)N(t)\exp(gt + \rho(t,\psi))$$

$$\text{(3–9)} \quad y_2{}^{*}(t) = (g + \rho + k)\xi N(t)\exp(gt + \rho(t,\psi))$$

The economic interpretation of equations (3–8) and (3–9) at an instant of time is the same as the interpretation for equations (2–26) and (2–27). In a dynamic economy, equations (3–8) and (3–9) say that the equilibrium time path of allocation of the final commodity to consumption and investment is determined by the sum of population growth rate and the rate of change in per capita demand.

Changes in the rate of population growth, per capita demand and labour productivity affect total employment, equations (2–28) and (2–29), as follows:

$$\text{(3–10)} \quad L_1 = \beta(t)\xi(t)N(t)\exp((gt + \rho(t,\psi)) - \eta t)$$

$$\text{(3–11)} \quad L_2 = (g + \rho + k)b(t)\xi(t)N(t)\exp((gt + \rho(t,\psi)) - \eta t)$$

3.3. Equilibrium conditions as an explicit goal of forest policy

When the preferred time path of per capita consumption is according to equation (3–3'), to maintain the dynamic macroeconomic condition (3–7) in equilibrium, the time path of per capita investment must evolve as follows:

$$\text{(3–12)} \quad (g + \rho + k) = \frac{1 - (\beta(t)/\alpha(t))\xi(t)\exp(\rho(t,\psi)-\eta t)}{(b(t)/\alpha(t))\exp(\rho(t,\psi)-\eta t)}$$

where $(g + \rho + k) = x(t)$

Condition (3–7) – which formally says that the determinant of the coefficient matrix of the system of homogeneous equations (3–6) must be singular in order to have non-trivial solutions to the system – is satisfied through time if, and only if, the time derivative of equation (3–12) is zero. When equation (3–12) is differentiated with respect to time we obtain:

$$\text{(3–13)} \quad 0 = [-\alpha^2(t)m'm(\cdot)\exp(m(\cdot)) + \alpha'(t)\,\beta(t)\,\xi(t) - \xi'(t)\,\alpha(t)\beta(t) - \alpha(t)\,\beta'(t)\,\xi(t)] / b(t)\alpha(t)$$

where

$$m(\cdot) = \rho(t,\psi) - \eta t$$
$$m' = \rho(t,\psi) - \eta$$

Only the L.H.S of equation (3–12) vanishes upon differentiation. The R.H.S. can only become zero if the production and the consumption coefficients remain constant and move at the same rate but in opposite directions – see Appendix A.

Since both sides of equation (3–12) do not become zero upon differentiation, this means that

the determinant, i.e., equation (3–7) of the coefficient matrix of the system of homogeneous equations (3–6), does not satisfy the singularity condition through time.

The non-satisfaction of the singularity condition and hence the dynamic macroeconomic condition (3–7) means that even if we start from a state of equilibrium characterized by full employment of the labour force and other economic resources, as the national economy develops through time unemployment becomes inevitable. Hence, the balance of demand and supply equilibrium must be pursued explicitly as a goal of economic policy. The goal of economic policy in this respect is comprised of

- short run demand management policies, and
- long run growth strategy.

The main focus of stabilization policy (i.e., demand management policies) is on various nominal flow-of-funds balances in the national economy: balance of payments, government accounts, saving-investment balance. It aims at attaining balance of demand and supply equilibrium which is often thought of as a state in which the price level is stable, full employment prevails, and the nation's balance of payments is in equilibrium.

The purpose of stabilization policy is to offset the negative aspects of business cycle phenomena during the process of economic development. By itself, it cannot lead to the resumption of growth.

The other aspect of technology referred to as embodied technological change makes it possible to achieve growth as a goal of economic policy. This means that for the economy to resume growth, policymakers must formulate an efficient economic policy that promotes the use of new production coefficients introduced in the national economy due to inventions. A solution to the structural problem of expanding the supply of the forest sector production in order to solve the emerging unemployment due to increase in labour productivity and population and also in order to satisfy the emerging potential demand constitutes an explanation of the determination of new investment. The problem of choosing the most efficient method of production, i.e., choice of optimum technology, arises with reference to new investment and replacement of worn-out capital (depreciation) at the level of forest stand, logging unit, and wood processing firm in the forest sector.

At the forestry level, growth as a goal of economic policy defines the problem of investment policy that determines the level of optimum capital intensity in timber production. In the following section, the expansion of timber supply as a function of the rate of change in embodied silvicultural technology (new technology built into or embodied in new physical and human capital) is examined.

4. DERIVATION OF A DYNAMIC SILVICULTURAL INVESTMENT CRITERION

4.1. Forest growth as a function of growing stock and silviculture

Forest growth is considered to be a product of positive interaction between biological timber capital stock (growing stock) and the inanimate capital stock (silviculture). Forest growth as a function of growing stock and silviculture is defined by the following macro growth function (Vehkamäki 1986a, pp. 58–59):

$$(4\text{–}1)\quad G(t) = \Gamma(V(t), K(t))$$

where

G = forest growth measured in cubic metres
Γ = forest growth function
V = growing stock measured in cubic metres
K = silviculture measured in current prices
t = time

In reality, growth functions for single forest stands are encountered as a result of site quality differences, variations in growth behaviour of tree species, state of silviculture, etc. To justify the derivation of a macro growth function from individual stand functions (Vehkamäki 1986a, pp. 58–59), we assume rational timber production behaviour (Sato 1975).

The growth function (4–1) is assumed to be continuous and twice differentiable with the first and second partial derivatives with respect to its arguments strictly positive and negative respectively.The curvature of the unconstrained growth function (4–1) is defined by the Hessian:

$$(4\text{–}2)\quad H = \begin{pmatrix} \Gamma_{vv} & \Gamma_{vk} \\ \Gamma_{kv} & \Gamma_{kk} \end{pmatrix}$$

The first and second principal minors of the Hessian (4–2) are negative and positive respectively, i.e., $\Gamma_{vv} < 0$ and $\Gamma_{vv}\ \Gamma_{vk} - \Gamma_{vk}{}^2 > 0$, where Γ_{vk} is squared because by Young's Theorem in calculus, $\Gamma_{vk} = \Gamma_{kv}$. Because the principal minors alternate in sign, the Hessian (4–2) is negative definite. This means that the forest growth function (4–1) is strictly concave and has a unique maximum value, Figure 3.

4.1.1. The level of silvicultural intensity

In the explanation of new investment at the forest industry level, timber production was by means of labour alone. We shall now consider explicitly, timber production by means of labour supplemented by capital (forest machinery and equipments), raw material, and energy. This constitutes what we shall refer to as silvicultural services, or silvicultural inputs, or simply silviculture.

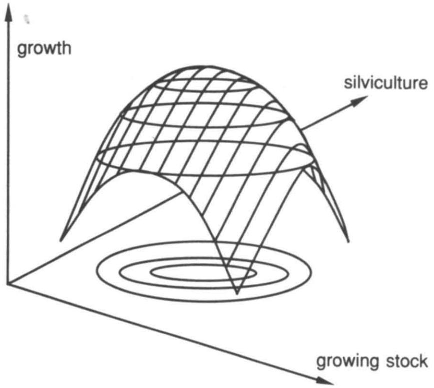

Fig. 3a. Forest growth as a function of growing stock and silviculture.

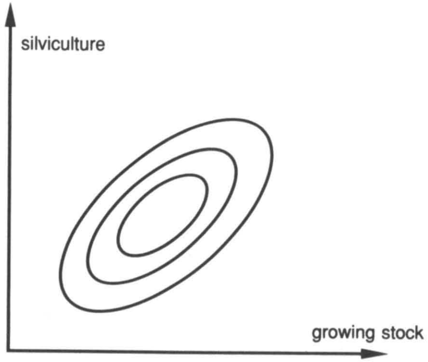

Fig. 3b. Isoquants of the forest growth function.

Silviculture is assumed to be a concave function of capital ($\check{K}$) and labour (L)

$$(4\text{–}3)\quad K(t) = K(\check{K}(t), L(t)).$$

The technical form that silviculture takes determines the level of capital intensity (i.e., the ratio of capital to labour) in timber production. This means that to determine silviculture (i.e., the amount of silvicultural inputs per unit area of forest land), we substitute equation (4–3) in the growth function (4–1) to obtain,

$$(4\text{–}4)\quad \bar{G} = \bar{\Gamma}(\bar{V}, \bar{K}(\check{K}, L) = \Gamma(\cdot) / J,$$

where

$\bar{G}$ = mean growth
$\bar{\Gamma}$ = mean growth function
$\bar{V}$ = mean growing stock
$\bar{K}$ = mean silviculture
J = forest land area

The explicit dependence of variables in equation (4–4) on time has been suppressed. Later on, we shall also find it desirable sometimes to assume the time variable for the sake of clarity in formal specifications.

Intensive (extensive) forest land use occurs when a large quantity (a small quantity) of silvicultural inputs per unit area is used. Note that intensity does not refer directly to forest growth, the product of forest land use.

The examination of the determination of the level of silviculture in timber production is based on an assumption of a small labour surplus economy. This is an economy that is a pricetaker on the world market and is characterized by disequilibrium between the marginal product of labour in the forest sector and its opportunity cost (Marglin 1976, p.11).

The problem of determining the level of silviculture in timber production is examined from the point of view of the national economy. The aim of investing in timber production is to maximize aggregate consumption. We assume that the rate of domestic savings is a constraint to investment in timber production. We further assume that the existence of foreign trade frees the level of silvicultural investment from the constraint of domestic saving. However, in order to maintain a state of balance of equilibrium, external borrowing is financed by expanding forest sector exports. This strategy of forest sector development is made explicit in Appendix B. In the long run, which we shall interpret in this study as the planning period, silvicultural production function (4–3) may be posited as a continuum of efficient silvicultural techniques, i.e, techniques that meet the target of timber production with as few inputs as possible (Debreu 1983, p. 31). A technique (or technology) is defined as a package of capital goods and appropriate know-how and skill needed in order to optimize the use of resources in timber production.

The amount of capital per unit of labour will vary from technique to technique on the silvicultural production function (4–3). The choice of a particular technology implies a certain strategy of economic development with a specific effect on the performance of the national economy. We shall assume that the technology that maximizes forest labour productivity is adopted so that a target of timber production consistent with forest industry capacity is attained. The problem of determination of optimum silvicultural technology is depicted in Figure 4.

4.1.1.1. Model

The problem of choice of optimum silvicultural technology is interpreted as a question of choice between current and future consumption (Sen 1957, 1960). Since the marginal product of labour in the forest sector is not equal to its opportunity cost and the rate of domestic savings is sub-optimal, the problem of choice of optimum silvicultural technology is further interpreted as a choice question in the theory of second best (Sen 1969, Marglin 1976).

Assumptions

1) We examine the problem of optimum labour employment for a marginal forestry project given the amount of capital.
2) The nominal wage rate per unit of labour employed is assumed to be independent of the size of the labour force and capital stock and constant over time.

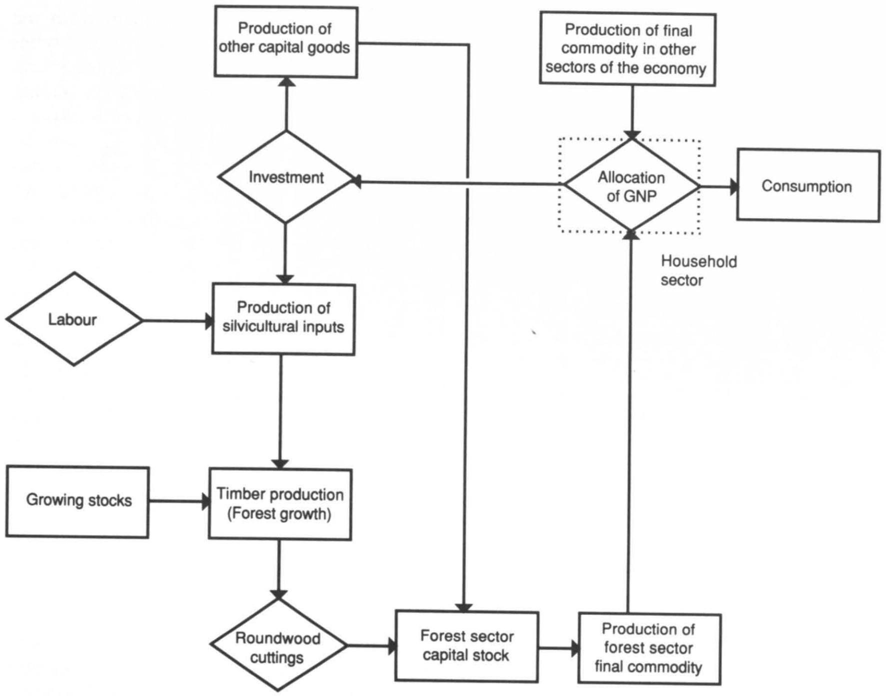

Fig. 4. A diagrammatic representation of the problem of choice of silvicultural technology in timber production.

3) The supply of labour at the exogenously fixed wage rate is infinitely elastic until the labour force is fully employed.
4) The real wage rate is defined in terms of a basket of consumption goods.
5) Political and institutional constraints prevent the savings rate from rising to the level required to finance investment consistent with full employment.
6) Because of the existence of a sub-optimal savings rate, an additional weight is attached to that part of the income that is saved and invested as against that part that is consumed immediately.
7) The propensity to consume out of wage income is greater than the propensity to consume out of non-wage income. This means that the proportion of additional income that is saved depends on the distribution of the additional income between wages and profit. This assumption establishes a direct link in allocation of income between savings and consumption to the level of employment at present and the proportion of the future additional income that will be saved.
8) Keynesian problems of translating savings into productive investment are non-existent. Hence, there is no distinction between the terms savings and investment. The two terms will be used interchangeably to refer simultaneously to that portion of output that is not consumed and to the net additions to the silviculture.
9) An infinite planning horizon is adopted. It is considered to be a sufficiently remote finite planning horizon. For the purpose of deriving an investment criterion, the choice between finite and infinite time horizon does not make much difference because the investment policy decision rule can be derived from either approach.
10) To simplify the analysis we shall assume that gross national product is made of forest sector final commodity only.

The aim of timber production is to maximize the standard of living (welfare) defined as a function of aggregate consumption and specified by the functional

$$\text{(4–5)}\quad \max U \int_{t=0}^{t=\infty} \exp(-rt)\, U\{y_1^*(t)\}\, dt$$

where
U = a measure of standard of living (welfare)
r = a positive social rate of time preference
y_1^* = forest sector final commodity allocated to consumption

To be consistent with the requirement of optimal consumption allocation in an intertemporal setting, the optimality criterion (4–5) is interpreted as a separable functional (Blackorby et al 1978, pp. 341–381). Given a positive social rate of time preference and assuming that welfare is a concave function of consumption, the integrand of (4–5) converges (Arrow 1968); hence, the improper integral itself is convergent.
The objective functional (4–5) is maximized subject to:
a) state equations

$$\text{(4–6)}\quad \dot{V} = \Gamma(V, K) - \omega y^*(L)$$

$$\text{(4–7)}\quad \dot{K} = I(t) - kK$$

where
$\dot{V}$ = time derivative of the growing stock
$\dot{K}$ = time derivative of the silviculture
I = gross silvicultural investment
k = depreciation rate of silviculture
ω = roundwood input required per unit of forest sector final commodity

b) the employment constraint

$$\text{(4–8)}\quad \upsilon_{min} \leqq \upsilon \leqq \upsilon_{max}$$

where
υ = employment per unit of capital (capital : labour ratio)
υ_{min} = defines the value for which the marginal product of labour is equal to the nominal wage rate
υ_{max} = defines the value for which the average product of labour is equal to the nominal wage rate

The current state of forestry stocks (i.e., growing stock and silviculture) constitute initial conditions embodied with all the relevant information required to determine the optimal control to apply on the time horizon envisaged. Specifically, they constitute a complete description of the current forest growth possibilities, past values of roundwood cuttings and silvicultural investment, land use and environmental factors. As far as current and the setting of future targets of timber production are concerned, past values of cuttings and investment add no information beyond that contained in the initial forestry stocks. This result is a consequence of the particular time separable structure of timber production as a renewable resource.

Target forestry stocks are required to be consistent with the existing or planned forest industry capacity.

Optimization

Current consumption is valued for its own sake. On the other hand, savings is valued for the future consumption it provides. This means that satisfaction derived from future consumption that silvicultural investment makes possible has only instrumental significance as an intermediate good in the production of future consumption.

To measure the welfare of current consumption and the instrumental welfare of silvicultural investment simultaneously, we define the discounted Hamiltonian function

$$\text{(4–9)}\quad h = \exp(-rt)\,[\underbrace{U\{y_1^*\}}_{\text{immediate consumption}} + \underbrace{p_1(t)(I - kK) + p_2(t)(\Gamma(V, K) - \omega y^*(L))}_{\text{— consumption postponed to the future}}]$$

h = discounted Hamiltonian function
p_1 = welfare (shadow) price of silvicultural investment
p_2 = welfare (shadow) price of growing stock

Forest sector final commodity per unit of capital is allocated to consumption and investment respectively, as follows:

$$(4\text{–}10)\ y_1^* = \gamma_2\, y^*(\upsilon) + (\gamma_1 - \gamma_2)\, w\upsilon$$

$$(4\text{–}11)\ y_{1F}^* = (1 - \gamma_2)\,(y^*(\upsilon) - w\upsilon) + (1 - \gamma_1)\, w\upsilon$$

where

y^* = forest sector final commodity per unit of capital

y_1^* = forest sector final commodity allocated to consumption

y_{1F}^* = forest sector final commodity allocated to silvicultural investment

w = nominal wage rate per unit of labour employed

γ_1 = the propensity to consume out of wage income

γ_2 = the propensity to consume out of non-wage income

By using equations (4–10), (4–11) and (2–16) to eliminate y_1^*, I, and $\omega y^*(L)$ respectively from equation (4–9), we obtain:

$$(4\text{–}12)\ \mathit{h} = \exp(-rt)\,[U\{\gamma_2\, y^*(\upsilon) + (\gamma_1 - \gamma_2)\, w\upsilon\} + p_1(((1 - \gamma_2)\,(y^*(\upsilon) - w\upsilon) + (1 - \gamma_1)\, w\upsilon) - kK) + p_2\,(\Gamma\,(V, K) - B)].$$

In order that an optimum exists for the control problem, we shall assume Mangasarian infinite horizon sufficiency theorem (Seierstad and Sydsæter 1987, pp. 234–235, Theorem 13).

The necessary conditions for an optimum solution for the Hamiltonian (4–12) are

a) Hamiltonian equations

$$(4\text{–}13)\ \dot{\hat{V}} = \Gamma\,(\hat{V},\hat{K}) - \hat{B}$$

$$(4\text{–}14)\ \dot{\hat{K}} = \hat{I} - k\hat{K}$$

$$(4\text{–}15)\ \dot{p}_1 = p_1 k - p_2\,\Gamma_k\,(\hat{V},\hat{K})$$

$$(4\text{–}16)\ \dot{p}_2 = -\,p_2\,\Gamma_v\,(\hat{V},\hat{K})$$

b) the Maximum principle (Pontryagin et al. 1962)

$$(4\text{–}17)\ \exp(-rt)\,[U\{\gamma_2\, y^*(\hat{\upsilon}) + (\gamma_1 - \gamma_2)\, w\hat{\upsilon}\} + p_1(((1 - \gamma_2)\,(y^*(\hat{\upsilon}) - w\hat{\upsilon}) + (1 - \gamma_1)w\hat{\upsilon}) - k\hat{K}) + p_2\,(\Gamma\,(\hat{V}, \hat{K}) - \hat{B})] \geqq \exp(-rt)\,[U\{\gamma_2\, y^*(\upsilon) + (\gamma_1 - \gamma_2)\, w\upsilon\} + p_1(((1 - \gamma_2)\,(y^*(\upsilon) - w\upsilon) + (1 - \gamma_1)w\upsilon) - k\hat{K}) + p_2\,(\Gamma\,(\hat{V}, \hat{K}) - \hat{B})]$$

c) initial conditions of the forestry stocks.

The circumflex (^) denotes optimal values.

Before solving the optimum condition of employment (4–17), the exponential term in the Hamiltonian (4–12) is eliminated by defining current value Hamiltonian and current value of welfare price of silvicultural investment and growing stock respective, as follows:

$$(4\text{–}18)\ \mathcal{H} \equiv \exp(-rt)\ \mathit{h}$$

$$(4\text{–}19)\ \lambda_i \equiv \exp(-rt)\, p_i\,, \qquad i = 1,2.$$

Assuming that the employment constraint (4–8) is not binding, the current value Hamiltonian of equation (4–12) is differentiated with respect to the control variable υ, and set equal to zero to obtain

$$(4\text{–}20)\ U'(y_1^*)(\gamma_2\, y^{*\prime}(\upsilon) + (\gamma_1 - \gamma_2)w) + \lambda_1[y^{*\prime}(\upsilon) - w]\,(1 - \gamma_2) + \lambda_1 w\,(1 - \gamma_1) = 0$$

or

$$(4\text{–}21)\ U'(y_1^*)(\gamma_2\, y^{*\prime}(\upsilon) + (\gamma_1 - \gamma_2)w) = -\,\lambda_1[y^{*\prime}(\upsilon) - w]\,(1 - \gamma_2) - \lambda_1 w\,(1 - \gamma_1)$$

From equation (4–21) we obtain

$$(4\text{–}22)\ \frac{U'(y_1^*)}{\lambda_1} = \frac{(1 - \gamma_2)[y^{*\prime}(\upsilon) - w] - (1 - \gamma_1)w}{\gamma_2\, y^{*\prime}(\upsilon) + (\gamma_1 - \gamma_2)w}$$

The left hand side of equation (4–22) defines the social marginal rate of indifferent substitution between consumption and silvicultural investment. Denoting the premium on savings by m and defining $1 + m = \lambda_1 / U'(y_1^*)$, we may rewrite equation (4–22) as follows:

$$(4\text{–}23)\ \frac{1}{1 + m} = \frac{(1 - \gamma_2)\, y^{*\prime}(\upsilon) + (\gamma_1 - \gamma_2)w}{\gamma_2\, y^{*\prime}(\upsilon) + (\gamma_1 - \gamma_2)w}$$

or

$$(4\text{–}24)\ \gamma_2\, y^{*\prime}(\upsilon) + (\gamma_1 - \gamma_2)w = (1 + m)\,(-y^{*\prime}(\upsilon)(1 - \gamma_2) + (\gamma_1 - \gamma_2)w$$

Defining the real wage rate in terms of the forest sector final commodity by $W \equiv y^{*\prime}(\upsilon)$, by collecting terms and solving for W, we obtain

from equation (4–24) silvicultural investment criterion

$$(4\text{–}25)\ W = [(\gamma_1 - \gamma_2)m \div 1 + (1 - \gamma_1)m]w$$

The investment criterion (4–25) depends critically on the savings function in a given national economy under consideration. For example, if we relax assumption 5 (which is the same as saying that the social marginal rate of indifferent substitution between consumption and investment is equal to unity), the premium on savings becomes zero and w in equation (4–25) becomes zero; this means that there is no change in forest labour productivity. Similarly, if we relax assumption 7 and replace it with the assumption that saving propensities are a function of instruments of fiscal and monetary policies (which permits the possibility of setting $\gamma_1 = \gamma_2$), w in equation (4–25) becomes zero.

The lack of change in forest labour productivity means that the national economy remains on the same production possibility frontier. Therefore, to solve long-term unemployment, the assumption of attaching a positive and greater than unity premium on current consumption postponed to the future in a labour surplus economy in order to be able to finance capital necessary for increasing labour productivity, is defensible because this allows shifts to higher production possibility frontiers which constitutes growth propitious to a higher rate of absorption of unemployed labour force.

4.1.1.2. Instantaneous intertemporal consistency conditions

The intertemporal consistency conditions (4–15) and (4–16), do not directly affect the instantaneous maximization of the Hamiltonian function. They only govern the path of optimal forestry stocks. Let us examine the meaning of condition (4–15).

The current value of welfare (shadow) price equation (4–15) is defined as follows

$$(4\text{–}15')\ \dot{\lambda}_1 = \lambda_1 k - \lambda_2\,\Gamma_k\,(\hat{V}, \hat{K}) + r\lambda_1.$$

If we divide (4–15') through by λ_1, we obtain

$$(4\text{–}26)\ \underbrace{\frac{\dot{\lambda}_1}{\lambda_1}}_{\text{I}} = \underbrace{(r + k)}_{\text{II}} - \underbrace{\lambda_1^{-1}\lambda_2\,\Gamma_k\,(\hat{V}, \hat{K})}_{\text{III}}.$$

The terms in equation (4–26) are interpreted as follows:

I = the percentage rate at which the marginal welfare of silvicultural investment declines over time,

II = capital costs comprised of the social rate of time preference and depreciation rate of silviculture arising from the marginal growth of forest with respect to the silviculture,

III = the welfare value of the marginal growth of forest with respect to silviculture

The opportunity cost of the value of the marginal growth of forest is the market rate of interest. Utilizing this piece of information and assuming that the depreciation rate of silviculture is zero, in order to make explicit the meaning of condition (4–26), we may write

$$(4\text{–}26')\ \frac{\dot{\lambda}_1}{\lambda_1} = r - \delta\,,$$

where

δ = market rate of interest.

A straightforward interpretation of condition (4–26') is that, on the optimal path of capital accumulation in timber production, the percentage rate at which the marginal welfare of silvicultural investment declines over time must be equal to the difference between the social rate of time preference and the rate of interest (the opportunity cost of forest marginal growth).

The rate of interest that serves as a opportunity cost of forest marginal growth is defined as the marginal rate of transformation between present and future consumption, with the rate of transformation being given by (1+ δ), in the absence of political and institutional constraints. It represents the boundary of feasible combination of wages and profits in an economy in which the wage rate is a choice variable. The social rate of time preference is defined in an analogous manner except that it incorporates political and institutional constraints and represents the boundary of feasible combination of wages and profits in an economy in which the wage rate is exogenously fixed.

Transformation function $(1+\delta)$, unlike $(1+r)$, defines combinations of consumption and silvicultural investment that are technologically possible but politically infeasible. This means that as long as there exists a disequilibrium between the social rate of time preference and the market rate of interest the rate of savings is not optimal. The optimal level of silvicultural investment in such an economy is the level at which the forest marginal growth is equal to the marginal social rate of time preference (Marglin 1963).

The premium on savings m, depends on the social rate of time preference and the rate of interest as follows (Marglin 1963, p. 279; Feldstein 1973, p. 5):

$$\text{(4–27)}\ m = \frac{\delta}{r}\ .$$

In an economy with sub-optimal savings, the aim of interfering with the market mechanism is to obtain $r < \delta$ so that a positive and greater than unity premium is attached to savings as against consumption. By transferring a relatively higher amount of current consumption to the future, the number of alternative techniques from which to choose an optimal silvicultural technology is increased as compared to what the situation would be in the absence of such an interference.

Because the welfare functional (4–5) is concave in consumption, on the optimal path of capital accumulation in timber production, when the percentage rate at which the marginal welfare of silvicultural investment declines over time becomes zero, we obtain from condition (4–26')

$$\text{(4–28)}\ \delta = r.$$

The equality between the interest rate and social rate of time preference is a necessary condition for optimal capital accumulation, but it it sufficient only under special assumptions. For example, let us suppose that condition (4–28) is only observed when the national economy is in a state of full employment, which means that the rate of silvicultural investment is consistent with a state of full employment. If we interpret a state of full employment as a limit, we may express condition (4–27) as follows:

$$\text{(4–29)}\ \lim_{r \to \delta} m = 1$$

In a state of full employment characterized by unconstrained optimal path of growth (since $r = \delta$), a weight of unity is attached to the part of consumption postponed to the future because the share of savings in the national income is optimum. A weight of unity attached to savings means that at the margin consumption and investment are equally valuable. In this case, allocation of an optimal amount of savings among techniques with varying degrees of labour intensity constitutes the problem of choice of silvicultural technology.

Stability behaviour

If we linearize the Hamiltonian equations (4–13) – (4–16) around the steady-state equilibrium, we obtain a Jacobian matrix (matrix of second order partial derivatives with respect to the state and costate variables). The stability of the system depends on the sign of the characteristic roots of the Jacobian matrix. Since Hamiltonian equations define a saddle-point equilibrium only (Shell and Stiglitz 1967, Magill 1977), it is concluded that the characteristic roots of the Jacobian matrix alternate in sign and hence, the system is stable in the neighbourhood of the steady-state equilibrium.

By integrating linearized version of equations (4–13) – (4–16), decision rules governing the adjustment process of silviculture and growing stock are obtained

$$\text{(4–30)}\ p_1 = p_1(0)\exp(kt) + p_2(0)\frac{\check{\Gamma}_k(\hat{V},\hat{K})}{\check{\Gamma}_v(\hat{V},\hat{K})+k}\left(\exp(-\check{\Gamma}_v(\hat{V},\hat{K})\,t)-\exp(kt)\right)$$

$$\text{(4–31)}\ p_2 = p_2(0)\exp(-\check{\Gamma}_v(\hat{V},\hat{K})\,t)\ ,$$

where

$\check{\Gamma}_k$ = the coefficient of silviculture in the linearized forest growth function

$\check{\Gamma}_v$ = the coefficient of growing stock in the linearized forest growth function

The optimal adjustment of silviculture and growing stock based on decision rules (4–30) and (4–31) depend critically on the determination of their correct welfare prices, Figures 5 and 6. According to Figure 5, given any initial forestry stocks, by selecting initial welfare prices $p_1(0)$ and $p_2(0)$ on the optimal path FF, target growing stock $\hat{V}$ and silviculture $\hat{K}$ are attained. The feedback control law F'F' in Figure 6 describes the change in forestry stocks that must occur at each instant of time so that the target stocks of timber production are obtained. The dashed trajectories in Figures 5 and 6 represent solutions to equations (4–30) and (4–31) with incorrect initial welfare prices.

The value of initial welfare prices depend on 1) the prevailing degree of scarcity of timber growing stock, 2) the social rate of time preference, 3) the depreciation rate of silviculture.

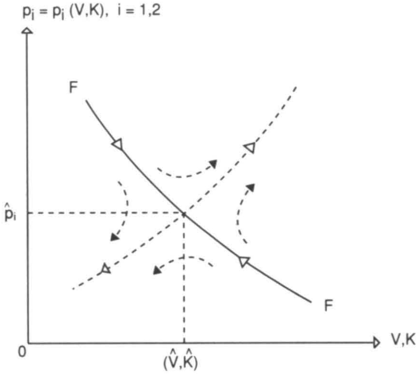

Fig. 5. Long run optimal adjustment of forestry stocks.

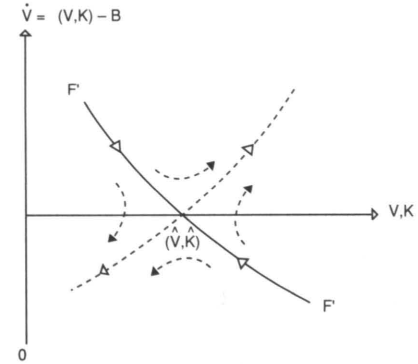

Fig 6. Forestry stocks feedback control law

Changes in parameters, e.g., the social rate of time preference and the depreciation rate of silviculture shift the position of the optimal path FF in the phase-space. Thus, in the short run dynamics characterized by fixed exogenous parameters, the position of the optimal path depends on the values of the social rate of time preference and the depreciation rate of silviculture used in the determination of the optimal timber production policy.

Assuming that changes in the social rate of time preference and the depreciation rate of silviculture take place in a slow and steady way, the system will trace out a trajectory along the equilibrium manifold. Since the optimal path is asymptotically stable, the short run dynamics ensure that in the long run the system moves along the stable submanifolds.

The process of capital deepening in timber production may be assumed to be monotonically continuous in the long run except at certain times when parameter changes may lead the system to a bifurcation equilibrium point (Magill 1977, pp. 195–200). At a bifurcation equilibrium point, a change, e.g., in the rate of time preference has an unpredictable response such that we cannot say *a priori* that a lower rate of social time preference in timber production will lead to a higher capital-labour ratio. At such an equilibrium point, the level of capital intensity in timber production will depend on the position of the stable submanifold since the optimal path moves along the stable submanifolds.

The fulfillment of the formal stability criterion of the steady-state equilibrium point has only analytical significance in the specified optimization problem. In the real world, it is important to ascertain whether or not it is possible to attain target forestry stocks (growing stock and silviculture) by allowing disequilibrium in actual timber production behaviour (i.e., variations in roundwood cuttings and silvicultural investment). In this case, the concept of controllability (Aoki 1976) is more important than the stability of a saddle point equilibrium.

4.1.2. Dynamic controllability of growing stock and silviculture

Current roundwood cutting and silvicultural investment behaviour determine the quantity of timber growing stock available in the long run. The inherent link between short run timber production behaviour and the quantity of timber available in the long run led to the specification of the laws of motion in timber production on the basis of state variable theory. For the purpose of forest policy, the targets of timber production constitute policy goals, and the means by which the goals are attained constitute policy instruments (Tinbergen 1952).

In this study, growing stock and silviculture consistent with forest industry capacity are interpreted as the goals of timber production. Since the state equations (4–6) and (4–7) depicting the evolution of forestry stocks are influenced by the derived demand for roundwood and silvicultural investments, we conclude that variations in roundwood cutting and silvicultural investment are sufficient to obtain the targets of timber production.

By examining the qualitative property of controllability of the dynamic timber production model, we ascertain the possibility of nudging forestry stocks to follow the trajectory of the optimal path over time by manipulating forest owners cutting and silvicultural investment behaviour.

Let us suppose that the state equations (4–6) and (4–7) behave like linear systems or alternatively, that their behaviour can be approximated by their corresponding linear systems obtained by Taylor series expansion. We assume that roundwood cutting and silvicultural investment are piecewise continuous functions of time, i.e., discontinuities in their application are allowed whenever the prevailing state of the national economy makes this behaviour rational.

We define cutting and investment programmes by

$$(4\text{–}32)\ \mathbf{E} = \begin{pmatrix} h & 0 \\ 0 & v \end{pmatrix}$$

We may define cutting and investment programmes within a finite time period respectively, as follows:

$$(4\text{–}33)\ h = \operatorname{diag}(h_0\ h_1\ \cdots\ h_T)$$

$$(4\text{–}34)\ v = \operatorname{diag}(v_0\ v_1\ \cdots\ v_T)$$

The influence of cutting and investment on the evolution of forestry stocks is defined by the coefficient matrix

$$(4\text{–}35)\ \mathbf{D} = \begin{pmatrix} \chi & 0 \\ 0 & \iota \end{pmatrix}$$

and changes in growing stock and silviculture within a finite time period are defined respectively, by

$$(4\text{–}36)\ \chi = \operatorname{diag}(\chi_0\ \chi_1\ \cdots\ \chi_T)$$

$$(4\text{–}37)\ \iota = \operatorname{diag}(\iota_0\ \iota_1\ \cdots\ \iota_T)$$

The adjustment of forestry stocks starting from an arbitrary initial state (V_0, K_0), to the target stocks (V_T, K_T), is defined by

$$(4\text{–}38)\ \begin{pmatrix} V_T \\ K_T \end{pmatrix} - \mathbf{D}\begin{pmatrix} V_0 \\ K_0 \end{pmatrix} = \mathbf{F}\begin{pmatrix} B \\ I \end{pmatrix}$$

where **F** depends on (4–32) and (4–35) as follows:

$$(4\text{–}39)\ \mathbf{F} = [\,\mathbf{E}\ \ \mathbf{DE}\,]$$

For equation (4–38) to have a solution, **F** must be of full rank (Aoki 1976, p. 80, Theorem I).

To verify the property of controllability, we specify timber production dynamic model in state space from as follows:

$$(4\text{–}40)\ \begin{pmatrix} V_T \\ K_T \end{pmatrix} = \mathbf{D}\begin{pmatrix} V_0 \\ K_0 \end{pmatrix} + \mathbf{F}\begin{pmatrix} B \\ I \end{pmatrix}$$

To determine the rank condition, we derive matrix **F**

$$(4\text{–}41)\ F = \begin{pmatrix} h & 0 & \chi h & 0 \\ 0 & v & 0 & \iota v \end{pmatrix}$$

Provided that $hv \neq 0$ and $\chi h \iota v \neq 0$, the rank of F is 2; hence, the dynamic model (4–40) is controllable. Observe that rank $\mathbf{E} = 2$, and since the dimension of the state space model (4–40) is 2, by Theorem I of Aoki (1976, p. 80) **E** alone spans the two-dimensional state space. This also confirms the preceding conclusion that system (4–40) is controllable.

5. DISCUSSION

The topic of timber supply from the national economy point of view has a long research tradition in the literature of forest management. Prior to the publication of the classical article on this topic by Vaux and Zivnuska (1952), timber supply studies were mainly preoccupied with mensurational aspects of forest management (Duerr 1977). Since the investigation of this topic based on the Marshallian partial equilibrium methods version of neoclassical theory by Vaux and Zivnuska (1952), numerous timber supply and demand models have been developed. The basis of these studies is still essentially the traditional Marshallian model. In these studies, the issue of choice of silvicultural technology (inputs of physical machines, intermediate inputs, and quantities of labour required in timber production) is not treated explicitly. However, based on the nature of these models, it is plausible to conclude that neoclassical theory of production is implied as the basis of choice of silvicultural technology.

The justification of economic policy is a situation in which a state of disequilibrium exists in the labour, commodity, and money markets in a given nation. Neoclassical theory in its most pristine form does not admit of the concept of disequilibrium in the national economy. In its post-Keynesian form, it deals with the possibility of disequilibrium by positing that suitable fiscal and monetary policies effectively guarantee a balance of demand and supply equilibrium in the labour, commodity and money markets. In this study, a) disequilibrium in demand and supply of forest products has been derived as a fundamental justification of national forest policy in a developing economy, b) choice of capital intensive silvicultural technology has been derived as the main determinant of growth as a goal of forest policy in a developing economy characterized by labour surplus and a shortage of investible resources. The two main results of the study present a complete departure in terms of policy implications when compared to policy implications of models based on the traditional theory of studying the topic of timber supply.

Consistency between forest industry productive capacity and the supply of roundwood as the main raw material input in the production of forest sector final commodity was the starting point of ascertaining optimum capital intensity in timber production. An explanation of new investment in terms of forest industry productive capacity has been interpreted as the basis of setting the target of timber growing stock and silviculture. The target growing stock and silviculture determined by the forest industry capacity constitute the goal of forest policy. The purpose of forest policy is to ensure consistent changes in roundwood supply and forest industry capacity.

Keynes' macroeconomic condition has been derived as the starting point of examining the behaviour of balance of demand and supply equilibrium in the forest sector through time. The macro equilibrium condition was defined as a determinant of a singular matrix. The matrix was comprised of production and aggregate demand coefficients. Production coefficients are determined by the state of technology; hence, they were considered to be exogenous to the economic analysis. Aggregate demand (consumption and investment) coefficients are magnitudes which depend on economic agents decisions. They were considered to be linearly dependent on the production coefficients because per capita consumption levels cannot be independent of the matrix of technical coefficients.

In a stationary economy the balance of demand and supply equilibrium requirement was satisfied because the determinant of the singular matrix was zero. This means that in a static or stationary economy (an economy in which the rate of expansion in supply and demand of commodities is constant or proportional), there is no need of forest policy since its purpose is to maintain a balance of demand and supply equilibrium in the forest sector when the purposeful behaviour of economic agents is incapable of producing this state of affairs.

To study the dynamic behaviour of the macro equilibrium condition, laws of motion responsible for the real process of economic changes were defined. The laws of motion were defined by means of state equations of population, technology and consumers' preferences. In this context, the singular matrix from which the balance of demand and supply equilibrium was derived did not have a zero determinant. This condition could only be satisfied if a national economy exhibited a state of proportional development.

The equilibrium condition derived in a static or stationary economy could not be derived in a dynamic economy. To obtain this equilibrium condition in a dynamic economy, forest policy becomes indispensable.

Changes in technology and consumers' preferences do not bestow a specific status to the forest sector macro equilibrium condition as a target of long-term adjustment process. The macro equilibrium condition is not an asymptotic position of the process of adjustment of the forest industry capacity because the process of economic development is characterised by expansion waves and pauses as a result of the interplay of technical improvements and demand evolution. Moreover, since the level of income and state of technology is different in different stages of economic development, expansion and pauses characterizing the behaviour of a developing economy will be different quantitatively and qualitatively in each successive recurrence. This implies that no unique forest policy package can be conceived as a solution to the structural changes influencing the demand and supply of forest products in a developing economy since they are different from one stage of development to another.

We assumed timber growing stock created by deliberate investment of an economic agent known as a forest owner to be the source of roundwood supply to the forest industry. The cost items borne by a forest owner that include engineering improvements, administration, stand establishment, stand improvement, protection, interest, etc., were defined as silviculture.

Silvicultural expenditure is incurred on various kinds of inputs comprised of physical machinery, intermediate commodities, and different kinds of labour services. A specific package of machinery, intermediate commodities, and different skills of labour was defined as silvicultural technique (or silvicultural technology). Silvicultural technology (which is the same as the level of silviculture) determines the level of capital intensity in timber production.

Choice of capital intensity in timber production was defined as the problem of choosing an optimal technique arising with reference to new silvicultural investment and replacement of the rate of depreciation of silviculture at the level of a forest stand at a given point in time. The determination of the degree of capital intensity in timber production is a major policy decision in forest sector development strategy. As a policy decision, it affects directly the level of employment in timber production and affects indirectly through the supply of roundwood the level of employment in the forest industry.

In the examination of the choice of optimum silvicultural technology, we explicitly considered an economy characterized by labour surplus and sub-optimal domestic savings rate. A labour surplus economy was defined as an economy in which an employed labour marginal productivity and its opportunity cost are not equal in the sense of valuation of inputs and outputs (Marglin 1976, p. 11). Sub-optimal savings rate was interpreted as a level of domestic savings available that falls short of the amount required to finance investment consistent with a state of full employment of labour and other economic resources in the national economy. Political and institutional constraints that lead to a state of labour surplus and sub-optimal savings rate were not examined in this study. However, these constraints have been examined elsewhere, e.g., by Dobb (1960), Sen (1960), Goodwin (1961), Chakravarty (1962), Marglin (1963, 1976).

In the derivation of silvicultural investment criterion, maximization of aggregate consumption was posited as an explicit goal of timber production. Growth as a goal of economic policy was considered to be propitious to a higher rate of absorption of unemployed labour force and other economic resources.

Because of the assumption of growth as the target of economic policy, a link between choice of capital intensity in timber production at a given point in time and financing of future silvicultural investment out of profit (investible surplus) was defined by a savings function which depends on the propensities to consume out of wage and non-wage incomes. The propensity to consume out of wage income was assumed to be higher than the propensity to consume out of non-wage income. Consequently, the proportion of additional income that is saved depends on the share of wages and profit out of the additional income. Since by assumption a higher consumption rather than savings will follow the choice of a labour-intensive instead of capital-intensive silvicultural technology, and this is inconsistent with maximization of the rate of growth of the national economy, a positive and greater than unity premium was attached to that portion of current consumption postponed to the future in order to prefer capital-intensive to labour-intensive silvicultural techniques. This implies that at the margin a unit of investment was considered to be more valuable than a unit of consumption when the state of sub-optimal domestic savings rate prevails in the national economy.

The traditional approach to the study of the problem of choice of optimum technology and the influence of factor prices on the volume of employment is based on neo-classical production theory. The basic tool underlying the study of the two problems in the concept of production function.

In the application of the concept of production function in economic analysis, it is desirable to distinguish between production function as defining all alternative methods of production that are available at a given point in time and production function as defining actual production structure, i.e, what is actually in operation. Johansen (1972) makes the distinction between the two definitions of the concept of production function by referring to the former as long run or *ex ante* production function and to the latter as short run or *ex post* production function.

We have assumed in this study that at the forest stand level, rationalization of investment in new silvicultural inputs that represent the exploitation of new production techniques (production methods not describable by the existing production structure) is based on the long-run production function. If we refer to the input prices as simply wages, then current and expected changes in the capital wage rate and labour wage rate induce movements along the long run silvicultural production function. These movements facilitate the exploitation of *ex ante* substitution possibilities between capital and labour. We consider *ex ante* substitution possibilities between capital and labour to be the influence of wage flexibility on choice of capital intensity in timber production.

When we consider actual silvicultural activities in timber production as defined by the short run production function, wage flexibility determines the employment (utilization) of capital and labour subject to the optimum technology prevailing at that particular point in time. To understand the process of physical substitution between capital and labour in actual timber production activities, the influence of wage flexibility on the level of employment of capital and labour has to be decomposed into substitution and expansion effects in a way which more or less parallels the Slutsky decomposition in consumer demand theory (Johansen 1972, pp. 124–130).

Substitution effect measures the amount by which capital is reduced and substituted for by labour when it becomes relatively more expensive than labour, and the converse holds with respect to labour. Since substitution possibilities between capital and labour are constrained in the short run by the existing silvicultural technology, the expansion effect measures the increase in the use of capital when it becomes expensive due to imperfect substitutability between capital and labour as a result of the fixed silvicultural technology.

The influence of the expansion effect depends on the expansion effect multiplier whose sign is indeterminate. The expansion effect multiplier depends on the elasticity of substitution between capital and labour determined by the nature of silvicultural technology currently employed in timber production. If capital and labour substitute each other a) at a constant rate the power of the expansion effect in restoring the use of the expensive input is zero, b) at a diminishing rate the expansion effect will increase the use of the more expensive input. Moreover, (i) if the increase in the use of capital or labour when it becomes expensive is greater than the decrease in capital or labour due to the substitution effect, the expansion effect outweighs the substitution effect and more of the expensive input will be used, (ii) if the expansion effect simply restores the use of the expensive input to its original level, the two effects are neutral, (iii) if less of the expensive input is used after the change in its wage, the substitution effect is stronger, (iv) when the expansion effect is stronger than the substitution effect, a decrease in capital wage rate may lead to an increase in the labour wage rate and vice versa, because a wage decrease means more of both inputs will be used.

The standard marginal productivity theory does not make explicit distinction between change in the level of capital intensity in timber production due to new silvicultural investment and the process of physical substitution between silvicultural inputs in actual timber production activities because the definition and the purpose of the long-run and short-run production functions are not explicitly separated. The failure to distinguish between the two definitions of the concept of production function makes the choice of technique (determination of optimum capital intensity) and the choice of input proportions (utilization of silvicultural inputs subject to the existing technology) to be interchangeable concepts.

When the distinction between choice of silvicultural technique and choice of silvicultural input proportions is made explicit, it becomes clear that by manipulating input wages by means of fiscal and monetary policies, or if the forest sector is subject to a shock, e.g., a sudden change in the level of demand, it is possible to realize changes in the proportion of silvicultural inputs in timber production while the optimum silvicultural technology remains fixed in the short run.

The influence of wage flexibility at a particular point in time on the process of physical substitution between silvicultural inputs examined in a manner analogous to Slutsky decomposition in consumer demand theory shows that we cannot *apriori* assert an unambiguous direction of change in the input proportions and relative prices as the traditional marginal production theory implies. The lack of a clear direction of change in the input proportions and relative prices means that the prediction of gains in employment by manipulating factor wages by means of fiscal and monetary policies may not be realized as expected when such measures are actually instituted.

A study of the problem of choice of silvicultural technology as a conflict between present and future consumption based on neoclassical social marginal productivity theory assumes implicitly that consumption and investment are equally valuable at the margin (Sen 1960, Marglin 1976). The assumption of attaching equal weights to current consumption and savings makes the choice of silvicultural technology as defined in this study to be equal to the problem of allocating an optimal amount of savings among techniques with varying degrees of labour-intensity. The link between choice of silvicultural technique (capital intensity in timber production) and financing of future silvicultural investments out of profit becomes non-existent because an equal weight is attached to a unit of consumption and investment at the margin. In other words, the proportion of reinvestment out of future income is not relevant to the problem of choice of capital intensity in timber production at present.

Social marginal productivity theory assumes that the real cost of excess labour and other economic resources in a state of equilibrium is zero. The valuation of excess labour at zero price comes out explicitly when marginal productivity theory is interpreted in terms of the mathematical technique of linear programming. This approach to the valuation of resources makes a substantial amount of labour a free commodity in a labour surplus economy. Hence, labour-intensive silvicultural technology as a solution to the problem of choice of capital intensity in timber production becomes inevitable.

The interpretation of choice of silvicultural technology as a problem of allocating optimal amount of savings among techniques with varying degrees of labour-intensity, is only compatible with an economy in which the rate of domestic savings in equal to the amount of savings required to finance investment consistent with a state of full employment. However, if the amount of domestic savings is sub-optimal, it is not necessarily true that the real cost of surplus labour is zero. Since the amount of future silvicultural investment depends on the share of wages and profits out of the forest sector income formation, the real cost of an extra worker employed from the excess labour pool is the amount of investment foregone valued in terms of consumption because the propensity to consume out of wage income is higher than the propensity to consume out of non-wage income.

Another way of establishing a direct link between choice of technique and the proportion of income saved is to make the propensity to save endogenous in the derivation of investment criterion. This condition is fulfilled by relaxing the assumption that there are no political and institutional constraints restricting the savings function. The absence of these constraints means that the savings rate can be adjusted by means of fiscal and monetary policy to that level needed to finance investment consistent with full employment freely. If this is possible for the national economy, the choice of silvicultural technology becomes once more a problem of allocating optimal amount of savings to different techniques with varying labour-intensity since fiscal and

monetary policies banish sub-optimality of savings. If this is possible, the real cost of labour is zero and the recommendation of a labour-intensive technology in a labour surplus economy is a rational investment decision.

The rationale of appropriate technology in economic development literature that contests the rationality of transferring current consumption to the future (i.e., the approval of capital-intensive techniques in labour surplus economies) assumes non-existence of sub-optimal savings by exhorting the virtue of fiscal and monetary policies. The aim of silvicultural investment in this context is to maximize immediate consumption with the future consumption not taken into account explicitly.

Silvicultural investment is indispensable to gains in forest labour productivity. Because labour productivity depends on savings and the degree of sub-optimality of the savings rate, in a labour surplus economy we must allow savings to command a premium in order to maximize forest labour productivity.

To conclude, we assume that before the national economy is operating in the neighbourhood of full employment, timber production is the main goal of forest management. To ensure that timber production is controllable, all other goals of forest management must be interpreted as constraints to the level of roundwood cuttings and silvicultural investment. When timber production is dynamically controllable, i.e., it is possible to ascertain a trajectory leading to the target growing stock and silviculture of forest policy, it is also stabilizable. The stability property makes it possible to damp out effects of a shock or policy error within the relevant time horizon such that the target variance is minimized.

6. SUMMARY

The basis of traditional forest production goal studies is Marshallian price theory. Following Marshall's typology of markets that involves alteration of the time frame of analysis, apparent dynamic timber supply models are derived. Because the assumption of *laissez-faire* atomistic competition is so critical to the Marshallian model, production goal studies based on it usually treat the case of market economy institution only. This is an unnecessary restriction because a model of timber supply is independent of any particular institutional set-up; a supply function represents an economic efficiency criterion which is objectively given for all types of economic systems. Moreover, since the dynamic timber production behaviour is examined within the framework of a fictitious time domain, disequilibrium is not derived as an on going process characterizing a developing economy so that the case for forest policy is established. In other words, economic models that exhort the virtues of price adjustment as a necessary and sufficient condition for maintaining a state of full employment are not consistent with the formulation of forest policy models which represent policymakers' deliberate interference in the functioning of the forest sector, with the aim of achieving balance of demand and supply equilibrium.

Another traditional approach to the study of timber supply is based on Faustmann optimum stand age model (Faustmann 1849). The purpose of these studies is to derive optimal steady state of timber growing stock as an asymptotic equilibrium position of the process of adjustment behaviour in timber production. The derivation of steady state timber growing stock that is independent of the influence of time is incongruous with the behaviour of the forest industry operating in a time domain economy. Therefore, studies of timber supply based on the stand age model do not represent a serious theory of forest policy.

In this study, the starting point of investigating the problem of timber supply was Adam Smith's Economic Principle as the mechanism of allocating resources in forest management. The nature of the problem posed by timber production was used as a criterion for selecting growth theory as the basis of examining the process of capital intensity in timber production.

Because of the inherent link between forest industry and timber production in the roundwood market, it was considered justifiable to interpret forest industry productive capacity as the determinant of national target forestry stocks (growing stock and silviculture). This interpretation led to the study of production and consumption of forest products in the context of a developing economy in order to understand the influence of changes in population, the state of technology, and consumers' preferences on the dynamics of timber production. Based on the three primary causes of dynamic changes in a developing economy, we have derived structural movements in the production of forest sector final commodity. As a requirement for the structural movements to take place in equilibrium, two types of necessary conditions emerged: 1) the forest industry new investment condition specifying the evolution of capital accumulation, 2) a macroeconomic effective demand condition specifying the demand for the forest sector final commodity in the context of aggregate demand in the national economy.

A dynamic investigation of the equilibrium conditions revealed that the macroeconomic effective demand condition has an inherent tendency to be undersatisfied through time. The failure to maintain the macro equilibrium condition in a dynamic setting makes the pursuit of balance of demand and supply equilibrium in the forest sector a continuous process by means of forest policy. This conclusion was considered to be the main justification of policymakers' intervention in the functioning of a national economy in which the purposeful behaviour of forest agents is guided by Adam Smith's Economic Principle.

Given the long period involved in timber production, a proportional expansion in the stocks of forestry would simplify the difficult task of planning for long-term timber supply. However, to be consistent with the nature of a developing economy, forest industry capacity must define the equilibrium values of growing stock and silviculture. These equilibrium values interpreted as the target of forest policy constitute an asymptotic position of the process of adjustment in timber production. Since forest industry capacity is non constant through time, forest policy target growing stock and silviculture represents a moving asymptotic equilibrium position.

The seemingly controversial question of employment in labour surplus economies has been examined in the course of the derivation of silvicultural investment criterion. In the study of silvicultural investment, there are two interrelated but distinct problems: a) the problem of choice of silvicultural technology in timber production, and b) the problem of choice of the level of utilization of silvicultural inputs in timber production. Unlike the traditional production theory that assumes choice of technique and choice of input proportions to be interchangeable concepts, in this study we dealt explicitly with the problem of choice of silvicultural technology alone by utilizing the definition of the long run of the concept of production function only.

According to the static traditional production theory, the technique that optimizes a forest owner's objective function when the marginal products of silvicultural inputs defined by it are in balance with their respective market prices, represents the optimal method of resource allocation in timber production. However, when the market prices of resources, e.g., wages and interest, do not reflect their true social opportunity cost, the use of marginal principles will result in misallocation of resources in timber production. In labour surplus economies, it has been observed that the prevailing market wage rate (interest rate) is usually greater than (less than) the real wage rate (real interest rate). To attain optimal allocation of resources in a labour surplus economy, the following measures have been put forward: a) social accounting of prices, b) changes in fiscal and monetary policies that should eliminate distortions in factor prices. The main aim of these measures is to encourage policymakers to choose labour-intensive techniques in economies where labour is plentiful.

The search for labour-intensive techniques should be subject to the amount of domestic savings available and the importance of alternative economic development objectives. This is necessary in order to avoid the interpretation of labour-intensive techniques in labour surplus economies as a *panacea* because given certain objectives they may not be desirable.

In economic analysis employment is a means of achieving certain objectives, e.g., income distribution, output creation (production). When aggregate consumption is defined as the objective of timber production, employment becomes subservient to forest sector income formation.

Given aggregate consumption as an objective, employment is desired not for its own sake but for the output it would generate, and it becomes an objective not to be pursued beyond a certain point. Optimum level of employment is defined at the margin where additional employment does not conflict with forest sector output expansion. Moreover, since additional output is comprised of additional immediate consumption and additional investment which is instrumental in increasing future additional consumption, a diversion of resources from investment to immediate consumption may be socially costly if we hold investment to be more valuable than consumption at the margin.

In this study, the reinvestible surplus criterion as developed by Sen (1960) was the basis of derivation of silvicultural investment criterion. Determination of capital intensity in timber production was formulated as a dynamic problem with the linkage between savings and choice of silvicultural technique defined via distribution of income between wages and profit. Maximization of aggregate consumption via output creation was posited as the objective of timber production. We assumed a state of sub-optimal savings rate as a justification for attaching a premium on investment as compared to immediate consumption at the margin. The assumption of sub-optimal savings rate implies that unemployed labour is not a free commodity in a labour surplus economy. Since labour is not a free commodity, intensive silviculture represents an efficient method of maximizing aggregate consumption as the goal of timber production.

The foregoing conclusions constitute in a concise form an economic explanation of the choice of capital intensity in timber production formulated in this study. The model formulated is original particularly with respect to its implications for the study of forest policy analysis. The notion of state employed in modeling dynamic aspects of a developing economy and timber production is basic to the representation of time series dynamics by Markovian models, e.g., Aoki (1987), and marks a complete departure from the traditional econometric forest policy analysis. The so-called structural models of econometric policy analysis essentially represent static relations among historical data, which remain valid as decision rules as long as economic agents' purposeful behaviour remains invariant with respect to their specification. However, in a developing economy it only makes sense to assume invariant policy models if dynamic econometric models are formulated explicitly in terms of the parameters characterizing economic agents' preferences and technologies.

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APPENDIX A

The condition at which the rate of change of the production and consumption coefficients are stationary is derived. When this condition is satisfied we obtain that special case of proportional economic development through time.

Given

$$\text{(A–1)}\quad x(t) = \frac{1-(\beta(t)/\alpha(t))\xi(t)\exp(\rho(t,\psi)-\eta t)}{(b(t)/\alpha(t))\exp(\rho(t,\psi)-\eta t)}$$

we are required to ascertain the rate of change of the R.H.S terms in (A–1) such that the following is true:

$$\text{(A–2)}\quad \frac{dx(t)}{d(t)} \equiv 0$$

Denote the numerator and the denominator of the R.H.S. of (A–1) by u(t) and v(t), respectively. Then

$$\text{(A–3)}\quad x(t) = \frac{u(t)}{v(t)} = \frac{1-a(t)e^{m(\cdot)}}{g(t)\,e^{m(\cdot)}}$$

where

$$a(t) = \frac{\beta(t)}{\alpha(t)}\xi(t)$$

$$g(t) = \frac{b(t)}{\alpha(t)}$$

$$m(\cdot) = \rho(t,\psi)-\eta t),$$

and

$$\text{(A–4)}\quad \frac{dx(t)}{dt} = \frac{u'(t)v(t)-v'(t)u(t)}{v^2(t)}$$

Because of (A–2), we are only concerned with the numerator of (A–3). From equation (A–4),

$$\text{(A–5)}\quad u'(t) = -\,[a'(t)e^{m(\cdot)} + a(t)m'(\cdot)e^{m(\cdot)}]$$

and

$$\text{(A–6)}\quad v'(t) = g'(t)e^{m(\cdot)} + g(t)m'(\cdot)e^{m(\cdot)}$$

Therefore,

$$\text{(A–7)}\quad u'(t)v(t) - v'(t)u(t) = a'(t)g(t)e^{2m(\cdot)} - g'(t)e^{m(\cdot)} + a(t)g'(t)e^{2m(\cdot)} + g(t)m'(\cdot)e^{m(\cdot)}$$

To satisfy condition (A–2), the R.H.S of (A–7) is set equal to zero:

$$\text{(A–8)}\quad e^{m(\cdot)}[a(t)g'(t) - a'(t)g(t)] - g(t)m(\cdot) - g'(t) = 0$$

or alternatively,

$$\text{(A–9)}\quad e^{m(\cdot)}[a(t)g'(t) - a'(t)g(t)] = g(t)m(\cdot) - g'(t)$$

Applying the quotient rule of equation (A–4) to equation (A–9) we obtain

$$\text{(A–10)}\quad \frac{d}{dt}\;\frac{g(t)}{a(t)} = \frac{g'(t)^{-m(\cdot)} + g(t)m'(\cdot)e^{-m(\cdot)}}{a^2(t)}$$

By inspection of (A–10) we obtain

$$\text{(A–11)}\quad a(t) = e^{-m(\cdot)}$$

When (A–11) is substituted in (A–3), condition (A–2) is satisfied.

APPENDIX B

The existence of foreign trade frees the level of silvicultural investment from the constraint of domestic savings. However, the meaning of the assumption that imported capital goods must be purchased with forest sector exports implies that a developing economy ultimately, depends on its own savings. This may be characterized as the process of self-contained development which is indispensible to growth as a goal of economic policy.

The aim of this Appendix is to show how maximization of real domestic absorption may be realized by expanding forest sector production. By domestic absorption we mean taking economic resources from the domestic capital, commodity and labour markets, as well as from foreign sources, i.e., imports (Alexander 1952). To facilitate the use of Keynesian macroarithmetic, we shall adopt discrete-time and construe all variables as annual magnitudes caused by change in policy (Feldstein 1973, Vehkamäki 1986 b).

Let us suppose that total domestic absorption is in balance with forest sector income formation (production):

$$\text{(B–1)}\quad y_1^* + y_2^* = y^*$$

where

y_1^*, y_2^* = total absorption comprised of domestic and foreign resources

y^* = forest sector production (income formation).

Equation (B–1) in terms of balance of payments equilibrium condition (2–20) becomes:

$$\text{(B–1')}\quad \zeta - \mu_c + \mu_i + \mu_k = (y^* - \xi) - x$$

where

ζ = additional forest sector exports

μ_c = additional import of consumption goods

μ_i = additional import of investment goods

μ_k = additional import of silvicultural inputs

ξ = additional consumption of forest sector products

x = forest sector investment

The left hand side of equation (B–1') defines the current account of the balance of payments equilibrium. Because there exist suboptimal savings in the economy, we introduce disequilibrium in the current account

$$\text{(B–2)}\quad \zeta < \mu_c + \mu_i + \mu_k$$

The deficit in the current account denoted by π, is financed by expanding forest sector production, i.e.,

$$\text{(B–3)}\quad \mu_c + \mu_i + \mu_k - \zeta = \pi$$

Because of the assumption of self-contained development, the condition for internal-external balance is defined by

$$\text{(B–4)}\quad \xi + x = y^* - \pi$$

$$\text{(B–5)}\quad \mu_c\xi + \mu_i x = \zeta - \mu_k$$

Notice that π in (B–4) is the value of extra imports in export-equivalent, i.e., the proportion of the additional amount of forest sector exports measured in domestic prices that must be sent abroad to meet the deficit created in the current account of balance of payments equilibrium by the extra imports.

To determine optimum domestic absorption, we solve for the values of ξ and x that simultaneously satisfy equations (B–4) and (B–5):

$$\text{(B–6)}\quad \xi = [\mu_i(y^* - \pi) - (\zeta - \mu_k)]\,/\,\mu_i - \mu_c$$

$$\text{(B–7)}\quad x = [(\zeta - \mu_k) - \mu_c(y^* - \pi)]\,/\,\mu_i - \mu_c.$$

assuming that $\mu_i \neq \mu_c$. This assumption is satisfied whenever marginal propensity to consume is less than unity, which implies that savings takes place out of additional income.

To obtain an intertemporal real domestic absorption criterion, we define

(B–8) $\Pi = \xi + m(x)$

where

Π = annual domestic absorption

m = premium on savings considered to be positive since we have assumed the savings rate to be suboptimal.

The value of m in (B–8), i.e., the additional weight attached to investment as against consumption, depends on the relative weights attached to current consumption as against future consumption as follows:

(B–9) $m = \dfrac{\delta}{r}$

where

δ = market interest rate

r = social rate of time preference.

By inserting (B–6) and (B–7) into (B–8) and summing up over the entire time horizon, a criterion for maximizing absorption intertemporally is obtained (Vehkamäki 1986b):

(B–10) $$\Pi = \sum_{t=0}^{t=T} \left[\frac{(\mu_1 - m\mu_c) y^* + (m-1)\mu_c + (m-1)\,\mu_i + (m-1)(\zeta - \mu_k)}{\mu_i - \mu_c} \right] (1+r)^{-t}$$

Future consumption is discounted by r because we want to increase future domestic absorption of economic resources. By increasing future absorption, the economy moves towards the desirable state of full employment, and the value of r approaches δ, hence:

(B–11) $\lim\limits_{r \to \delta} m = 1$

Assuming that in a state of full employment the share of savings is optimum (by optimum savings level we mean an amount of savings that is sufficient to finance investment consistent with a state of full employment), it is defensible to attach the same weight to the part of forest sector income formation that is consumed immediately and to the part that is invested.

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ISBN 951-651-082-5
ISSN 0001-5636

Karisto Oy:n kirjapaino
Hämeenlinna 1988