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RISTO OJANSUU

PREDICTION OF SCOTS PINE INCREMENT USING A MULTIVARIATE VARIANCE COMPONENT MODEL

MÄNNYN KASVUN ENNUSTAMINEN MONIMUUTTUJA- JA VARIANSSIKOMPONENTTIMALLILLA

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ACTA FORESTALIA FENNICA 239

PREDICTION OF SCOTS PINE INCREMENT USING A MULTIVARIATE VARIANCE COMPONENT MODEL

Männyn kasvun ennustaminen monimuuttuja- ja varianssikomponenttimallilla

Risto Ojansuu

To be presented, with the permission of the Faculty of Agriculture and Forestry of the University of Helsinki, for public criticism in Auditorium M II, Metsätalo, Unioninkatu 40 B, Helsinki, on 17 September 1993, at 12 o'clock noon.

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Diameter and volume increment as well as change in stem form of Scots pine (*Pinus sylvestris* L.) were analysed to predict tree increment variables. A stem curve set model is presented, based on prediction of the diameters at fixed angles in a polar coordinate system. This model consists of three elementary stem curves: 1) with bark, 2) without bark and 3) without bark five years earlier. The differences between the elementary stem curves are the bark curve and the increment curve. The error variances at each fixed angles and covariances between the fixed angles are divided into between-stand and within-stand components. Using principal components the between-stand and within-stand covariance matrices are condensed separately for stem curve with bark, bark curve and increment curve. The two first principal components of the bark curve describe the vertical change in Scots pine bark type and the first principal component of the increment curve describes the increment rate. The elementary stem curves, bark curve and increment curve as well as the corresponding stem volumes, bark volume and volume increment can be predicted for all trees in the stand with free choice of sample tree measurements. When only a few sample trees are measured, the stem curve set model gives significantly more accurate predictions of bark volume and volume increment for tally trees than does the volume method, which is based on the differences between two independent predictions of volume. The volume increment of tally trees can be predicted as reliably with as without measurement of sample tree height increment.

Männyn (*Pinus sylvestris* L.) läpimitan ja tilavuuden kasvua sekä muodon muutosta analysoidaan puun ja koealan menneen kauden kasvun ennustamisen kannalta. Työssä esitetään runkokäyrästömalli, jolla ennustetaan napakoordinaatiston kiinteitä kulmia vastaavat rungon läpimitat. Malli käsittää kolme yhtäaikaisesti määriteltyä perusrunkokäyrää: runkokäyrät kuorineen ja kuoretta mittaushetkellä ja kuoretta viisi vuotta aikaisemmin. Perusrunkokäyrien erotuksina saadaan kuorikäyrä ja kasvukäyrä. Kiinteiden kulmien jäännösvarianssit ja kulmien väliset kovarianssit jaetaan metsiköiden väliseen ja sisäiseen komponenttiin. Metsiköiden väliset ja sisäiset kovarianssimatriisit tiivistetään tärkeimpiin vaihtelusuuntiinsa pääkomponenttien avulla erikseen kuorelliselle runkokäyrälle, kuorikäyrälle ja kasvukäyrälle. Kuorikäyrän kaksi ensimmäistä pääkomponettia kuvaavat männyn kuorityypin muuttumista puun tyveltä latvaan ja kasvukäyrän ensimmäinen pääkomponentti kuvaa puun kasvunopeutta. Mallilla voidaan ennustaa metsikön puiden runkokäyrät ja niistä johdetut tilavuudet, kuoren tilavuus sekä menneen viisivuotiskauden tilavuuskasvu sekä koepuille että lukupuille vapaasti valittavalla koepuiden mittaustavalla. Lukupuiden kuoren tilavuuden ja tilavuuskasvun ennusteiden luotettavuus on huomattavasti parempi kuin erotusmenetelmässä. Koepuiden pituuskasvun mittauksesta luopuminen ei huononna lukupuiden tilavuuskasvuennusteiden luotettavuutta.

Keywords: bark volume, crown height, variance component models, multivariate models, stem curve models, tree form change, volume increment.
FDC 174.7 *Pinus sylvestris* + 56

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Contents

Preface

Professor Pekka Kilkki gave me the idea of this study. His enthusiastic scientific support in the beginning of the work was considerable. Dr. Juha Lappi guided me during the whole process with encouraging criticism and comments. Discussions with Dr. Helena Henttonen helped me to clarify many details of the work. Advice of Professor Hannu Niemi were very valuable in the final outlining of the study. Professor Pertti Hari, Dr. Jouko Laasasenaho, Professor Timo Pukkala and Dr. Lauri Valsta made useful comments to the manuscript.

The study was mostly carried out in the former Section of Growth and Yield of the Finnish Forest Research Institute. Professor Yrjö Vuokila and his successor Professor Kari Mielikäinen, the heads of the section, gave me the possibility to concentrate on this work. After the reorganization of the Finnish Forest Research Institute, I finished this study in the project "Forest models", lead by Dr. Lappi.

I obtained the primary data from Professor Vuokila. Mr. Kari T. Korhonen (Lic. For.) helped me to select the test material from the tree analysis data of the Department of Forest Resources of the Finnish Forest Research Institute.

The language was checked by Dr. Joann von Weissenberg. Ms. Marja-Liisa Herno and Ms. Hannele Alhola drew the final figures.

I wish to express my deep gratitude to all the persons who have helped me in my work.

Finally, I would like to thank my wife Saija, and my children Elina and Eero, for their patience and understanding.

Helsinki, June 1993

Risto Ojansuu

1 Introduction

1.1 Generalization of the sample tree data

The prediction of variables for a set of trees is normally divided into two stages. First, the distribution of diameter at breast height is measured or estimated for each tree species. Some trees, however are measured more accurately. Trees measured to determine the diameter distribution are called tally trees and the more accurately measured trees, sample trees. In the second phase, the values of the sample tree variables are predicted for the tally trees. It is called "generalization of the sample tree data".

Forest inventory, or determination of values for variables describing a set of trees of a sample plot, stand or forest area, can be improved by developing measurement techniques, sampling methods and models used for calculating the results (Päivinen 1987). Those components are mutually connected so that improvement in one can lead to changes in the others. To find effective use of sample trees, information on the effects of different measurement strategies of the sample trees is needed. The measurable variables, measurement errors and costs are related to the measurement techniques. The measurement error in the independent variable of a model can lead to increasing random errors and in many cases also to bias of the prediction. The main questions about utilization of the sample tree are how many sample trees should be measured, how they should be divided among the sample plots and which dimension of the sample trees shall be measured.

Regression models are often used for generalization of the sample tree data to the tally trees. Another method of generalization is the grid method (Holm et al. 1979). In the grid method, tally and sample trees are sorted into classes of stand variables and tree variables common for both tree types. Then, to derive values for the sample tree variables, a sample tree located in the same class is allotted for each tally tree.

The interesting tree variables, such as volume, value and their increments, are not directly measurable from standing trees. Tree models, as volume functions or stem curve models, are used to predict these variables as a function of sample tree variables. For tally trees these predictions can be made based on information generalized from measurements of sample trees. Values of the interesting variables can also be predicted for the tally trees directly from the corresponding predictions for the sample trees using methods for generalization of sample tree data. The standard error of the predictions can be decreased by increasing the number of sample trees or by including more independent variables in the tree models.

In normal practice, generalization of sample tree data and prediction of the values of interesting tree variables are done separately. The generalization phase does not use any a priori information from the statistical point of view, and prediction of the interesting tree variables is based purely on the a priori information in the form of the fixed part of tree models for the interesting variables as function of the measured sample tree variables. Fog and Jensen showed already in 1953 that a priori information about the between- and within-stand variation can be used to improve the reliability of the predictions of tally tree and stand volumes by calibration of the volume equation.

Later, a priori information on random variation has been used in different modelling problems in forestry. Burk & Ek (1982) and Green & Strawderman (1985) used empirical Bayes estimation to increase the reliability in small strata by using informative prior distribution of the slope coefficient of different models. Gertner (1984) showed the efficiency of the sequential Bayes estimation in calibrating a regional growth model on a local level. Pekkonen (1983) used between-stand and within-stand variances to weight sampling information and prior information to estimate stand volume. A variance component model, which is a mixed linear model with random classification variable, was first used in forestry by Andersen (1982) to predict tree and stand volumes using tree models. Lappi (1990) gives a more general view of the use of mixed linear models with random parameters for stand level in forest mensuration.

A mixed linear model gives effective calibration of a tree model on the stand level, with respect to the measurements made in a particular stand. Stand-wise calibration also gives more reliable results on the regional level than does estimation without calibration. Mixed linear models can also be calibrated to a stand with only one measured sample tree. On the other hand, the predictions of methods using a priori information are shrunk to the sample mean of the data set used for model estimation. A variance component model with random stand effects gives biased prediction for a particular stand but unbiased prediction for the whole population of the stands.

1.2 Increment prediction

A tree is depicted quantitatively using tree variables. Increment is the difference between values of a variable at two different points in time. The direct and most accurate way to study the past development of the tree stem in the form of tree dimension increments is stem analysis (e.g. Spurr 1952, p. 226–239, Prodan 1965, p. 197 and Loetsch et al. 1973, p. 244), which produces the without bark stem curves of a tree at fixed years. The stem curves are determined using measured diameters at fixed intervals on the vertical axis of the stem. The diameters in given years are measured from cores made with an increment borer or from cross section cuttings of felled trees. Diameters at given years can also be measured from standing trees with repeated measurement. This method of measurement produces increments with bark. It is not normally used because time interval between the measurements is usually 1–10 years and the shorter the time interval between the measurements the relatively less accurate are the measures of diameter increment.

Diameters between the measured points can be interpolated to obtain continuous stem curves. The accuracy of the interpolated stem curve depends on the number of diameters measured, their distribution on the stem and the method of interpolation when the effects of noncircularity of the cross-section of the stem and measurement errors are excluded (Lahtinen & Laasasenaho 1979 and Lahtinen 1988). The volume can also be determined directly as the sum of the volumes of stem sections determined with the measured diameters and their distances. Formulas for stem section volumes are normally based on conic sections (Prodan 1965, p. 53–58 and Loetsch et al. 1973, p. 146).

Indirect methods for determination of stem volume increment using tree models without stem analysis can be classified into four groups (groups 2–4 according to Svensson 1988): 1) the derivative method, 2) the component method, 3) the volume method (also called the difference method), and 4) the increment method. The three first methods are based on indirect determination of volume increment.

The derivative method uses partial derivatives of a volume function. The simplest example is the tariff-difference method (Loetsch et al. 1973, p. 250). Volume increment is the product of diameter increment and the first derivative of the volume function based only on diameter, the so-called tariff function. If the diameter increment is small and the model is based on a sample of a stable population, the derivative method is nearly unbiased. On the other hand, the predictions of the derivative method are only rough estimates, since the method assumes that each tree follows the regression line based on a cross-section of the population. The effects of changes in the surroundings of the tree cannot be taken into account.

The component method is based on the multiplicative components of tree volume: basal area at breast height, tree height and form factor of the tree. It can be showed that the relative volume increment can be approximated very well by the sum of relative basal area, height and form factor increments (Prodan 1965, p. 450–453). The height and the form factor can also be linked together to form height. In the application of the component model, the form height increment or the form factor increment are not measurable variables and they have to be predicted by auxiliary models as a function of some measurable tree or stand variables. Earlier the component method was widely used in Scandinavia: In Finland, Ilvessalo's (1947) tables for calculation volume increment based on basal area increment at breast height, height increment and form factor increment, in Sweden the tables of Jonsson (1928) and in Norway the functions of Strand (1968) based on basal area increment at breast height and form height increment.

In the volume method the increment is calculated as the difference between present and past volumes. These volumes are usually predicted by standard volume equations or stem curve models. Usually volume models are used where the independent variables are diameter at breast height and tree height. The values of the independent variables of the used volume equation have to be known at the beginning and at the end of the increment period. They can be measured or predicted with auxiliary models. The prediction errors of the auxiliary models have to be

taken into account, when the values of the independent variables of tree models are predicted using them (Kilkki 1979). The measurement error of height increment also has an effect on the accuracy of the volume method, because its variance is large and systematic errors occur (Päivinen et al. 1992).

In the increment method, the volume increment is predicted by a regression model directly as a function of the measurements without direct association with the volume prediction. The frequently used independent variables are diameter at breast height and tree height at the end of the increment period and measured past increment in basal area at breast height (Svensson 1988 and Strand & Li 1990). The height increment, measured or predicted with an auxiliary model, can be omitted as an independent variable. The regression model can also be formulated without basal area increment as an independent variable. Reliability of the model can be increased by including tree, site and competition variables into the independent variables.

Increment with bark is often of interest in forestry. Normally, stem analysis gives information from stem curves and increments without bark. Only the stem curve at the time of measurement can be determined with bark. The bark volume can be determined using the difference between the stem curves with and without bark. To calculate increment with bark, the bark increment is needed. Because the bark increment is an unmeasurable variable on temporal sample plots, indirect approximations for it are used. Most of the approximation methods are based on the assumption that the ratio of a particular tree variable with and without bark is constant or conditional on some tree and stand variables. As an example (e.g. Loetsch et al. 1973, p. 116), we assume that in a population there exists a bark equation $B = b_0 + b_1 D$, where B is double bark thickness, D diameter with bark, and b_0 and b_1 are parameters. The bark diameter increment can then be approximated with the first derivative of the bark equation, b_1. The diameter increment with bark is $[1 / (1 - b_1)]i_{Du}$, where i_{Du} is the diameter increment without bark and $[1 / (1 - b_1)]$ is the bark increment coefficient.

For volume increment, more complicated methods are used. In the Swedish National Forest Inventory, the bark volume increment is determined using the formula (Jacobson 1978):

$$I_{BV} = \frac{V_B - V_{B5}}{V_B}(V - V_u) \qquad (1.2.1)$$

where I_{BV} = prediction of bark volume increment
V_B = prediction of bark volume as a function of diameter at breast height without bark
V_{B5} = prediction of bark volume five years before the measurement as a function of diameter at breast height without bark at the same point in time
V = prediction of volume with bark as a function of sample tree and stand variables
V_u = prediction of volume without bark as a function of sample tree stand variables

The term $V - V_u$ is the predicted bark volume as a function of sample tree and stand variables. Because of some systematic errors in bark predictions, Svensson (1988) predicted the bark volume directly with a regression model as a function of sample tree variables without bark and stand variables as independent variables.

In the Finnish National Forest Inventory estimations of the bark increment and the change in stem form are aggregated using a modification of form height defined as the ratio of volume with bark and basal area at breast height without bark (Kujala 1980). This ratio is assumed to follow a regression line conditional to tree height.

In studies dealing with changes in stem form, the main emphasis has been placed on the effect of one particular factor at a time. In his broad review Larson (1963) gives empirical results of the effects of crown size and exogenous factors on stem form and changes in it, as well as reviews the most important theories of stem form.

Graphical description has often been used in analyzing changes in stem form (e.g. Nyyssönen 1952, Vuokila 1960a and Assmann 1970). For analyzing changes in stem form by mathematical or statistical methods, the difference method has been most common. The form measure used has been form factor (e.g. Saramäki 1980) or form quotient (e.g. Hagberg 1966).

Sloboda (1977a) used curvlinear coordinate systems to describe the development of stem form. The stem is a result of the accumulation of diameter growth at different heights. On that basis Kilkki & Varmola (1981) illustrated changes in stem form by the partial derivative matrix of diameters at relative heights. In their modification of the derivative method, the diameter increments at relative heights were predicted as a function of one known diameter increment.

1.3 Purpose of the study

Volume increment is often the most interesting variable when tree stem increment is analyzed in order to predict single tree and sample plot variables. Increment of tree dimensions can also be of interest in finding changes in timber assortment volumes and timber value. For an individual tree, stem analysis produces the information needed to calculate interesting increment and bark variables, but it demands very accurate and expensive measurements.

Most of the indirect methods for prediction of volume increment give no information on stem form or the change in stem form. When a stem curve model is used, only the volume method gives stem form at the beginning and at the end of the increment period. When the volume method is used, the same stem dimensions have to be known at the beginning and at the end of the increment period. In many cases not all the available information on stem form can be used. For example, in the Finnish National Forest Inventory, the measured dimensions of sample trees are diameter with bark, double bark thickness and diameter increment in the past five years at breast height, diameter with bark at six meters height, tree height and height increment in the past five years. When the volume method is used, only information on breast height diameter and stem height can be utilized for increment prediction.

One of the main arguments for using the increment method is that the unreliable measurement of height increment can be omitted (Svensson 1988 and Strand & Li 1990). Information on the upper diameter with bark can also be used as an independent variable. In other words, the increment method is flexible, because it allows free choice of independent variables. The disadvantage of the increment method is that it predicts only volume increment without any information on changes in stem form.

Tree models have usually been examined separately, and their relevance for prediction is studied in terms of total residual sum of squares. It is often possible to formulate different statistical models with the same independent variables, which gives the same accuracy to the predictions. An example of this is the volume prediction models by Laasasenaho (1982), which are based on direct volume prediction with volume equations and volume determination using predicted stem curves. Because in many cases a new formulation of the fixed part of the model cannot significantly reduce the total residual sum of squares, the choice of independent variables has great importance for the relevance of a model.

When stand variables are predicted using tree models, it is important to find independent variables which can explain the between-stand variation in order to improve the prediction (Kilkki 1983). A variance component model with random stand effect separates the between-stand and within-stand variation and gives a practical tool for analyzing the effects of different independent variables on the between-stand variation. It also correctly describes a data set consisting of sample plots with mutually correlated trees.

The purpose of this study is to analyze Scots pine (*Pinus sylvestris* L.) stem increment using a statistical model with special emphasis on increment prediction. A model, called the stem curve set model, is presented, which consists of three elementary stem curves: stem curves with and without bark in year t and the stem curve without bark in year t–5. The difference between the stem curves with and without bark in year t is called the bark curve, and the difference between the stem curves without bark in the years t and t–5 is called the increment curve. The model is used to study the effects of different strategies of sample tree measurements on both bark and increment predictions on the tree and stand level. The effects of measurement errors of the standard measurement methods are also studied. The bark curve should be predictable as a function of diameters with or without bark in order to determine the increments with bark. For this purpose the stem curve set model should fulfil the following three main requirements: 1) free choice of sample tree measurements, 2) ability to take into account possible measurement errors and 3) efficiency of calibration at the stand level.

The free choice of measurements means that the stem curve set can be predicted if any dimensions of the elementary stem curves are measured. Firstly, with this kind of model it is easy to study the effect of different measurements on the prediction errors. Secondly, it is possible to predict the stem curve with bark as a function of the tree dimensions without bark.

If the model is able to take into account measurement errors, the elementary stem curve deviates from the measurement point depending on the measurement error variance. If no measurement error occurs, the elementary stem curves pass through the measurement points.

The efficient calibration means that the general model can be calibrated efficiently for a stand with only one measured sample tree. This requirement leads to a choice of methods using a priori information, which is unbiased only over all stands in the population, but is biased for a particular stand. If a priori information is not used to obtain reliable results for generalization of the sample tree information to tally trees, the number of sample trees needed for each plot is 20–40 (e.g. Vuokila 1965).

1.4 Choice of the modelling approach

The existing stem curve models are a natural base for developing of the stem curve set model. Here, stem curve models are reviewed only as needed to understand the background of the choice of the modelling approach used. Sterba's (1980) more comprehensive review of the stem curve models covers stem curve studies made before 1980.

The most natural way to describe the stem curve is as a continuous function. Stem curves expressed as continuous functions are often based on the form quotient (the proportion of two diameters measured at different heights) with base diameter at breast height (e.g. Höjer 1903, Kozak et al. 1969 and Max & Burkhard 1976). Some of these functions are derived from volume functions to obtain compatible systems of taper and volume functions (e.g. Demaerschalk 1972 and Clutter 1980). Volume functions have also been derived from stem curves (Byrne & Reed 1986). Common for all those models is that diameter at breast height and tree height are the only independent variables. Roiko-Jokela (1976) presented a stem curve model based on broken conic sections as a function of two fixed diameters ($D_{1.3}$ and D_7) and tree height.

Stem curve models based on the form quotient give the same shape for all trees with the same diameter at breast height and height. Laasasenaho (1982) developed a polynomial continuous stem curve model based on the natural form quotient with base diameter $D_{.2H}$ where the tree shape can vary as a function of tree height and additional diameters. The base model can be used if tree height and one diameter at any given height, D_1, are known. The base diameter can be derived from the measured diameter using the base model predictor f_b of the diameter at the corresponding relative height H_1 as follows:

$$\hat{D}_{.2H} = D_1 / f_b(H_1) . \qquad (1.4.1)$$

The base model can be adjusted to greater accuracy with a correction polynomial. The correction polynomial can be calculated for a single tree if three diameters at different heights and tree height are known. It can also be determined using auxiliary functions with diameters at fixed heights as independent variables. These functions should be fitted for every measurement combination from the same data set that was used to estimate the base model. Laasasenaho gives auxiliary functions for cases where the measured variables are $D_{1.3}$ and H, or $D_{1.3}$, D_6 and H.

If an adequate number of diameters are available, the stem curve can be described as a continuous function by interpolation (Sloboda 1977b). According to Lahtinen & Laasasenaho (1979), if the height and seven correctly positioned diameters are known, it is possible to obtain a reliable description of a stem curve using cubic spline functions. The correct positioning of the measurement heights of diameters is most important at the base of the stem. In the method of Lahtinen & Laasasenaho (1979), the additional conditions needed at the terminal points of the stem curve are derived from each stem, using its own measurements. The cubic spline function can give a nonmonotonic stem curve even if the measurements are monotonic. However, it is always possible to obtain a monotonic stem curve from monotonic measurements with a quadratic spline function (Lahtinen 1988).

Prediction of a continuous stem curve can be divided into two phases (Kilkki et al. 1978): prediction of some fixed points of the stem and interpolation between the points. Determination of fixed points is closely related to the concept of measurement function used in allometry,which is the study of the relationship between the size and shape of an organism (Sprent 1972). The concepts of measurement function, standard size variable and shape vector are used to describe an object in an operational way (Mosimann 1970). The measurement function is formed from linear distance measurements made between homologous points, here the fixed points on the stem curve. The shape vector is obtained by dividing the measurement function by the standard size, which is derived from the measurement function, e.g. as the weighted mean of the measurement function. Two individuals have the same shape if their measurement functions are proportional.

The description of a tree stem by the measurement function, standard size and shape vector requires that the biologically homologous points can be determined exactly. The birth point and the terminal bud of a tree are genuine homologous points. Determination of other homologous points is, however, theoretically problematic. The exact measurement function for a tree stem can be derived by replacing biologically homologous points with geometrically analogous points. Two common ways of determining analogous points of a tree stem are the diameters at relative heights (e.g. Cajanus 1911 and Laasasenaho 1982) and the ray lengths or corresponding diameters at constant angles in a polar coordinate system (e.g. Sloboda 1977a).

Kuusela (1965) expressed a measurement function using a natural form quotient, which was predicted as a function of the natural form factor. For practical applications this method demands regression models for the form factor as a function of the available measurements. Kilkki et al. (1978) predicted the measurement function using a multidimensional model based on simultaneous equations for diameters at relative heights. In the equations each element is regressed to all other elements of the measurement function. Exogenous variables, e.g. crown height, can also be added to the equations, which can be linear or nonlinear (Kilkki & Varmola 1979). A multidimensional model also gives estimates of the variances of the measurement function elements. Diameters and variances between measurement function elements can be estimated by interpolation. To predict basal area and volume without bias, estimates of the variances of the predicted diameters are needed.

Cajanus (1911) stated that, to separate stem size and stem form, a genuine variable for tree form should be independent of the absolute measures of the tree. Fries & Matérn (1966) and also Liu & Keister (1978) used principal component analysis to define the size and the form of a tree stem statistically independently. The first principal component of the stem measurement function describes the variation in stem size and the other principal components describe variation in stem form. Stem size described by the first principal component is uncorrelated with the shape of the individual. This places restrictions on studies dealing with the relationship between tree size and shape (Mosimann 1970).

Lappi (1986) developed a multidimensional stem curve model in which the logarithmic measurement vector is regressed on tree size determined as the first principal component of logarithmic measurement function. The measurement function is determined with diameters corresponding to fixed angles in a polar coordinate system. Lappi (1986) used variance components to separate between-stand and within-stand variation, and the model can be calibrated to a plot with only one sample tree. He also showed, how knowledge of measurement errors can be taken into account with the help of the measurement error variances and the within stand variances.

The model of Lappi (1986) was taken as the starting point for development of the new model, since it fulfills the requirements of free choice of measurements, ability to take into account possible measurement errors and efficiency at stand level calibration. Lappi's model still has two advantages. Firstly, the polar coordinates used are suitable for analysis of form changes (Sloboda 1977a). All analogous points and changes in their locations can be derived independently of each other since the tree height is the length of the ray at an angle of 90°. Secondly, the variance component structure of Lappi's (1986) model corresponds to the structure of the data used, where measured trees are concentrated on sample plots. When we are interested in the properties of the phenomenon, not only the prediction errors for a single tree, it is necessary to take into account the structure of the data set.

1.5 Computational aspects and notation

The programming language was FORTRAN-77. IMSL subroutines (IMSL...1982) were used for the matrix operations and interpolations. The spline interpolations of the stem curves were made by spline-subroutines based on the study of Lahtinen & Laasasenaho (1979).

In the terminology of mixed models, the fixed variables are estimated and the random parameters are estimated or predicted. Here the term "estimate" is used for both. The term "prediction" is used only to denote the determination of stem dimensions and volumes using models. For tree variables, capital letters denote arithmetic scale and lower case letters logarithmic scale. A list of the general symbols is given in Appendix E.

The concepts stem curve and taper curve have both been used for the continuous description of tree diameter as a function of its position on the vertical axis of the stem. The concept stem curve is preferred here because it is more informative and since it includes no assumption about the monotony of the stem form. The concepts stand and sample plot are used synonymously.

2 Data

2.1 Primary data

The data of Vuokila & Väliaho (1980), which were collected for stand-growth models, are used for analysis and modelling (primary data). The primary data were collected during the years 1970–1974 and consist of subjectively placed temporary sample plots that fulfilled the following conditions:

- Basal area of Scots pines over 90 % of the total basal area
- Homogeneous site and growing stock on the compartment
- No openings in the growing stock
- Stand established by sowing or planting
- No understorey
- Dominant height over 7 m
- At least 3 years since the last thinning
- Treated only with thinnings from below
- No hold-overs
- Good health

The geographical distribution of the primary data set covers nearly the whole of Finland (Fig. 2.1).

The sample plots were delineated to consist of at least 100 trees, with the exception of the youngest stands, which included at least 200 trees. The plot area was between 0.10–0.25 ha. Diameters at breast height were measured from two directions at right angles to each other from all trees with a diameter over 5 cm. The base height for all measurements was ground level.

For increment analysis, eight sample trees were measured from each plot. These trees were selected in two phases. First, 30 sample tree candidates were chosen from healthy unforked pines. Then every third tree of the sample tree candidates was selected. After that, the basal area distribution of the selected trees was compared with the basal area distribution of the whole plot. If necessary, the sample trees were changed to get an approximately even covering over the whole basal area distribution. Selection of neighboring trees was avoided.

The sample trees for increment analysis were measured as standing trees by climbing them. The diameters over bark, double bark thickness and also annual diameter increments at 1.3 and 6 m heights and at 9 relative heights, 1, 2, 6, 10, 20, 30, 50, 70 and 85 %, were measured. The highest measurement was omitted if it was impossible to climb to that height. The diameters, bark thickens and annual radial increments were measured from two directions at right angles to each other. The bark thickness was measured with a Swedish bark gauge and the annual radial increments by boring radial increment cores. Tree height, height of the living crown and height increment in five-years periods to the last thinning were also measured. Separate living branches under the living crown were not included in the crown if there were two or more death whorls between them.

Only sample trees with complete measures from

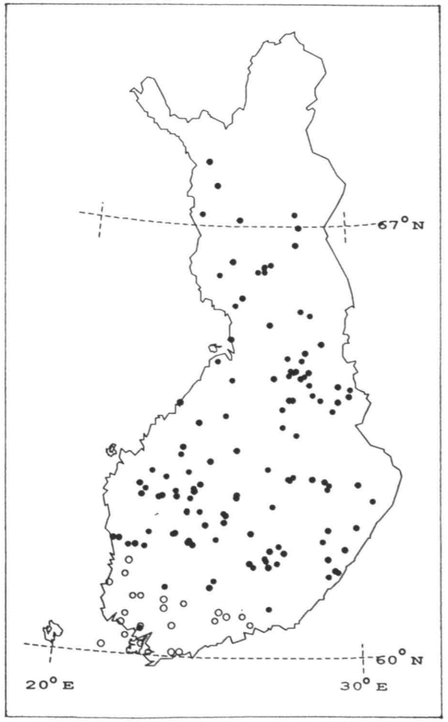

Fig. 2.1. Locations of the sample plots in the data sets: primary data (●) and test data (○).

Table 2.1. Diameter height distribution of the primary data.

D, cm	Height, m: 5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	34	25	26	27	28	Total
	Number of trees																								
5		2																							2
6			2	1																					3
7	1			1		2	1																		5
8	1		2	4	5	7																			19
9					6	6	2	1																	15
10			2		2	6	9	8																	27
11				3	1	4	5	11	3	1	1														29
12			1		2	3	4	3	3	8	4	1													29
13					1	5	6	7	1	10	7	2													39
14						4	9	3	3	7	6	2	3												37
15					1	5	2	3	6	13	12	5	3												50
16							5		3	4	18	12	5	1											48
17						2	1	1	2	11	15	15	6	6	1										60
18							3	1	3	5	12	8	10	8	1		1								52
19						1	2	1		8	8	10	8	5	5	1									49
20								1		1	8	17	7	5	5		1								45
21							1	1		1	3	5	5	9	2				1						28
22										3	5	1	11	7	4	1	2								34
23											2	1	6	5	4	1	1		2						22
24										1	1	2	6	1	6	1			1						19
25								1		1	1	1	3	4	4	3		1	1	1					21
26										1		1	1	3	4						1	1			12
27											1		1	3	2	1	2	2		1		1			14
28										1		1			3		1	1	3		1				11
29															1	1			2		1	1			6
30													1			2		1	1		2			1	8
31																		2		3	1	1			7
32														1					1	2	1				5
33																1									1
34																		1				1	1		3
35																									0
36																			1		1		1		3
37																									0
38																									0
39																						1			1
40																							1		1
41																									0
42																		1							1
Total	2	2	7	9	18	45	50	42	24	76	104	84	76	58	42	12	8	9	13	7	8	6	3	1	706

1, 2, 6, 10, 20, 30, 50 and 70 % relative heights and tree height were included in the primary data. The minimum accepted increment period was five years. The original data set consisted of 223 plots. Of these, 82 plots were rejected due to insufficient increment measurements and 12 due to the short measurement period. The final data set consisted of 129 plots and 706 Scots pines. The diameter height distribution of the primary data set is presented in Table 2.1.

The stem curve expressed with diameters at the fixed angles in the polar coordinate system was interpolated from the diameters at relative heights by cubic spline functions. Some statistics on the reliability of the transformation of the coordinate system are presented in Table 2.2. These were calculated at the above-mentioned relative heights from the differences between the original measurements and diameters interpolated from the diameters at the knot angles. All

Table 2.2. Reliability of the transformation from diameters (cm) at relative heights to the corresponding diameters interpolated from the diameters at knot angles. mean = bias and s_t = standard deviation.

Measurement height, %	Diameters: With bark in year t		Without bark in year t		Without bark in year t–5		Bark		Increment	
	mean	s_t	mean	s_t	mean	s_t	mean	s_t	mean	s_t
1	–.01	.01	–.01	.01	–.00	.01	–.00	.01	–.00	.01
2	.01	.01	.01	.01	.00	.01	.00	.01	.00	.01
6	–.04	.10	.04	.10	–.02	.06	–.01	.11	–.00	.07
10	.02	.06	.01	.06	.01	.06	.01	.08	.00	.06
20	–.02	.07	–.02	.08	–.02	.08	–.01	.10	–.00	.07
30	.01	.05	.00	.05	.01	.04	.01	.07	–.00	.05
50	–.00	.01	–.00	.01	.00	.01	–.00	.02	–.00	.01
70	.00	.01	–.00	.00	–.01	.02	.00	.02	.00	.02

stem curves and also the differences between them, the bark curve and the increment curve, are nearly unbiased. The standard deviations are usually less than 0.5 mm.

2.2 Test data

As independent test material, tree analysis data collected by the Department of Forest Resources of the Finnish Forest Research Institute were used (Korhonen & Maltamo 1990). This is a subsample from the sample plots of the 8th National Forest Inventory sample plots with a potential mean annual increment of over 0.1 m^3/ha/a. The sample plots measured in 1988 and 1989 in southernmost Finland were available for this study (Fig. 2.1).

Five felled sample trees were selected systematically from each angle-count sample plot. Diameters over- and under bark at heights of 1.3 and 6 m and at 16 relative heights were measured from cross-sectional cuttings in two directions at right angles to each other. Diameter increments were measured from two directions at right angles to each other at the following eight relative heights: 2.5, 7.5, 15, 30, 50, 70, 85 and 95 %. Scots pines that had reached a height of more than 1.3 m five years before measurement were accepted for the test data set. The final data set consisted of 30 sample plots and 103 Scots pines. In the test data, stem form and its change are known accurately. Although the data set is small, it can be used to illustrate how the stem curve set model can be used in an independent data set. The diameter height distribution of the test data set is presented in Table 2.3.

3 Stem curve set model

3.1 Delineation of the stem curve set model

The stem curve set model consists of three elementary stem curves determined in a polar coordinate system: the stem curves with bark and without bark in year t and the stem curve without bark in year t–5. The elementary stem curve model for stem curve with bark is identical with the stem curve model of Lappi (1986). In the stem curve set model, each of the elementary stem curves is expressed by a measurement function consisting of either rays R(u) or diameters D(u) corresponding to fixed angles u in a polar coordinate system (Fig. 3.1). Diameters are given in centimeters and heights in meters. The ray corresponding to the 90° angle is tree height. Fixed angles 1–13 are called "knot angles". The knot angles used are 0.25°, 0.7°, 1.5°, 3°, 5°, 8°, 14°, 21°, 31°, 41°, 56°, 72° and 90°, which are the same as those used by Lappi (1986). A continu-

Table 2.3. Diameter height distribution of the test data.

D, cm	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	34	25	26	27	28	Total
	Tree height, m																								
	Number of trees																								
5			2																						2
6		2	3																						5
7	1		2																						3
8	1		2		2																				5
9			1	1																					2
10		2	1	2		1					1														7
11			2			2		1																	5
12		1	2				1				1														5
13					1	1	1			1															4
14				2			1	1	1	1	1														7
15						1					1														2
16							1	2		1		1	1												6
17														1											1
18							1																		1
19											1	3													4
20										1	1														2
21												1		2		1									4
22										1			1	1											3
23													1				1								2
24														1											1
25												2	1	1	1				1						6
26											1		1	1		1		2	1						7
27													2												2
28																		1		1					2
29																							1		1
30													1			1	1								3
31												1									1				2
32															1										1
33								1																1	2
34																						1			1
35																									0
36													1					1							2
37																1									1
38																		1							1
39																									0
40																									0
41																									0
42																									0
43																	1								1
Total	2	5	15	5	3	5	5	5	1	5	7	8	9	7	2	4	3	5	2	1	1	1	1	1	103

ous elementary stem curve is interpolated between the elements of the measurement function. Crown height is included in the model as an exogenous variable.

The stem curve set is determined using the logarithmic delineation vector **d**. The elements of the delineation vector are called delineation variables. Denote the logarithmic delineation variables by $d_g(u)$, where u is the knot angle in polar coordinates and g indicates the stem curve: g = 1 for the stem curve including bark in year t, g = 2 for the stem curve without bark in year t and g = 3 for the stem curve without bark in year t–5. The corresponding delineation vector **d** consists of the measurement vector of the stem curve including bark in the year t $[d_1(1), \ldots, d_1(13)]$ and

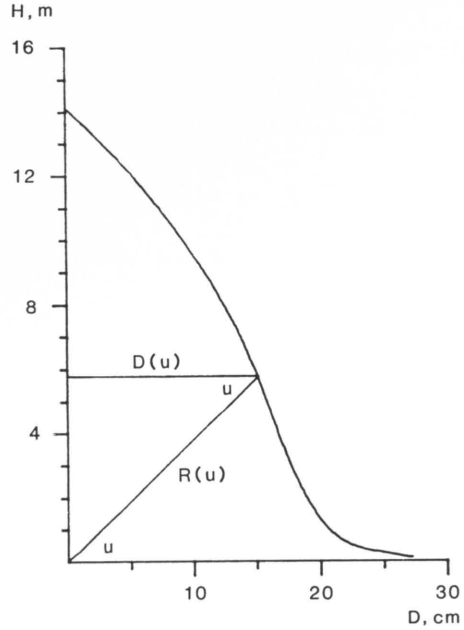

Fig. 3.1. Polar coordinate system where the stem dimension for angle u is either ray R(u) or diameter D(u) (Lappi 1986).

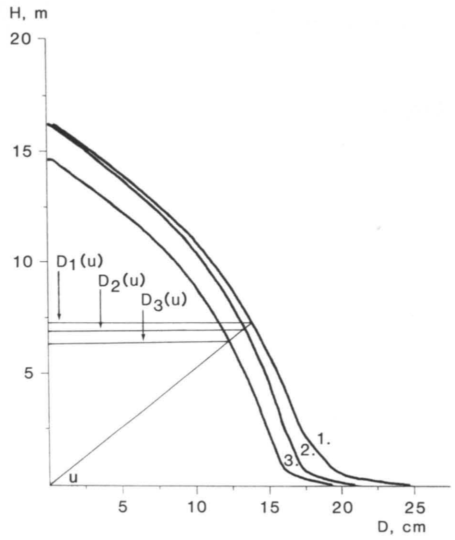

Fig. 3.2. The stem curve set in the polar coordinate system. 1 = stem curve with bark in the year t, 2 = stem curve without bark in the year t and 3 = stem curve without bark in the year t–5. The diameters at angle u are $D_1(u)$, $D_2(u)$ and $D_3(u)$, respectively.

the diameters at the same knot angles of stem curves without bark in the years t $[d_2(1), \ldots, d_2(13)]$ and t–5 $[d_3(1), \ldots, d_3(13)]$ and the crown height in the year t (h_c):

$$\mathbf{d} = [d_1(1), \ldots, d_1(13), d_2(1), \ldots, d_2(13), d_3(1), \ldots, d_3(13), h_c]. \quad (3.1.1)$$

The delineation variables at angle u are illustrated in Fig. 3.2. The difference between the stem curves with and without bark in year t is the bark curve (g = B), and the difference between the stem curves without bark in years t and t–5 is the increment curve (g = I).

The expected elementary stem curves are examined as a function of tree size s and the average tree size $\bar{s}$ in the stand. The tree size is defined in logarithmic scale as a weighted average of the elements of the logarithmic measurement function of the stem curve with bark:

$$s = \sum_{u=1}^{13} w(u) d_1(u). \quad (3.1.2)$$

The weight vector [w(1),...,w(13)] is the first eigenvector of the logarithmic sample covariance matrix for the measurement function of stem curve with bark scaled so that the sum of the elements is one. The logarithmic diameter of elementary stem curve g at angle u for tree i in stand k is

$$d_g(u)_{ki} = a_{g0}(u) + a_{g1}(u)s_{ki} + a_{g2}(u)s_{ki}^2 + a_{g3}(u)(s_{ki} - \bar{s}_k) + v_g(u)_k + e_g(u)_{ki} \quad (3.1.3)$$

where a_{g0}, a_{g1}, a_{g2} and a_{g3} are fixed parameters. The random variation is divided into random stand and tree effects; $v_g(u)_k$ is the random effect of stand k and $e_g(u)_{ki}$ is the random effect of tree i in stand k. It is further assumed that cov(**v**,**e**) = 0.

The term $(s_{ki} - \bar{s}_k)$ in Equation 3.1.3 is the relative size of tree i in stand k. The effect of relative size $(s_{ki} - \bar{s}_k)$ can also be divided into the effects of tree size and the average tree size, and the model can be written

$$d_g(u)_{ki} = a_{g0}(u) + [a_{g1}(u) + a_{g3}(u)]s_{ki} + a_{g2}(u)s_{ki}^2 - a_{g3}(u)\bar{s}_k + v_g(u)_k + e_g(u)_{ki} \qquad (3.1.4)$$

Lappi (1986) pointed out the problem of the low reliability of the relative size determination when small sample plots are used to describe the stand. Effects of the average size and the individual tree size can also be mixed in the fixed part of the model, since the correlation coefficient between them is as high as 0.79. Therefore the following model without average size was also studied:

$$d_g(u)_{ki} = a_{g0}(u) + a_{g1}(u)s_{ki} + a_{g2}(u)s_{ki}^2 + v_g(u)_k + e_g(u)_{ki} \qquad (3.1.5)$$

Replacement of the squared logarithmic tree size s^2 by the arithmetic tree size $S = \exp(s)$ was also studied. The logarithmic variances at knot angles were similar to those of model 3.1.3. The predictions in arithmetic scale were biased because of the inaccuracy of the Taylor expansion of S used to linearize the nonlinear constraint between s and S when the model is applied (Equation 4.1.2). The models with average size (3.1.3) and without average size (3.1.5) are examined in greater detail.

3.2 The stem curve set model as a mixed linear model

The standard linear model technique is used to estimate parameter values for the stem curve model. The general linear model is

$$\mathbf{y} = \mathbf{Xa} + \mathbf{Zb} + \mathbf{e}, \qquad (3.2.1)$$

and the normal equations are

$$\begin{bmatrix} \mathbf{X}^T\mathbf{R}^{-1}\mathbf{X} & \mathbf{X}^T\mathbf{R}^{-1}\mathbf{Z} \\ \mathbf{Z}^T\mathbf{R}^{-1}\mathbf{X} & \mathbf{Z}^T\mathbf{R}^{-1}\mathbf{Z} + \mathbf{D}^{-1} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{a}} \\ \hat{\mathbf{b}} \end{bmatrix} = \begin{bmatrix} \mathbf{X}^T\mathbf{R}^{-1}\mathbf{y} \\ \mathbf{Z}^T\mathbf{R}^{-1}\mathbf{y} \end{bmatrix}, \qquad (3.2.2)$$

where the matrices are the following:

y = vector of the dependent variables
X = matrix of the fixed independent variables
a = vector of the fixed parameters
Z = matrix of the random independent variables
b = vector of the random parameters
e = vector of random errors
R = covariance matrix of the random errors, var(**e**)
D = covariance matrix of the random parameters, var(**b**)

Lappi (1986) showed how model 3.1.3 with both ordinary fixed parameters (a's) and random stand effects (v's) as random parameters can be expressed as a variance component model in the case of the stem curve model. Here the parameter values for each element of the delineation vector **d** were estimated separately. The matrices for parameter estimation of the stem curve set model are the following:

y = elements of the delineation vector for the stem curve g at the angle u, $d_g(u)_{ki}$, Nx1, where N is the total number of trees.
X = matrix of the fixed independent variables [1, s_{ki}, s^2_{ki} and $\bar{s}_k$], Nxq, q = 4.
a = vector of the fixed parameters, qx1.
Z = incidence matrix of the stands, NxK, where K is the number of stands.
b = random stand effects v, Kx1, cov(v, v') = 0.
e = vector of random errors e, Nx1, cov(**b**,**e**) = 0
R = covariance matrix of the random errors, var(**e**), NxN
D = covariance matrix of the random stand effects, var(**b**), KxK. (3.2.3)

The matrices used in the normal equations are given in more detail in Appendix A1.

The stem curve set model is also a multivariate model, which consists of the elements of the measurement vector. The fixed part of the model describes the dependence of the stem form, bark thickness and diameter increment on tree size and relative size. The regularities of stem form, bark thickness and diameter increment deviations from the fixed part of the model are described with the covariance matrices between knot angles of the random stand effects (**b**) and the random tree effects (**e**). Those matrices separate and describe the variations between and within stands. The variances and covariances of the random parameters (random stand effects) and the random errors (random tree effects) are estimated using the fitting constant method (Henderson's method 3; see Searle 1971). Pairwise sums of the knot angles and the covariance formula for the sum of two variables were used to calculate the covariances of the random effects between knot angles u and u': cov(u,u') = 0.5[var(u – u') – var(u) – var(u')].

3.3 Fixed part of the stem curve set model

The weight vector **w** for the logarithmic size (Formula 3.1.2) was estimated from the primary data using the measurement vector of the stem

curve with bark. The estimated weights for the 13 knot angles are 0.0829, 0.0812, 0.0798, 0.0794, 0.0788, 0.0793, 0.0794, 0.0784, 0.0770, 0.0750, 0.0718, 0.0693, 0.0677. Volume and size are closely related in the primary data set, and the following allometric equation exists between them

$$V = 0.074016\ S^{2.969}. \qquad (3.3.1)$$

The relative standard error of the volume prediction is 3.2 %.

The average size used in the analysis was calculated from the sample trees. The estimates of the fixed parameters of models with average size (3.1.3) and without average size (3.1.5) are presented in Table 3.1. The effect of tree size on the stem form, bark thickness and diameter increment is presented in Fig. 3.3 according to the

Table 3.1. Estimates of the fixed parameters for models 3.1.3 and 3.1.5. g is the index of the elementary stem curve and u is the knot angle.

		Model with average size (3.1.3)				Model without average size (3.1.5)		
g	u	a_0	a_1	a_2	a_3	a_0	a_1	a_2
1	1	0.9782	0.5775	0.0861	0.2145	0.4292	0.8675	0.0596
1	2	0.7941	0.6744	0.0645	0.1871	0.3295	0.9083	0.0473
1	3	0.6418	0.7438	0.0481	0.1870	0.1859	0.9707	0.0321
1	4	0.4800	0.8273	0.0317	0.1833	0.0348	1.0387	0.0204
1	5	0.4373	0.8207	0.0337	0.1441	0.0933	0.9852	0.0255
1	6	0.4722	0.7421	0.0512	0.1464	0.1241	0.9085	0.0429
1	7	0.2911	0.8228	0.0379	0.1037	0.0531	0.9401	0.0329
1	8	0.0584	0.9427	0.0147	0.0313	–0.0012	0.9838	0.0109
1	9	–0.3460	0.9999	–0.0263	0.0781	–0.1215	1.0465	–0.0146
1	10	–0.8121	1.3914	–0.0709	–0.2028	–0.2651	1.1594	–0.0591
1	11	–1.3365	1.5582	–0.1052	–0.3374	–0.4403	1.1895	–0.0920
1	12	–1.9661	1.6092	–0.1170	–0.4138	–0.8704	1.1534	–0.1007
1	13	–0.7538	1.6539	–0.1267	–0.4619	0.4759	1.1347	–0.1060
2	1	0.7801	0.5386	0.1078	0.1994	0.2841	0.7910	0.0884
2	2	0.6062	0.6503	0.0820	0.1787	0.1707	0.8607	0.0700
2	3	0.4554	0.7344	0.0620	0.1708	0.0446	0.9364	0.0493
2	4	0.2998	0.8186	0.0455	0.1749	–0.1224	1.0213	0.0344
2	5	0.1182	0.9436	0.0209	0.1488	–0.2374	1.1142	0.0124
2	6	0.1447	0.9025	0.0283	0.1403	–0.1876	1.0643	0.0195
2	7	0.1877	0.8331	0.0409	0.1168	–0.0841	0.9652	0.0346
2	8	–0.0235	0.9493	0.0174	0.0480	–0.1229	1.0086	0.0131
2	9	–0.4428	1.1875	–0.0295	–0.0615	–0.2607	1.1011	–0.0204
2	10	–0.8864	1.4141	–0.0732	–0.1828	–0.3887	1.1999	0.0609
2	11	–1.4030	1.5857	–0.1089	–0.3256	–0.5392	1.2282	–0.9572
2	12	–1.9956	1.6201	–0.1183	–0.4099	–0.9093	1.1682	–0.1020
2	13	–0.7538	1.6536	–0.1267	–0.4619	0.4759	1.1347	–0.1060
3	1	0.5067	0.6017	0.1109	0.1306	0.1946	0.7725	0.0953
3	2	0.4278	0.6516	0.0958	0.1151	0.1614	0.7915	0.0855
3	3	0.3435	0.6987	0.0808	0.1101	0.0917	0.8330	0.0703
3	4	0.2428	0.7409	0.0719	0.1177	–0.0278	0.8826	0.0615
3	5	0.0126	0.8982	0.0423	0.0947	–0.2022	1.0126	0.0341
3	6	0.0143	0.8737	0.0467	0.0769	–0.1552	0.9672	0.0398
3	7	0.0288	0.8173	0.0580	0.0496	–0.0720	0.8776	0.0537
3	8	–0.1155	0.8743	0.0468	–0.0269	–0.0176	0.8410	0.0491
3	9	–0.5532	1.1078	0.0039	–0.1509	–0.1225	0.0925	0.0143
3	10	–1.0691	1.3805	–0.0473	–0.2735	–0.3213	1.0734	–0.0357
3	11	–1.7091	1.6432	–0.1002	–0.4129	–0.6097	1.1977	–0.0877
3	12	–2.3453	1.7034	–0.1142	–0.4931	–1.0408	1.1712	–0.0999
3	13	–1.1494	1.7589	–0.1254	–0.5385	0.2800	1.1645	–0.1049
h_c		–2.4925	1.9526	–0.1032	–1.2689	0.8541	0.5506	–0.0563

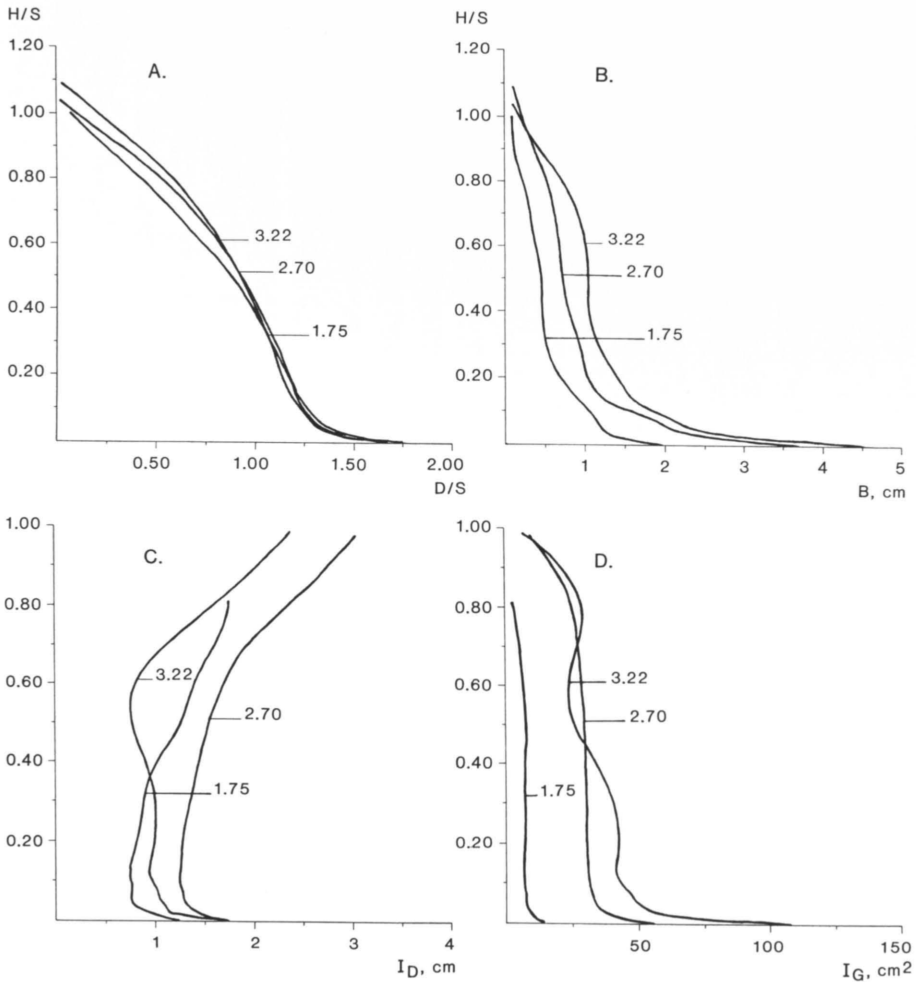

Fig. 3.3. Effect of tree size (s = 1.75, 2.70, 3.22) on the expected value of the stem curve set. A = stem curve with bark, B = bark curve, C = diameter increment curve and D = basal area increment (I_G) curve. Horizontal lines indicate crown height. Heights are divided by the arithmetic tree size S = exp(s), as are also the diameters in subfigure A. The model with average size (3.1.3) is used.

model (3.1.3). The average size has been assumed to be equal to the tree size. The sizes 1.75 and 3.22 correspond to the minimum and maximum values in the primary data, and the size 2.70 is near the mean value. Trees of average size are less thick than small and large trees. The crown height of small trees is relatively lower than that of larger trees. Thickness of the bark increases with increasing tree size. The diameter increment increases slowly below the crown height from low to high, except in the area where butt swellings are formed. Above the crown height, the diameter increment increases rapidly to the top. The absolute diameter increment is greatest in average sized trees. In extremely large trees, the bark and the increment curves fluctuate

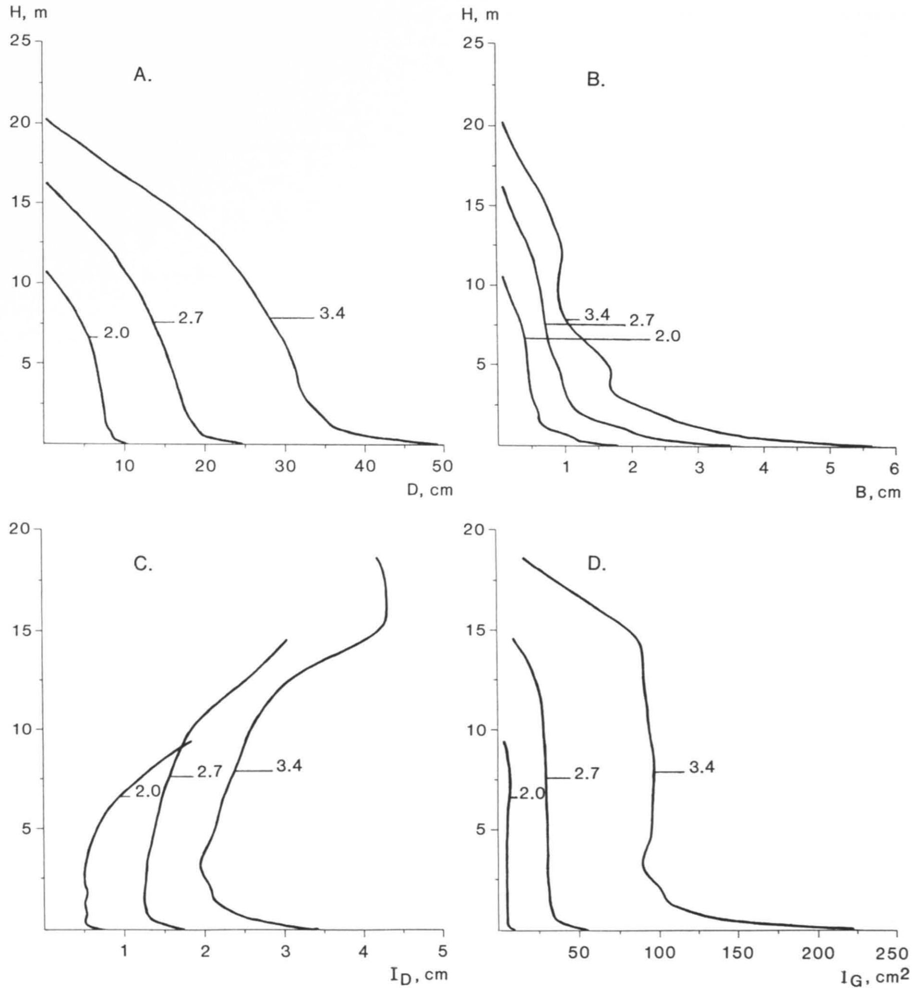

Fig. 3.4. Effect of relative size on the expected value of the stem curve set. The average size is 2.7 and the size is 2.0, 2.7 or 3.4. A = stem curve with bark, B = bark curve, C = diameter increment curve and D = basal area increment (I_G) curve. The horizontal lines indicate crown height. The model with average size (3.1.3) is used.

irregularly. This is easy to understand, because each point of those curves is a difference between predictions of two independently estimated polynomial regression models of the second order.

The prediction of basal area increment is consistent with the pipe model theory (Shinozaki et al. 1964), excluding the butt swelling area and the irregular fluctuations of extreme large trees (Fig 3.3.D). According to the theory, the foliar dry matter of a tree is proportional to the cross-sectional area of the active pipes, the water conduction tissue. Over a longer period the vertical distribution of the formation of new pipes should be balanced with the formation of the new dry matter of the leaves. This theory presumes con-

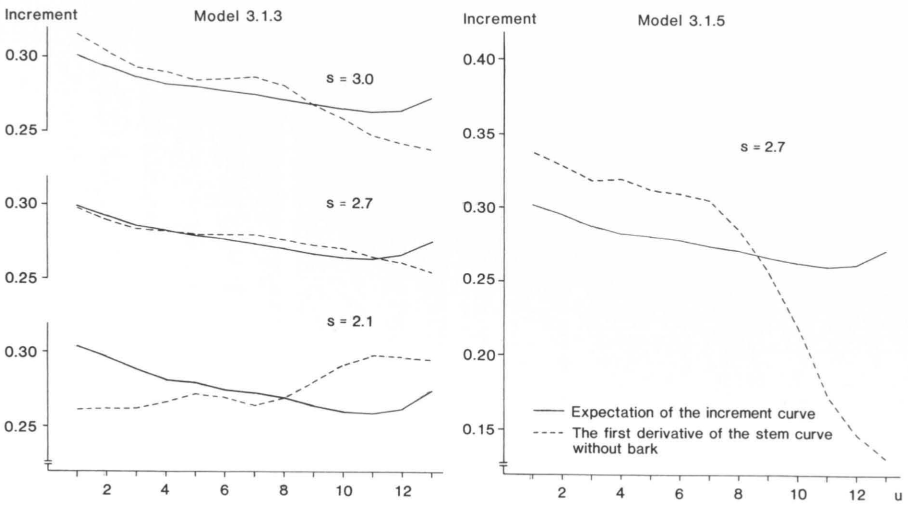

Fig. 3.5. Increment predictions of models 3.1.3 and 3.1.5 at the knot angles for different tree sizes in logarithmic scale determined as 1) the expectation of the increment curve and 2) the first derivative of the model with respect to size. The predictions are scaled so that the sum of squared increments at different knot angles is 1.

stant basal area growth below crown height. Growth in the basal area should increase from top to bottom in proportion to the formation of new leaf biomass.

Fig. 3.4 illustrates trees with different relative sizes in the same stand. Big trees are relatively thicker than small ones. The crown height is nearly constant for all trees; it is lower only for very small trees. The diameter and basal area increments increase as relative size increases.

Because of the structure of the stem curve set model, it is possible to check the consistency of the model with regard to the change in stem form. The change in stem form can be predicted independently in two ways from the fixed part of the stem curve set model as function of tree size: 1) as the expectation of the increment curve (the expectation method), and 2) as the first derivative of the stem curve without bark with respect to the size, multiplied by the size increment (the derivative method). The first derivatives for both models with average size (3.1.3) and without average size (3.1.5) have the same form:

$$\frac{\delta d}{\delta s} = a_{g1} + 2a_{g2}s, \qquad (3.3.2)$$

when it is assumed that $\bar{s} = s$ for the model with average size.

If the stem curve set model is consistent, the predictions made by the expectation and the derivative methods are identical for stem form change. The consistency of the model with average size is sufficient for trees with a size equal to the average size (2.7) in the primary data (Fig. 3.5). For small trees, the derivative method gives a larger height increment in relation to diameter increment than the expectation method does, and for big trees the opposite is true.

The model without average size is inconsistent. Compared to the expectation method, the derivative method leads to radical underprediction of height increment. This is caused by the different stem forms of the big and small trees in a stand. The big trees are relatively shorter than the small trees. The relative size of the biggest trees in the data set used for parameter estimation is always big and relative size of the smallest trees is always small. In other words, tree size and the average tree size are highly correlated (R = .79). For this reason the model without average size has a built in property, that small trees are always assumed also to be relatively small and big trees relatively big. The regression line of the model without average size does not follow a particular tree but moves from a relatively small tree to relatively big tree when the tree size is

Table 3.2. Estimated between-stand and within-stand standard deviations and correlations for the model 3.1.3. On the diagonal are the standard deviations*100. Only three knot angles are shown. g is index of the elementary stem curve and u is the knot angle.

g	u	With bark (t) 3°	31°	90°	Without bark (t) 3°	31°	90°	Without bark (t–5) 3°	31°	90°	h_c
		Between stands									
1	3°	3.109									
	31°	–.829	1.349								
	90°	–.943	.645	7.181							
2	3°	.755	–.705	–.764	4.205						
	31°	–.623	–.800	.421	–.409	1.655					
	90°	–.943	.645	1.000	–.764	.421	7.181				
3	3°	.695	–.628	–.707	.812	–.229	–.707	4.097			
	31°	–.430	.548	.336	–.565	.687	.336	–.048	3.869		
	90°	–.874	.644	.916	–.858	.457	.916	–.624	.606	8.635	
	h_c	–.694	.610	.772	–.869	.406	.772	–.666	.566	.831	27.54
		Within stands									
1	3°	3.249									
	31°	–.534	2.583								
	90°	–.729	.332	5.615							
2	3°	.816	–.385	–.596	3.902						
	31°	–.474	–.957	.271	–.313	2.750					
	90°	–.729	–.332	1.000	–.596	.271	5.615				
3	3°	.732	–.402	–.539	.896	–.339	–.539	4.924			
	31°	–.355	.702	.212	–.216	.772	.212	–.066	3.108		
	90°	–.697	.308	.951	–.552	.248	.951	.476	.250	2.574	
	h_c	–.365	.220	.463	–.294	.196	.463	–.185	.290	.933	16.47

increasing. This does not happen with the expectation method, because the relative tree size is the same at the beginning and end of the increment period.

The small above-mentioned inconsistency of the model with average size for small and big trees is also associated with the relative size of a tree in a stand. The model with average size assumes that the effect of the relative size is independent of the average size of the stand. The inconsistency indicates that relative size has a different effect in stand, where the average size is small than in the stand where the average size is big.

3.4 Random part of the stem curve set model

Because of the stability of the tree stem form, diameters near each other involve mainly parallel information about tree form. When the distance between two diameters increases, the parallelism decreses. The fixed part of the model eliminates the tree size dependent variation of the diameters. The random stand effects (v) as well as the random tree effects (e) are still mutually highly correlated, but after adjusting for the size effect, the highest absolute correlations occur for diameters at opposite parts of the stem (Table 3.2).

In the application of the stem curve set model, the covariance matrix of stand effects cov(**v**) has to be inverted (equation 3.2.2, matrix **D**). Because of the high dimension of the stem curve set model (40x40) and the strongly correlated elements, the inverse of cov(**v**) is algorithmically not positive definite when subroutines of the IMSL library are used. If the covariance matrix of stand effects is organized suitably, it is possible to decrease dimensions of the matrix using principal components and also interpret the result with regard to the stem form (Lappi 1986) and the stem form changes.

Denote the covariance matrices of the random stand effects at the knot angles by **B** and the covariance matrices of the random tree effects by **W**. They are partitioned into the submatrices according to the elementary stem curves:

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} & \mathbf{B}_{13} \\ \mathbf{B}_{21} & \mathbf{B}_{22} & \mathbf{B}_{23} \\ \mathbf{B}_{31} & \mathbf{B}_{32} & \mathbf{B}_{33} \end{bmatrix} \qquad \mathbf{W} = \begin{bmatrix} \mathbf{W}_{11} & \mathbf{W}_{12} & \mathbf{W}_{13} \\ \mathbf{W}_{21} & \mathbf{W}_{22} & \mathbf{W}_{23} \\ \mathbf{W}_{31} & \mathbf{W}_{32} & \mathbf{W}_{33} \end{bmatrix} \tag{3.4.1}$$

where:

$\mathbf{B}_{11}, \mathbf{W}_{11}$ = covariance matrices of the stem curve with bark in the year t

$\mathbf{B}_{22}, \mathbf{W}_{22}$ = covariance matrices of the stem curve without bark in the year t

$\mathbf{B}_{33}, \mathbf{W}_{33}$ = covariance matrices of the stem curve without bark in the year t–5

$\mathbf{B}_{gg'}, \mathbf{W}_{gg'}$ = covariance matrices between elementary stem curves g and g', g = 1,2,3, g' = 1,2,3, when $g \neq g'$.

The submatrices $\mathbf{B}_{gg}$ and $\mathbf{W}_{gg}$, g = 1, 2, 3, are symmetrical with diagonals var($\mathbf{v}_g$) and var($\mathbf{e}_g$).

The principal components of each submatrix are not correlated and they can be interpreted as in the case of the stem curve model (Lappi 1986). The principal components of the different submatrices are, however, highly mutually correlated (Table 3.3). Since the principal components of each submatrix describe only one particular elementary stem curve, it is difficult to find any interpretation for the bark and the increment curves. To simplify the model for interpretation and to obtain more accurate inversion of the stand effect matrix, owing to lower correlations between the principal components, the stand and tree effects were expressed by the stem curve with bark v_1, the bark curve $v_B = (v_1–v_2)$ and the increment curve $v_I = (v_2–v_3)$ instead of the elementary stem curves. The stem curve set model, where stand effects are expressed using bark and increment curves, is called the difference curve

Table 3.3. Estimated variances*100 of the principal components (k) of the stand effects (on diagonal) and correlations between the principal components of the elementary stem curves (g = 1, 2 and 3) for model 3.1.3.

g	k	Elementary stem curve (g) 1, Principal component (k) 1	1, 2	1, 3	1, 4	2, 1	2, 2	2, 3	2, 4	3, 1	3, 2	3, 3	3, 4	h_c
1	1	1.77												
	2		0.09											
	3			0.04										
	4				0.02									
2	1	.90	–.26	.03	–.25	1.74								
	2	.40	.37	.21	.51		0.20							
	3	–.01	.55	.08	–.53			0.10						
	4	.12	.21	–.82	–.16				0.05					
3	1	.93	–.22	–.06	–.22	.98	.12	.03	–.01	2.61				
	2	–.07	.14	.14	–.03	–.12	.21	.42	–.20		0.84			
	3	–.23	.04	.18	–.71	.06	–.60	.68	–.20			0.16		
	4	–.08	–.43	.82	.17	.02	.07	–.33	–.84				0.06	
h_c		–.81	.27	.04	.31	–.92	.02	–.06	–.03	–.90	.03	–.13	.02	7.58

Table 3.4. Estimates of the eigenvectors and the variances of the principal components for the stem curve with bark, the bark curve and the increment curve for models 3.1.3 and 3.1.5. u = the knot angle, s^2 = the variance estimate of a principal component*100, $\sum$ = cumulative percentages of the total variance absorbed by the principal components.

Between stands, model 3.1.3

u	Stem curve with bark 1	2	3	4	Bark curve 1	2	3	4	Increment curve 1	2	3	4
1	0.200	0.543	–0.235	–0.632	0.578	0.239	0.551	0.148	–0.258	0.400	0.333	0.326
2	0.170	0.354	0.080	0.015	0.501	0.137	0.180	–0.089	–0.234	0.342	0.220	0.213
3	0.189	0.354	0.334	0.452	0.426	0.034	–0.370	–0.140	–0.226	0.262	0.065	0.059
4	0.226	0.132	0.289	0.205	0.357	–0.029	–0.579	–0.363	–0.248	0.232	–0.027	0.038
5	0.208	–0.175	0.196	0.071	0.249	–0.182	–0.195	0.225	–0.238	0.209	–0.025	–0.066
6	0.229	–0.388	0.144	–0.103	0.156	–0.372	–0.268	0.715	–0.252	0.177	–0.097	–0.239
7	0.176	–0.417	–0.086	–0.189	0.108	–0.458	0.062	0.057	–0.264	0.097	–0.101	–0.149
8	0.087	–0.325	–0.290	–0.052	0.075	–0.492	0.227	–0.287	–0.281	0.067	–0.106	–0.205
9	–0.074	–0.110	–0.376	0.109	0.044	–0.397	0.172	–0.168	–0.301	–0.047	–0.204	–0.408
10	–0.226	0.064	–0.382	0.265	0.021	–0.306	0.169	–0.212	–0.309	–0.124	–0.353	–0.216
11	–0.383	0.126	–0.211	0.259	–0.008	–0.195	0.057	–0.191	–0.316	–0.297	–0.352	0.471
12	–0.474	0.052	0.098	–0.001	–0.013	–0.101	0.009	–0.078	–0.329	–0.415	–0.022	0.449
13	–0.531	–0.122	0.509	–0.388					–0.321	–0.484	0.717	–0.284
s^2	1.774	0.090	0.041	0.024	0.572	0.053	0.009	0.003	0.968	0.079	0.013	0.004
$\sum$	0.916	0.963	0.984	0.997	0.895	0.979	0.993	0.998	0.907	0.981	0.993	0.997

Between stands, model 3.1.5

u	Stem curve with bark 1	2	3	4	Bark curve 1	2	3	4	Increment curve 1	2	3	4
1	0.226	0.521	–0.253	–0.630	0.579	0.230	0.542	0.187	–0.258	0.400	0.331	0.324
2	0.197	0.361	0.073	0.015	0.501	0.126	0.197	–0.101	–0.235	0.342	0.218	0.214
3	0.209	0.236	0.327	0.453	0.427	0.040	–0.366	–0.116	–0.227	0.262	0.064	0.062
4	0.227	0.112	0.275	0.211	0.357	–0.026	–0.557	–0.400	–0.245	0.233	–0.023	0.040
5	0.196	–0.189	0.195	0.075	0.248	–0.183	–0.191	0.116	–0.234	0.210	–0.022	–0.067
6	0.208	–0.389	0.124	–0.103	0.156	–0.350	–0.260	0.755	–0.250	0.178	–0.097	–0.240
7	0.156	–0.423	–0.071	–0.193	0.106	–0.458	0.047	0.101	–0.263	0.097	–0.100	–0.150
8	0.067	–0.336	–0.278	–0.056	0.072	–0.497	0.221	–0.196	–0.281	0.067	–0.106	–0.206
9	–0.080	–0.110	–0.369	0.103	0.041	–0.397	0.177	–0.154	–0.303	–0.047	–0.205	–0.408
10	–0.226	0.063	–0.382	0.262	0.019	–0.305	0.181	–0.258	–0.311	–0.123	–0.354	–0.217
11	–0.383	0.130	–0.217	0.260	–0.009	–0.205	0.068	–0.228	–0.318	–0.297	–0.352	0.470
12	–0.472	0.075	0.098	–0.001	–0.014	–0.100	0.008	–0.085	–0.329	–0.414	–0.021	0.450
13	–0.527	–0.068	0.522	–0.388					–0.321	–0.483	0.718	–0.283
s^2	3.182	0.100	0.041	0.024	0.572	0.055	0.009	0.003	1.076	0.079	0.013	0.004
$\sum$	0.949	0.978	0.991	0.998	0.891	0.978	0.992	0.998	0.916	0.983	0.994	0.997

Continued on page 24.

set model. Denote the covariance matrices of random stand and tree effects for the difference curve set model by

$$\dot{\mathbf{B}} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{1B} & \mathbf{B}_{1I} \\ \mathbf{B}_{B1} & \mathbf{B}_{BB} & \mathbf{B}_{BI} \\ \mathbf{B}_{I1} & \mathbf{B}_{IB} & \mathbf{B}_{II} \end{bmatrix} \qquad \dot{\mathbf{W}} = \begin{bmatrix} \mathbf{W}_{11} & \mathbf{W}_{1B} & \mathbf{W}_{1I} \\ \mathbf{W}_{B1} & \mathbf{W}_{BB} & \mathbf{W}_{BI} \\ \mathbf{W}_{I1} & \mathbf{W}_{IB} & \mathbf{W}_{II} \end{bmatrix} \tag{3.4.2}$$

where

$\mathbf{B}_{11}, \mathbf{W}_{11}$ = covariance matrices of the stem curve with bark

$\mathbf{B}_{BB}, \mathbf{W}_{BB}$ = covariance matrices of the bark curve

$\mathbf{B}_{II}, \mathbf{W}_{II}$ = covariance matrices of the increment curve

$\mathbf{B}_{gg'}, \mathbf{W}_{gg'}$ = covariance matrices between elementary stem curves g and g', g = 1,B,I, g' = 1,B,I, when $g \neq g'$.

The submatrices $\mathbf{B}_{gg}$ and $\mathbf{W}_{gg}$, g = 1, B, I, are symmetrical with diagonals var($\mathbf{v}_g$) and var($\mathbf{e}_g$).

Table 3.4 continued.

Within stands, models 3.1.3 and 3.1.5

u	Stem curve with bark 1	2	3	4	Bark curve 1	2	3	4	Increment curve 1	2	3	4
1	0.294	0.691	–0.427	–0.189	0.602	0.649	–0.166	–0.121	–0.384	–0.300	0.715	0.054
2	0.275	0.323	0.145	0.217	0.518	0.111	0.270	0.175	–0.339	–0.234	0.269	0.048
3	0.283	–0.023	0.546	0.437	0.412	–0.431	0.539	0.206	–0.298	–0.154	–0.249	0.048
4	0.243	–0.133	0.225	0.023	0.313	–0.492	–0.071	–0.487	–0.299	–0.108	–0.437	0.074
5	0.180	–0.190	0.080	–0.389	0.248	–0.256	–0.564	–0.319	–0.298	–0.130	–0.139	0.137
6	0.157	–0.306	0.021	–0.443	0.171	–0.245	–0.343	0.422	–0.299	–0.089	–0.187	0.081
7	0.067	–0.295	–0.195	–0.130	0.090	–0.084	–0.298	0.345	–0.306	–0.061	–0.228	0.092
8	–0.022	–0.267	–0.295	0.084	0.047	–0.060	–0.197	0.344	–0.280	0.027	–0.118	–0.028
9	–0.147	–0.179	–0.334	0.319	0.022	–0.051	–0.140	0.313	–0.262	0.196	–0.076	–0.302
10	–0.252	–0.064	–0.232	0.339	0.015	–0.021	–0.122	0.220	–0.242	0.339	0.021	–0.429
11	–0.366	0.082	0.000	0.135	0.002	–0.002	–0.060	0.104	–0.218	0.450	0.128	–0.300
12	–0.434	0.164	0.183	–0.088	–0.003	0.002	0.024	–0.036	–0.166	0.481	0.135	0.158
13	–0.477	0.205	0.343	–0.335					–0.084	0.451	0.065	0.749
s^2	1.161	0.338	0.151	0.074	0.244	0.074	0.034	0.021	0.393	0.067	0.044	0.031
$\sum$	0.631	0.815	0.897	0.937	0.597	0.778	0.862	0.913	0.648	0.759	0.831	0.882

The estimated eigenvectors and principal component variances of covariance matrices of random stand and tree effects for the stem curve with bark, the bark curve and the increment curve are shown in Table 3.4 for models with average size (3.1.3) and without average size (3.1.5). Model 3.1.5 obviously has higher between-stand variation than model 3.1.3. Most of this variation is in the direction of the first principal component of the stand effects for the stem curve with bark. The average size has only a small effect on the bark and increment curves. The first four principal components explain 99.7 % of the total variation of stand effects at each elementary curve. The estimated eigenvectors and the variances of the principal components of the tree effects are identical for both models. The proportions of variance explained by the first four principal components for tree effects are 93.7 %, 91.3 % and 88.2 % for the stem curve with bark, the bark curve and the increment curve, respectively.

The first principal component of the covariance matrix of the stand effects for the stem curve with bark describes the slenderness of a tree (Fig. 3.6) and explains over 90 % of the between-stand variation in stem form. The crown height of a thick tree is lower than that of a thin tree. The eigenvectors of the tree effects are similar to those of stand effects. These results are well in line with those of Lappi (1986).

The first principal component of the covariance matrix of the stand effects for the bark curve contains the variation in bark thickness at the lower part of the stem, explaining nearly 90 % of the total between-stand variation. This would be typical for Scots pine, because the type of bark differs along the stem (Östlin 1963). Up to a certain stem height the bark is very rough, then changes rather abruptly into the relatively fine "mirror bark". The first principal components explain relatively more of the between-stand variation than the within-stand variation of bark thickness.

The first principal component of the covariance matrix of the random stand effects for the increment curve explains 90 % of the between-stand variation and the corresponding principal component of the tree effects explains 65 % of the within-stand variation. The first eigenvectors of stand and tree effects can both be called the "increment rate" component, because all elements of the eigenvectors have the same sign. Still, the eigenvectors have different shapes (Fig 3.7). The height increment variation within stands is relatively smaller than between stands. This is a consequence of the competition for light in a stand. Trees compete by height growth, which leads to greater relative variation in tree diameters than in tree heights (e.g. Lönnroth 1925). The height increment varies between stands parallel to the diameter increment in relation of the growth potential of the stands. The second eigenvectors of the increment curve for both stand

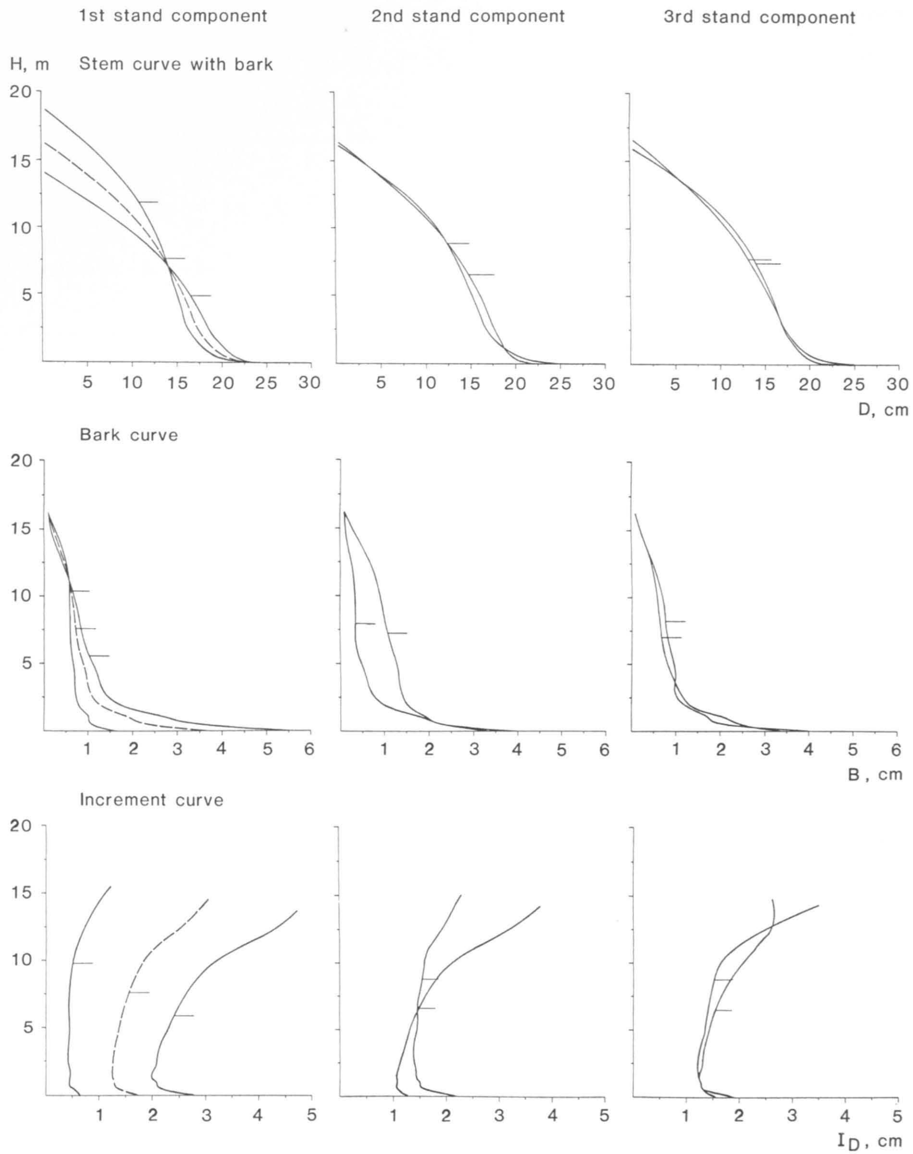

Fig. 3.6. Variation in stem form, bark and increment in the directions of the three first principal components of the covariance matrix for stand effects. The effects of each principal component are described using elementary stem curves conditional to tree size and average tree size 2.7 and ±two times the standard deviation of the principal component. In the subfigures for the first principal components, the expected stem curve is also shown (broken line). The horizontal lines indicate the crown height. The model with average size (3.1.3) is used.

Table 3.5. Estimated variances*100 of the between-stand and within-stand principal components (on diagonal) and correlations between the principal components of the stem curve with bark, bark curve and increment curve (g = 1, B, I) for models 3.1.3 and 3.1.5.

		Between stands												
		Stem curve with bark				Bark curve Principal component, k				Increment curve				h_c
g	k	1	2	3	4	1	2	3	4	1	2	3	4	
		Model 3.1.3												
1	1	1.77												
	2		0.09											
	3			0.04										
	4				0.02									
B	1	−.20	.61	.05	.52	0.57								
	2	.03	.02	−.03	.25		0.05							
	3	−.21	−.05	.41	.13			0.01						
	4	.32	.11	.07	.04				0.0					
I	1	−.24	.27	−.10	.31	.50	.36	−.15	.01	0.97				
	2	−.24	.34	−.23	.27	.32	−.09	−.06	−.13		0.08			
	3	−.12	−.01	−.14	.05	.07	.24	.27	−.02			0.01		
	4	−.12	.04	−.01	−.10	.06	.15	−.14	−.26				0.00	
h_c		−.81	.27	.04	.31	−.57	.09	.15	−.20	.46	.26	.25	.27	7.58
		Model 3.1.5												
1	1	3.18												
	2		0.10											
	3			0.04										
	4				0.02									
B	1	−.10	.64	.03	.52	0.57								
	2	.17	.05	−.03	.23		0.05							
	3	−.30	−.03	.40	.14			0.01						
	4	.49	.09	.11	.03				0.0					
I	1	−.38	.22	−.11	.30	.45	.28	−.06	.18	1.07				
	2	−.19	.36	−.23	.27	.31	−.10	−.04	−.14		0.08			
	3	−.11	−.03	−.13	.04	.06	.22	.28	−.01			0.01		
	4	−.08	.00	−.00	−.11	.07	.15	−.12	−.27				0.00	
h_c		−.86	.26	.03	.27	−.46	.04	.24	−.39	.54	.24	.23	.24	10.18

and tree effects have the same form as the first eigenvectors of the stem curve with bark. The second principal component of the increment curve can be called the "slenderness change" component.

The principal components of each elementary curve are not mutually correlated, but there is a correlation between the elementary curves (Table 3.5). The estimation of the covariance matrix between the principal components of elementary curves of the difference curve set model is presented in Appendix A2. Although the average size clearly reduces the between-stand variation, the correlations between the elementary curves are quite similar both with and without average size as an independent variable. The correlations between the principal components of the elementary curves of tree effects are quite low.

Later, the term stem curve set model will be used for the difference curve set model.

Table 3.5 continued.

		Within stands												
		Stem curve with bark				Bark curve Principal component, k				Increment curve				h_c
g	k	1	2	3	4	1	2	3	4	1	2	3	4	
		Models 3.1.3 and 3.1.5												
1	1	1.16												
	2		.34											
	3			.15										
	4				.07									
B	1	−.11	−.14	−.21	−.21	.24								
	2	.00	.15	−.14	−.13		.07							
	3	.07	−.07	.04	.05			.03						
	4	−.04	−.08	.18	.15				.02					
I	1	.17	−.02	.12	−.18	.05	−.02	.11	−.07	.39				
	2	.11	−.03	−.04	−.13	.05	.13	.01	.02		.07			
	3	.04	−.08	.18	.21	−.07	−.19	−.12	.07			.04		
	4	.12	.03	−.05	−.04	−.01	.05	.06	−.12				.03	
h_c		−.46	.10	.07	.03	−01	−.02	−.01	.02	.16	−.14	.01	−.04	2.14

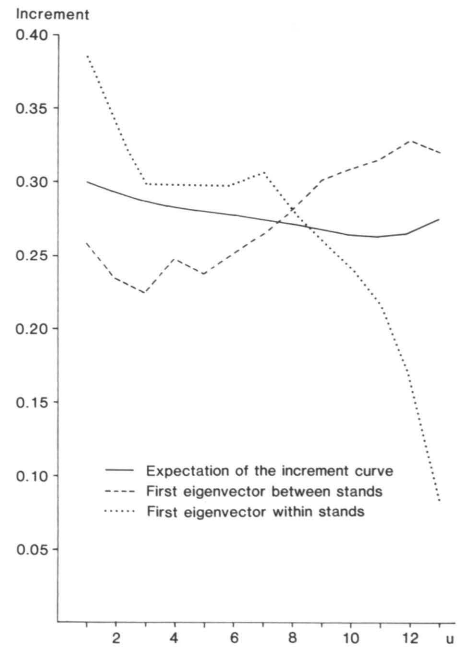

Fig. 3.7. The first eigenvectors of the estimated tree and stand effect covariance matrices and the expectation of increment curve in logarithmic scale for size 2.7. The predicted increment curve is scaled so that the sum of squared increments at different knot angles is 1. The model with average size (3.1.3) is used.

4 Application of the stem curve set model

4.1 Parameter estimation in the application

The fixed part of the stem curve set model (3.1.3) predicts the average logarithmic elementary stem curves as a function of the fixed regressors s and $\bar{s}$. The difference between the average stem curves of a stand and the population is described using random stand effects v. For tally trees, the average stem curve of the stand is used as such. For sample trees, the stand stem curve is corrected to pass through the measured points if it is assumed that there are no measurement errors. The stem curve set can be predicted for all trees in the stand for any combination of diameter and height measurements. Prediction with the stem curve set model can be divided into two phases: parameter estimation for a stand and stem curve set prediction for single trees in the stand.

Fixed sizes of the sample trees and the stand effects are estimated in the application phase. All measurements of one stand are used simultaneously. Estimates of the fixed parameters (a_{g0}, a_{g1}, a_{g2} and a_{g3}) of the elementary stem curves at knot angles and the covariance matrices of stand effects var(**v**) and tree effects var(**e**) are also known. One-dimensional cubic splines can be used for the fixed parameters and two-dimensional cubic splines for the covariances to predict the values at any angle between the knot angles.

Let $d_g(u_{ij})$ denote the j:th measured dimension of elementary stem curve g at angle u for tree i. According to equation (3.1.3), the prediction for measurement ij is

$$\hat{d}_g(u_{ij}) = a_{g0}(u_{ij}) + a_{g1}(u_{ij})s_i + a_{g2}(u_{ij})s_i^2 + a_{g3}(u_{ij})(s_i - \bar{s}) + v_g(u_{ij}) \tag{4.1.1}$$

The unknown fixed parameters are the tree size s_i for every sample tree i and the average size of the stand $\bar{s}$, and the unknown random parameters are the stand effects $v_g(u_{ij})$.

There exists a nonlinear constraint for the unknown size parameters s_i and s_i^2. Because coefficient a_{g2} of s_i^2 is small, the constraint is nearly linear. The linearity has been approximated by the first order Taylor series (Lappi 1986). The variance component model for a auxiliary variable $y_g(u_{ij})$ can be written

$$y_g(u_{ij}) = a_g(u_{ij})s_i + v_g(u_{ij}) + e_g(u_{ij}), \tag{4.1.2}$$

where the fixed sizes s and random stand effects v are unknown and

$$y_g(u_{ij}) = d_g(u_{ij}) - a_{g0}(u_{ij}) + a_{g2}(u_{ij})\dot{s}_i^2 - a_{g3}(u_{ij})(\dot{s}_i - \bar{\dot{s}})$$
$$a_g(u_{ij}) = [2a_{g2}(u_{ij})\dot{s}_i + a_{g1}(u_{ij})]$$

and $\dot{s}$ is the preliminary estimate of tree size.

The unknown parameters are estimated using the mixed linear model technique. Denote the sample tree index by i, the number of sample trees by n, the measurement index by j and the total number of measurements in the stand by M. The matrices of the normal equations (3.2.2) are

- **y** = vector of the measured dimensions, expressed as the auxiliary variable $y_g(u_{ij})$, Mx1
- **X** = matrix of the fixed parameters $a_g(u_{ij})$ interpolated to the measured angles u_{ij}, Mxn
- **a** = vector of the sizes s_i, nx1.
- **Z** = incidence matrix of the random effects $v_g(u)$, Mx40.
- **b** = vector of the random effects $v_g(u)$, 40x1
- **D** = covariance matrix of the stand effects v at the knot angles: cov(**v**), 40x40
- **R** = covariance matrix of the tree effects $e_g(u_{ij})$ interpolated to the measured angles u_{ij}, MxM

(4.1.3)

The models are written in terms of diameters, but the interpolations of parameter $a_g(u_{ij})$ and random stand and tree effects are based on the corresponding ray lengths. Solution of the normal equations gives estimates for the size vector of sample trees $\hat{\mathbf{a}}^T = (\hat{s}_1,\ldots,\hat{s}_n)$ and the stand effects at the knot angles $\hat{\mathbf{b}}^T = (\hat{v}_1(1),\ldots,\hat{v}(h_c))$.

In the application phase the dimension of the random part was reduced by replacing the stand effects with the first p principal components of each stem curve of the covariance matrix **D**. The first four principal components explained over 99 % of the between-stand variance of the stem curve with bark, the bark curve and the increment curve. The stem curve set model can be considered to consist of three different elementary stem curve models and the crown height model. The stand effects **v** of the elementary stem curves for tree i at angle u_{ij} are estimated for each elementary stem curve as follows:

$$v_1(u_{ij}) = \sum_{k=1}^{p} q_{1k}(u_{ij})c_{1k}$$
$$v_2(u_{ij}) = \sum_{k=1}^{p} q_{1k}(u_{ij})c_{1k} - \sum_{k=1}^{p} q_{Bk}(u_{ij})c_{Bk} \tag{4.1.4}$$
$$v_3(u_{ij}) = \sum_{k=1}^{p} q_{1k}(u_{ij})c_{1k} - \sum_{k=1}^{p} q_{Bk}(u_{ij})c_{Bk} - \sum_{k=1}^{p} q_{Ik}(u_{ij})c_{Ik}$$

The value of p may differ for different elementary curves. Here equal values of p have been used for all curves. The prediction equations for the elementary stem curves are:

$$y_1(u_{ij}) = a_1(u_{ij})s_i + \sum_{k=1}^{p} q_{1k}(u_{ij})c_{1k}$$
$$y_2(u_{ij}) = a_2(u_{ij})s_i + \sum_{k=1}^{p} q_{1k}(u_{ij})c_{1k} - \sum_{k=1}^{p} q_{Bk}(u_{ij})c_{Bk} \tag{4.1.5}$$
$$y_3(u_{ij}) = a_3(u_{ij})s_i + \sum_{k=1}^{p} q_{1k}(u_{ij})c_{1k} - \sum_{k=1}^{p} q_{Bk}(u_{ij})c_{Bk} - \sum_{k=1}^{p} q_{Ik}(u_{ij})c_{Ik}$$

Denote stand effects at the knot angles by $\mathbf{v}_g^T = [v_g(1),..,v_g(13)]$, where g = 1,2,3. The vectors of the principal components for the stem curve with bark, the bark curve and the increment curve are $\mathbf{c}_g^T = (c_g(1), ..., c_g(13))$, where g = 1,B,I, and the corresponding matrices for eigenvectors are $\mathbf{Q}_g$. The row k of $\mathbf{Q}_g$ is the k:th eigenvector $\mathbf{q}_{gk} = [q_{gk}(1),...,q_{gk}(13)]$. Let $\dot{\mathbf{c}} = [\mathbf{c}_1, \mathbf{c}_B, \mathbf{c}_I]^T$ and $\dot{\mathbf{v}} = [\mathbf{v}_1, \mathbf{v}_B, \mathbf{v}_I]^T$. According to the definitions of the bark and increment curves, $\mathbf{c}_B = \mathbf{Q}_B(\mathbf{v}_1 - \mathbf{v}_2)$ and $\mathbf{c}_I = \mathbf{Q}_I(\mathbf{v}_2 - \mathbf{v}_3)$. Since $\dot{\mathbf{c}} = \dot{\mathbf{Q}}\dot{\mathbf{v}}$, the matrix $\dot{\mathbf{Q}}$ can be written

$$\dot{\mathbf{Q}} = \begin{bmatrix} \mathbf{Q}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{Q}_B & -\mathbf{Q}_B & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_I & -\mathbf{Q}_I \end{bmatrix}, \tag{4.1.6}$$

where the submatrices $\mathbf{Q}_g$ are orthogonal. Now

$$\dot{\mathbf{c}} = \dot{\mathbf{Q}}\dot{\mathbf{v}} \tag{4.1.7}$$

and

$$\dot{\mathbf{v}} = \dot{\mathbf{Q}}^{-1}\dot{\mathbf{c}}. \tag{4.1.8}$$

The orthogonality of the submatrices can be used to calculate $\dot{\mathbf{Q}}^{-1}$:

$$\dot{\mathbf{Q}}^{-1} = \begin{bmatrix} \mathbf{Q}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{Q}_B & -\mathbf{Q}_B & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_I & -\mathbf{Q}_I \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{Q}_1^T & \mathbf{0} & \mathbf{0} \\ \mathbf{Q}_1^T & -\mathbf{Q}_B^T & \mathbf{0} \\ \mathbf{Q}_1^T & -\mathbf{Q}_B^T & -\mathbf{Q}_I^T \end{bmatrix}. \tag{4.1.9}$$

In the present application of the stem curve set model the matrices of the normal equations for the mixed linear model of the form (3.2.2) are the following:

- **y** = vector of the measured dimension, expressed as the auxiliary variables $y_g(u_{ij})$, Mx1
- **X** = matrix of the fixed parameters $a_g(u_{ij})$ interpolated to the measured angles u_{ij}, Mxn
- **a** = vector of the sizes s_i, nx1
- **Z** = interpolated values of the eigenvectors $\mathbf{q}_{gk}$ at measurement angles u_{ij}, g = 1, B, I, k = 1,...,p, and the crown height, Mx(3p+1) matrix.
- **b** = vector of the principal components c_{gk}, g = 1, B, I, k = 1,...,p, and the crown height, (3p+1)x1
- **D** = covariance matrix of the principal components of the stand effects and the crown height for the elementary stem curves, var(**c**), (3p+1)x(3p+1).
- **R** = covariance matrix of the random tree effects $e_g(u_{ij})$, MxM

(4.1.10)

A more detailed description of the matrices is given in Appendix B.1.

If there are any measurements from some of the elementary stem curves, the stand effect matrix can be partitioned into measured and unmeasured parts. Denote the principal components of the stand effects of the measured elementary stem curves by $\mathbf{c}_m$ and those of the unmeasured elementary stem curves by $\mathbf{c}_u$. Now $\dot{\mathbf{B}}$ is partitioned as follows:

$$\dot{\mathbf{B}} = \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix} \tag{4.1.11}$$

where $\mathbf{B}_{11}$ = var($\mathbf{c}_m$), $\mathbf{B}_{22}$ = var($\mathbf{c}_u$) and $\mathbf{B}_{12}$ = cov($\mathbf{c}_m$, $\mathbf{c}_u$). Let

$$\dot{\mathbf{B}}^{-1} = \begin{bmatrix} \mathbf{B}^{11} & \mathbf{B}^{12} \\ \mathbf{B}^{21} & \mathbf{B}^{22} \end{bmatrix}. \tag{4.1.12}$$

Then the normal equations can be written by partitioning the random part in the measured and the unmeasured elements as follows:

$$\begin{bmatrix} \mathbf{X}^T\mathbf{R}^{-1}\mathbf{X} & \mathbf{X}^T\mathbf{R}^{-1}\mathbf{Z}_m & \mathbf{0} \\ \mathbf{Z}_m^T\mathbf{R}^{-1}\mathbf{X} & \mathbf{Z}_m^T\mathbf{R}^{-1}\mathbf{Z}_m + \mathbf{B}^{11} & \mathbf{B}^{12} \\ \mathbf{0} & \mathbf{B}^{21} & \mathbf{B}^{22} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{a}} \\ \hat{\mathbf{c}}_m \\ \hat{\mathbf{c}}_u \end{bmatrix} = \begin{bmatrix} \mathbf{X}^T\mathbf{R}^{-1}\mathbf{y} \\ \mathbf{Z}_m^T\mathbf{R}^{-1}\mathbf{y} \\ \mathbf{0} \end{bmatrix} \quad (4.1.13)$$

The zero matrices in equation 4.1.13 result from the zero $\mathbf{Z}_u$ matrices. The simultaneous estimation according equation 4.1.13 is simpler to carry out in practical applications than the separate estimation of measured and unmeasured random effects (Lappi 1991), because it is not necessary to partition the random part for each measurement combination of sample trees (Appendix B.2).

4.2 Prediction of the stem curve set and volume

When the parameters $\hat{s}_i$, i = 1,...,n and $\hat{c}_{gk}$, where g = 1,2,3 and k = 1,...,p have been estimated, the stem curve set is predicted in two phases: 1) The values for the auxiliary variable y at the knot angles are predicted and 2) the diameters at knot angles are calculated and diameters between the knot angles are interpolated.

The estimate of y (Equation 4.1.2) consists of three additive components: 1) the conditional population mean, 2) the stand effect expressed by the principal components of the stand effect covariance matrix and 3) the tree effects. The estimated population mean at the angle u for tree i is

$$a_g(u)\hat{s}_i \quad (4.2.1)$$

The estimates of stand effects of elementary stem curves $v_g(u)$ are linear combinations of the principal component estimates $\dot{\mathbf{c}}_g$ and eigenvectors $\dot{\mathbf{q}}_g$:

$$\hat{v}_1(u_{ij}) = \sum_{k=1}^{p} q_{1k}(u_{ij})\hat{c}_{1k}$$
$$\hat{v}_2(u_{ij}) = \sum_{k=1}^{p} q_{1k}(u_{ij})\hat{c}_{1k} - \sum_{k=1}^{p} q_{Bk}(u_{ij})\hat{c}_{Bk} \quad (4.2.2)$$
$$\hat{v}_3(u_{ij}) = \sum_{k=1}^{p} q_{1k}(u_{ij})\hat{c}_{1k} - \sum_{k=1}^{p} q_{Bk}(u_{ij})\hat{c}_{Bk} - \sum_{k=1}^{p} q_{Ik}(u_{ij})\hat{c}_{Ik}$$

The predictor for the auxiliary variable y for tree i at angle u of elementary stem curve g is

$$\hat{y}_g(u_i) = \hat{s}_i a_g(u) + \hat{v}_g(u) + \mathbf{w}_i^T \mathbf{W}_i^{-1} \hat{\mathbf{r}}_i \quad (4.2.3)$$

The last term is the estimated tree effect where $\mathbf{w}_i$ is the covariances of the tree effects between the measured angles and the knot angle u, and $\mathbf{W}_i$ is the tree effect covariance matrix of the measured angles. $\mathbf{w}_i$ and $\mathbf{W}_i$ are interpolated with cubic splines from the tree effect covariance matrix. $\hat{\mathbf{r}}_i$ is the vector of the residual estimates

$$\hat{\mathbf{r}}_i = \mathbf{y}_i - \hat{s}_i \mathbf{a}_i - \mathbf{Z}_i \hat{\mathbf{c}}, \quad (4.2.4)$$

where $\mathbf{y}_i$ is a vector of the measurements of tree i, $\mathbf{a}_i$ is a vector of the values of variable $a_g(u_{ij})$ defined for Equation (4.1.2) corresponding to the measurement angles and $\mathbf{Z}_i$ is the matrix of the interpolated values of the eigenvectors $\mathbf{q}_k$ at measurement angles for tree i.

For each tally tree the size estimator is

$$\hat{s}_0 = a_g(u_0)^{-1}[y_g(u_0) - \hat{v}_g(u)] \quad (4.2.5)$$

and the predictor for the auxiliary variable y at angle u is

$$\hat{y}_g(u_0) = a_g(u)\hat{s}_0 + \hat{v}_g(u) \quad (4.2.6)$$

The unbiased transformation from the logarithmic scale to the arithmetic scale is based on the assumption of normality in the error term (Lappi 1986, p. 26). The continuous stem curve in the arithmetic scale is interpolated from the diameters at knot angles in height coordinates. Diameter predictions are unbiased at all angles but are biased at a given height, e.g. breast height. However, this bias is small and has been omitted here.

The variance of the estimate of the logarithmic volume can be approximated using the close statistical relationship between tree size and volume with bark (Lappi 1986). The approximated variance of the volume predictor is estimated with the first order Taylor series:

$$\mathrm{var}[\ln(V) - \ln(\hat{V})] \approx \mathrm{var}[(V - \hat{V})/E(V)] \approx b^2\,\mathrm{var}(\hat{s}), \quad (4.2.7)$$

where the estimate for coefficient b is from the regression $\hat{V} = aS^b$, where S = exp(s). In the case of volume with bark R^2 is 0.999. Due to the very low correlations between the size and the bark volume or the volume increment (R^2 = 0.226 or R^2 = 0.156), this method is not used for bark

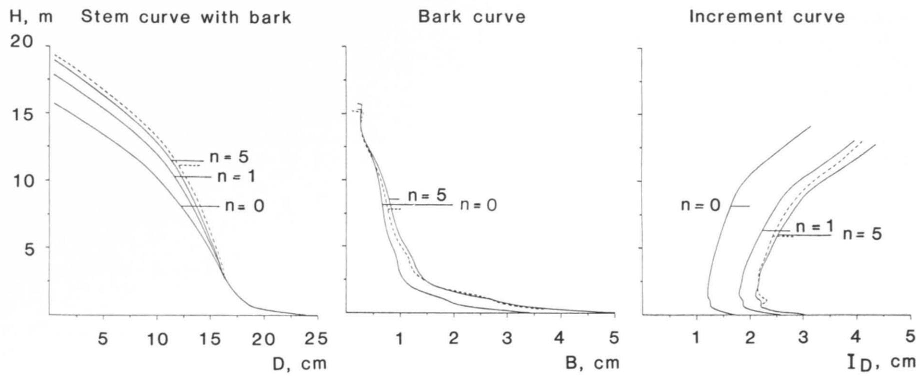

Fig. 4.1. Tally tree predictions as a function of the number of identical sample trees, when tree height (H), double bark thickness ($B_{1.3}$) and diameter increment ($I_{1.3}$) are measured from the sample trees. Diameter at breast height with bark is 17.8 cm for all tally and sample trees. Solid lines with n = 0 are the tally tree predictions, when no sample trees are measured on the plot. Broken lines are the sample tree predictions for trees where H, $B_{1.3}$, and $I_{1.3}$ are population means plus two times their standard deviations, respectively. Solid lines with n > 0 are tally tree predictions with 1 or 5 identical sample trees on the plot. The horizontal lines indicate the crown height. The model with average size (3.1.3) is used.

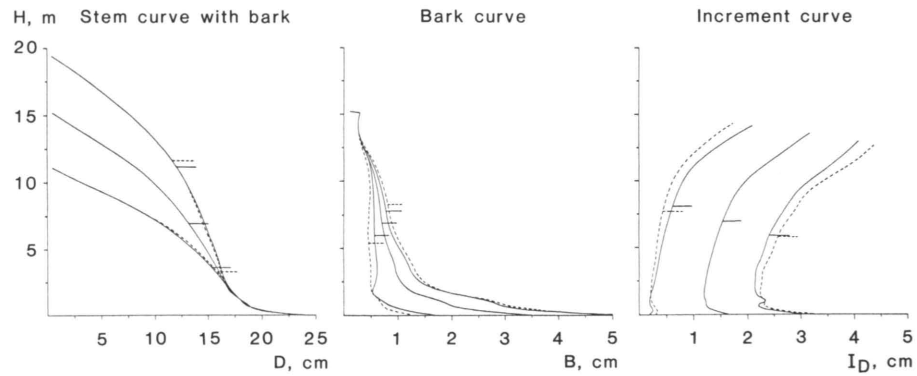

Fig. 4.2. Sample tree predictions when the values of the measured sample tree variables change. Diameter at breast height with bark is 17.8 cm for all trees. The values of the sample tree variables are for stem curve prediction H = (11.0, 15.2 or 19.4 m), $B_{1.3}$ = 1.5 cm and $I_{1.3}$ = 1.3 cm, for bark curve prediction H = 15.2 m, $B_{1.3}$ = (0.5, 1.5 or 2.5 cm) and $I_{1.3}$ = 1.3 cm and for increment curve prediction H = 15.2 m, $B_{1.3}$ = 1.5 cm and $I_{1.3}$ = (0.5, 1.5 or 2.5 cm). The solid lines indicate one sample tree on the plot and the broken lines five identical sample trees on the plot. The horizontal lines indicate the crown height. The model with average size (3.1.3) is used.

volume and volume increment. The method is also unsatisfactory for volumes without bark since size with bark is not very closely related to volume without bark in the years t (R^2 = 0.996) or t–5 (R^2 = 0.985).

The variance of the volume prediction can be estimated using diameter and height predictions and their estimated covariances (Kilkki & Varmola 1981). The volume variance estimation is, however, not included in this study.

Predictions of the stem curve set are illustrated here using a basic tree with variations in the sample tree measurements. The basic tree is determined using the means of some sample tree variables in the primary data. The mean diameter at breast height with bark is 17.8 cm. The means of tree height (H), double bark thickness at breast height ($B_{1.3}$) and diameter increment at breast height ($I_{1.3}$) and their standard deviations in the diameter class 16.8–18.8 cm are as follows: H = 15.2 m (2.1 m), $B_{1.3}$ = 1.5 cm (0.5 cm) and $I_{1.3}$ = 1.3 cm (0.5 cm). In all examinations, $D_{1.3}$ is assumed to be constant, but H has been varied for stem curve with bark, $B_{1.3}$ for bark curve and $I_{1.3}$ for increment curve prediction ± two times the standard deviation.

Stand level calibration of the stem curve set is examined first (Fig. 4.1). The stem curve set of a tally tree predicted without any sample trees on the plot is based only on the a priori information about stem form, bark and increment. The measured sample trees give stand level information which is combined with the a priori information. When the number of measured sample trees increases, the sample tree information becomes more accurate and carries more weight in the prediction. The effect of the first sample tree on the plot is the strongest, and five identical sample trees gives a stem curve set for the stand, which nears the measured points of the sample trees. Stem, bark and increment curves are stable and do not have noticeably more fluctuations than the corresponding curves of the basic tree.

Sample tree prediction is examined with one or five identical sample trees on the plot (Fig. 4.2). Variation in tree height also causes a stable change in stem form for extreme tree heights. When only one sample tree is measured on the plot, the bark and the increment curve predictions for extreme values of bark thickness and diameter increment give curves with a bend at the breast height measured. When the number of sample trees increases, the more accurate stand information diminishes the bend.

5 Prediction results

5.1 Test criteria

Variance components of the diameters predicted by the stem curve set model can be derived analytically, assuming that the model formulation is correct. In the model application the independent and the measured variables are not the same. Tree size is a fixed independent variable in the model. The model is unbiased in relation to size, if the fixed part of the model is assumed to be correct. In the practical applications, the diameters and heights are known instead of the size. As the size is used as a fixed variable, the diameter estimates are biased for the measurements. In addition to the statistical properties of the stem curve set model, there also exist errors caused by the interpolation of the parameter vectors, eigenvectors and covariance matrices of the random effects. Empirical tests are needed to determine whether these sources of error significantly influence the reliability of the stem curve set model.

The prediction error $y_{ki} - \hat{y}_{ki}$ is decomposed into bias (mean), random error in the stand k (b_k) and random error of tree i in stand k (e_{ki}):

$$y_{ki} - \hat{y}_{ki} = \text{mean} + b_k + e_{ki}. \qquad (5.1.1)$$

The studied statistics are bias (mean), standard deviation between stands s_b, standard deviation within stands s_w, total standard deviation s_t and root mean square error RMSE, (standard error) and also standard error between stands $RMSE_b$. Estimation of the reliability statistics has been carried out using the formulas of Searle (1971, p. 474). The variables studied are the diameters of the elementary stem curves, the bark curve and the increment curve as well as stem, bark and increment volumes (V_1, V_2, V_3, B_V, I_V). The stem volumes are volumes above stump height. The stump height has been determined by the stump height model (Laasasenaho 1982, equation 81.1), which has the diameter at breast height and tree height as independent variables. The minimum stump height was assumed to be 10 cm. The tests have been done utilizing the stand structure of the data. The first four principal components of the stem curve with bark, the bark curve and the increment curve were used to estimate the stand effects (p = 4 in equation 4.1.4).

The applicability of the model depends on its reliability in the arithmetic scale. The natural criterion for testing a logarithmic model in arithmetic scale is the relative error

$$e_r = (y - \hat{y}) / E(y) \qquad (5.1.2)$$

which is the first degree Taylor approximation for the logarithmic error $\ln(y) - \ln(\hat{y})$. E(y) is the expected value of y conditional on s and $\bar{s}$. The relative error as defined in equation 5.1.2 measures the reliability of the prediction in relation to the independent variables of the model. If the interest is in the reliability in relation to the measured dimensions, the denominator is the prediction of y ($\hat{y}$).

The reliability of the diameter prediction is studied on the logarithmic and on the arithmetic scale and the reliability of the bark thickness and the diameter increment predictions only on the arithmetic scale. The denominator $\hat{D}$ is used for the relative error. The reliability of the stem, bark and increment volume predictions are studied empirically using absolute and relative errors. In Table 5.1 some statistics for the relative errors of the volume predictions are presented using the real volume V, the predicted volume $\hat{V}$ and the expectation of the volume E(V) as denominator. The values of E(V) are predicted by the following allometric equations:

$$E(V_1) = 0.07402 * S^{2.969}, \quad R^2 = .999, \quad RMSE\% = 3.2$$
$$E(V_2) = 0.05322 * S^{3.037}, \quad R^2 = .996, \quad RMSE\% = 5.7$$
$$E(V_3) = 0.02812 * S^{3.184}, \quad R^2 = .985, \quad RMSE\% = 11.7$$
$$E(B_V) = E(V_1) - E(V_2)$$
$$E(I_V) = E(V_2) - E(V_3) \qquad (5.1.3)$$

The predictions of the bark and increment volumes are clearly biased if V is used as denominator. V can only be used when size and volume are closely related. The difference of the estimates of the reliability statistics are small between $\hat{V}$ and E(V) as denominators. The use of E(V) gives slightly smaller estimates for the variance components. In studies of reliability, E(V) has been used as the denominator.

To obtain complete test material in relation to the measurements studied, 24 trees higher than 7 m without D_6 measurement were rejected from the primary set of data. The final test material for sample trees consisted of 682 trees. To the tally tree test material only plots consisting of at least six trees with complete tree information were accepted, altogether 68 stands and 467 trees. The average stem, bark and increment volumes are near the same for the primary data set and the sample tree data subset; but for the tally tree data subset, the averages are higher (Table 5.2).

In tables of model reliability the sample tree measurements are denoted as follows: diameter at breast height with bark = $D_{1.3}$, diameter at six

Table 5.1. Some statistics for the reliability of volume predictions when $D_{1.3}$ and H are known. The real volume V, the predicted volume $\hat{V}$ and the expectation of the volume E(V) have been used as the denominator for the relative errors. Mean of the volume variable is $\bar{x}$ in dm^3, and the reliability statistics (mean, s_b and s_w) are given as percentages. Model 3.1.3 has been used.

Stem		$(V_g - \hat{V}_g) / V_g$			$(V_g - \hat{V}_g) / \hat{V}_g$			$(V_g - \hat{V}_g) / E(V_g)$		
curve	$\bar{x}$	mean	s_b	s_w	mean	s_b	s_w	mean	s_b	s_w
1	235.9	0.23	2.75	5.58	0.62	2.76	5.55	0.31	2.68	5.42
2	206.9	0.11	4.25	6.38	0.68	4.15	6.28	0.37	4.02	6.16
3	167.2	–0.61	9.82	8.03	0.85	9.07	7.59	0.49	8.91	7.46
B	29.1	–4.34	18.34	13.18	0.28	18.29	11.57	–0.01	17.01	10.77
I	40.6	–12.87	37.40	25.79	1.33	30.92	19.27	–0.03	27.87	18.50

Table 5.2. Average stem, bark and increment volumes of the primary data set and the test data subsets for sample trees and tally trees.

Material	n	$\bar{V}_1$	$\bar{V}_2$	$\bar{V}_3$	$\bar{B}_V$	$\bar{I}_V$
Primary data set	706	237.7	208.5	167.9	29.2	40.6
Sample tree data subset	682	235.9	206.9	166.2	29.1	40.6
Tally tree data subset	467	267.2	234.6	189.6	32.7	45.0

Table 5.3. Reliability of the diameter, bark thickness and diameter increment predictions for sample trees. The measured dimensions are $D_{1.3}$ and H. The logarithmic statistics are multiplied by 100 and the arithmetic statistics are in cm, except the height (u = 13) and the crown height (h_c), which are in m. RMSE% is the relative standard error given as percentages. Model 3.1.3 has been used.

	Stem curve with bark								
	Logarithmic scale				Arithmetic scale				
u	mean	s_b	s_w	RMSE	mean	s_b	s_w	RMSE	RMSE%
1	0.45	2.52	5.81	6.35	0.14	0.60	1.49	1.61	7.01
2	0.40	1.59	3.73	4.07	0.08	0.29	0.78	0.84	3.96
3	0.37	1.43	3.26	3.58	0.07	0.25	0.61	0.66	3.43
4	0.38	1.07	2.10	2.39	0.06	0.16	0.34	0.38	2.09
5	0.31	0.56	1.76	1.87	0.05	0.13	0.32	0.35	1.99
6	0.29	1.02	2.35	2.58	0.04	0.22	0.40	0.46	2.75
7	0.28	1.41	2.59	2.96	0.04	0.26	0.42	0.50	3.14
8	0.26	1.49	2.92	3.28	0.04	0.26	0.43	0.50	3.47
9	0.22	1.55	3.12	3.49	0.04	0.23	0.39	0.46	3.60
10	0.14	1.76	2.97	3.45	0.02	0.21	0.32	0.38	3.51
11	0.07	1.75	2.42	2.98	0.01	0.15	0.19	0.24	3.08
12	0.03	1.09	1.54	1.89	0.00	0.05	0.07	0.08	1.97
13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
h_c	–0.07	17.50	14.72	22.82	–0.10	1.23	0.91	1.53	20.49

	Bark curve, arithmetic scale					Increment curve, arithmetic scale				
u	mean	s_b	s_w	RMSE	RMSE%	mean	s_b	s_w	RMSE	RMSE%
1	0.01	0.96	0.70	1.19	37.55	0.00	0.59	0.60	0.84	46.47
2	0.02	0.81	0.52	0.96	34.80	0.00	0.51	0.45	0.68	43.94
3	0.02	0.63	0.48	0.79	35.12	0.00	0.44	0.39	0.59	43.24
4	0.01	0.50	0.37	0.62	31.07	0.00	0.46	0.39	0.60	45.07
5	0.00	0.32	0.30	0.44	28.61	0.00	0.42	0.35	0.55	42.14
6	0.00	0.24	0.26	0.35	29.88	0.00	0.44	0.35	0.56	42.82
7	0.00	0.22	0.18	0.29	29.22	0.00	0.45	0.37	0.59	45.83
8	0.00	0.23	0.16	0.27	31.65	0.00	0.49	0.37	0.61	40.79
9	0.00	0.20	0.14	0.24	31.82	0.00	0.58	0.41	0.70	41.01
10	0.00	0.18	0.14	0.23	31.46	0.00	0.68	0.45	0.81	40.25
11	0.00	0.15	0.12	0.19	34.03	–0.02	0.84	0.50	0.97	30.56
12	0.00	0.10	0.10	0.14	44.34	–0.04	1.02	0.48	1.12	37.42
13						–0.02	0.50	0.43	0.66	44.36

meter height with bark = D_6, tree height = H, crown height = H_c, double bark thickness at breast height = $B_{1.3}$, diameter increment at breast height without bark in the last five years = $I_{1.3}$ and height increment in the last five years = I_H.

5.2 Sample trees

The reliability of the predictions of diameter with bark, bark thickness and diameter increment for sample trees was studied as a function of the knot angles (Tables 5.3 and 5.4). Two different measurement combinations of the sample trees were used: 1) diameter at breast height with bark ($D_{1.3}$) and tree height (H) or 2) $D_{1.3}$, double bark thickness at breast height ($B_{1.3}$), diameter increment at breast height ($I_{1.3}$) and H. The relative standard errors are also given in the arithmetic scale.

Predictions of the bark thickness and the diameter increment at knot angles include the diameter difference at fixed height (E_2 in Fig. 5.1) and a component caused by the difference between heights at the knot angle (E_1 or E'_1). The real difference between the elementary stem curves ($E = E_1 + E_2$ or $E = E'_1 + E_2$) depends on the knot angle. In studies of the reliability of the bark thickness and the diameter increment at knot angles, the elementary stem curves were assumed to be linear and parallel in the neighborhood of the knot angles. The difference between the di-

Table 5.4. Reliability of the diameter, bark thickness and diameter increment predictions for sample trees. The measured dimensions are $D_{1.3}$, $B_{1.3}$, $I_{1.3}$ and H. The logarithmic statistics are multiplied by 100 and the arithmetic statistics are in cm, except the height (u = 13) and the crown height (h_c), which are in m. RMSE% is the relative standard error as percentages. Model 3.1.3 has been used.

	Stem curve with bark								
	Logarithmic scale				Arithmetic scale				
u	mean	s_b	s_w	RMSE	mean	s_b	s_w	RMSE	RMSE%
1	0.40	2.39	5.73	6.22	0.13	0.58	1.47	1.59	6.90
2	0.36	1.36	3.66	3.92	0.08	0.26	0.77	0.82	3.87
3	0.31	1.16	3.22	3.43	0.06	0.21	0.60	0.64	3.29
4	0.31	0.82	2.07	2.25	0.05	0.12	0.34	0.36	1.96
5	0.34	0.43	1.73	1.82	0.06	0.08	0.32	0.33	1.88
6	0.37	0.44	2.32	2.39	0.06	0.12	0.39	0.41	2.46
7	0.36	0.92	2.54	2.72	0.05	0.17	0.41	0.45	2.87
8	0.37	1.31	2.85	3.16	0.06	0.23	0.42	0.48	3.33
9	0.33	1.53	3.06	3.44	0.05	0.23	0.39	0.45	3.56
10	0.25	1.70	2.94	3.40	0.04	0.21	0.31	0.38	3.54
11	0.15	1.61	2.41	2.90	0.02	0.13	0.19	0.23	2.98
12	0.08	0.97	1.54	1.82	0.01	0.05	0.07	0.08	1.85
13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
h_c	0.36	11.81	14.24	18.47	–0.04	0.82	0.86	1.19	16.24

	Bark curve, arithmetic scale					Increment curve, arithmetic scale				
u	mean	s_b	s_w	RMSE	RMSE%	mean	s_b	s_w	RMSE	RMSE%
1	0.06	0.34	0.63	0.72	22.80	0.02	0.14	0.42	0.45	25.01
2	0.07	0.28	0.45	0.53	19.15	0.02	0.08	0.27	0.28	18.09
3	0.06	0.20	0.40	0.45	19.93	0.02	0.07	0.22	0.23	17.00
4	0.06	0.14	0.25	0.29	14.56	0.02	0.08	0.20	0.21	15.80
5	0.04	0.06	0.22	0.23	14.92	0.02	0.01	0.17	0.17	13.10
6	0.02	0.13	0.23	0.26	22.01	0.02	0.05	0.17	0.18	13.79
7	0.01	0.17	0.18	0.25	25.45	0.01	0.06	0.21	0.22	17.24
8	0.01	0.19	0.16	0.24	27.72	0.02	0.10	0.22	0.24	15.94
9	0.01	0.17	0.14	0.22	28.69	0.02	0.18	0.29	0.34	19.78
10	0.00	0.15	0.14	0.21	28.48	0.01	0.25	0.34	0.43	21.26
11	0.00	0.13	0.12	0.18	31.90	0.00	0.13	0.12	0.18	22.29
12	0.00	0.09	0.10	0.13	41.47	–0.02	0.53	0.42	0.67	22.34
13						–0.02	0.30	0.23	0.37	24.87

ameters at the same height is $E = E_1 + E_2$, where E_2 is the difference between the diameters at the same angle and E_1 is the component caused by the height difference. Now $E_1 = E_2\tan(u)/\tan(w)$ and $E = E_2(1+\tan(u)/\tan(w))$. Denote $f^{-1} = (1+\tan(u)/\tan(w))$. Now $E = E_2 f^{-1}$. A satisfactory approximation for angle w can be derived from the stem curve with bark as function of angle u since the stem form variation in the direction of tree size is small. The function $f^{-1}(u)$ estimated by Lappi (1986) was used here.

When only $D_{1.3}$ and H are measured, the stand-

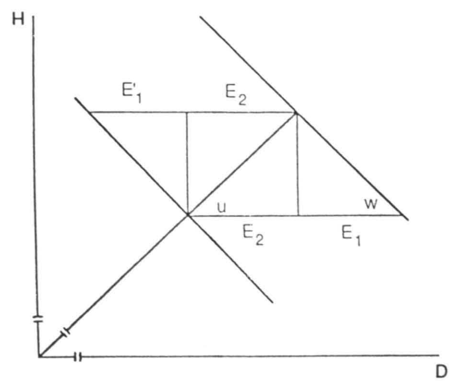

Fig. 5.1. Difference in diameter between two elementary stem curves at a given height. Explanation in the text.

ard error of the prediction of diameter with bark is largest at the stem base. The within-stand standard deviation is larger than the between-stand standard deviation at each angle. The measurements of $B_{1.3}$ and $I_{1.3}$ reduce slightly the standard error of the predicted diameters with bark. The reason for this is that stem form correlates weakly with bark thickness and the increment rate (Table 3.5).

In the prediction of bark thickness, when only $D_{1.3}$ and H are measured, the absolute standard error is largest at the stem base and decreases toward the top. The relative error is smallest in the middle of the stem and increases toward the base and the top. The between-stand and within-stand standard deviations are quite equal. The measurement of $B_{1.3}$ clearly reduces the standard deviation between stands at stem base and middle, but the reducing effect diminishes near the top. The $B_{1.3}$ measurement reduces the within-stand standard deviation only near breast height.

If only $D_{1.3}$ and H are measured, the relative standard error of the predicted diameter increment varies from 30 % to 47 % as a function of the knot angle. The proportion of between-stand variation increases from the stem base to the top. The small within-stand variation in height increment was also clear in the results of the principal component analysis of the increment curve (Fig. 3.7). The measurement of $I_{1.3}$ reduces the standard error to half. The between-stand standard deviation clearly decreases more than the within-stand standard deviation does. Measurement of $I_{1.3}$ reduces the relative standard error of the height increment from 44 % to 25 %.

The random variation between stands is clearly larger in the model without average size (Equation 3.1.5) than in the model with average size (Equation 3.1.3). If $D_{1.3}$ and H are measured, the standard errors of the predictions for the diameters with bark are equally large in both models. This can be reasoned on the basis of the covariance structure of the random stand and tree effects of the stem curve with bark. The higher between-stand variation of the model without average size is mainly in the direction of the first principal components of the stem curve with bark. The measurements of $D_{1.3}$ and H fix the first principal components of the stem curve with bark very effectively and cover the explanatory effect of the average size.

If only $D_{1.3}$ and H are measured, the standard errors of bark thickness and diameter increment predictions of both models are equal in size. If $B_{1.3}$ and $I_{1.3}$ are also measured, the prediction of the model 3.1.5 has larger standard error for the bark thickness at the upper part of the stem than does the corresponding prediction of model 3.1.3. For increment predictions, no difference was found between the models.

When at least $D_{1.3}$ and H are measured, the standard errors of the predictions for stem, bark and increment volume are similar for models 3.1.3 and 3.1.5. Reliability of the volume predictions for sample trees is studied here using model 3.1.3. The residuals are examined in Fig. 5.2 on the arithmetic scale with respect to the independent variable size S = exp(s), the measured diameter at breast height $D_{1.3}$ and the predictions for stem volume with bark $\hat{V}_1$, bark volume $\hat{B}_V$, and volume increment $\hat{I}_V$. The bias with respect to the measured dimension ($D_{1.3}$) can be find as a downwards concave delineator of the residual means of the predictions for volume with bark. The bark volume and the volume increment predictions seem to be unbiased with respect to S and $D_{1.3}$ because they are the differences between two volume predictions with the same source of bias. All the predictions are unbiased with respect to the predicted volume, and the standard errors seem to be proportional to the predictions.

All additional measurements from the stem curve with bark besides $D_{1.3}$ and H reduce the standard error of the prediction of stem volume with bark (Tables 5.5 and 5.6). The crown height measurement has no or only a small effect on the standard error of the prediction of stem volume with bark, when at least $D_{1.3}$ and H are measured. This result is consistent with those of Lappi (1986) and Korhonen (1991), although the standard errors here are smaller because the data are more homogeneous. The $B_{1.3}$ and $I_{1.3}$ measurements does not affect the reliability of the prediction of stem volume with bark.

With all measurement combinations from the stem curve with bark, the standard error of the bark volume prediction is about 20 %. Measurement of the double bark thickness at breast height reduces the standard error to 14 %.

The standard error of the volume increment prediction is about 33 % when the measurements are made only from the stem curve with bark and is independent of the measurement combination. Measurement of the crown height reduces the standard error between stands from 28 % to 23 % but has only a small effect on the standard error within stands. The measurement of $I_{1.3}$ reduces the total standard error from 33 % to 13 %. When $I_{1.3}$ is measured, the measurement of the

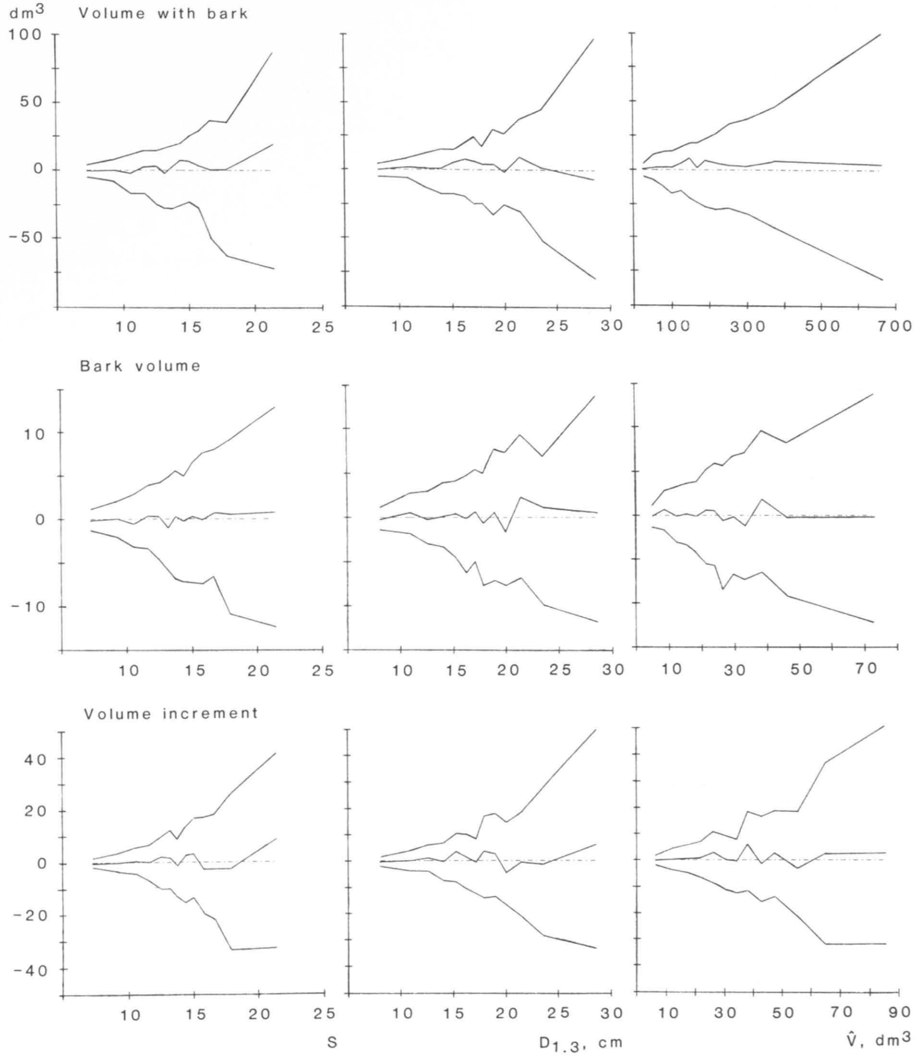

Fig. 5.2. Mean and mean ± standard deviation of $V - \hat{V}$ (dm³) within groups of 50 trees (82 in the last group) with respect to arithmetic size S = exp(s), diameter at breast height $D_{1.3}$ and predicted volume for volume with bark, bark volume and volume increment $\hat{V}$. The measured dimension is $D_{1.3}$ and the model with average size (3.1.3) is used.

Table 5.5. Reliability of the stem, bark and increment volume predictions with different measurement combinations and as a function of the logarithmic tree size s. All measurements are made from the stem curve with bark. The relative statistics are given as percentages and absolute statistics as dm³. Model 3.1.3 has been used. g is the index of the elementary stem curve.

Measure-ments	g	Absolute errors				Relative errors			
		mean	s_b	s_w	RMSE	mean	s_b	s_w	RMSE
s	1	0.3	4.5	9.3	10.3	0.57	1.62	3.06	3.51
	2	0.3	7.1	10.3	12.5	0.64	3.61	4.20	5.56
	3	–0.5	16.6	13.4	21.7	0.96	8.60	6.47	10.74
	B	–0.0	5.0	4.2	6.5	0.16	16.50	10.44	19.47
	I	0.9	17.2	11.1	20.5	–0.01	28.32	17.30	33.10
$D_{1.3}$	1	3.7	27.5	27.3	38.7	0.57	10.12	8.76	13.38
	2	3.7	26.1	26.0	36.8	0.70	10.95	9.67	14.61
	3	2.7	23.9	22.8	33.0	0.74	15.04	10.48	18.30
	B	0.1	5.4	4.4	7.0	–0.30	18.34	11.33	21.50
	I	1.0	16.8	11.9	20.5	0.58	27.87	20.07	34.27
$D_{1.3}$,H	1	1.2	13.2	15.2	20.1	0.31	2.68	5.42	6.06
	2	1.1	14.4	14.9	20.7	0.37	4.02	6.16	7.38
	3	0.6	15.9	15.8	22.3	0.49	8.91	7.46	11.62
	B	0.1	4.9	4.2	6.4	–0.01	17.01	10.77	20.07
	I	0.5	17.0	11.4	20.4	–0.03	27.87	18.50	33.36
$D_{1.3}$,D_6,H	1	1.6	8.1	10.1	13.0	0.34	1.90	3.12	3.65
	2	1.4	9.0	10.0	13.5	0.25	2.89	3.85	4.82
	3	1.2	14.6	13.1	19.6	0.81	8.96	6.45	11.05
	B	0.2	5.4	4.2	6.8	0.85	17.81	10.66	20.71
	I	0.1	16.0	10.9	19.3	–1.51	28.12	16.75	32.64
$D_{1.3}$,H H_c	1	1.0	12.3	15.0	19.4	0.27	2.66	5.40	6.04
	2	0.7	12.5	14.7	19.2	0.31	4.02	6.15	7.36
	3	0.8	15.5	15.3	21.7	0.45	8.40	7.06	10.97
	B	0.3	4.9	4.0	6.3	0.14	15.70	10.75	18.98
	I	0.0	13.2	10.3	16.7	–0.29	23.12	17.87	29.17
$D_{1.3}$,D_6,H H_c	1	1.8	9.2	10.3	13.9	0.32	1.75	3.10	3.56
	2	1.5	9.6	10.1	13.9	0.29	2.90	3.79	4.78
	3	1.6	13.9	12.7	18.8	0.59	7.89	6.03	9.92
	B	0.3	5.1	4.0	6.5	0.56	16.13	10.62	19.28
	I	–0.1	13.3	9.8	16.5	–0.82	23.33	15.92	28.19

crown height cannot reduce the standard error of the volume increment prediction. The I_H measurement reduces the standard error only from 13 % to 12 %.

5.3 Tally trees

The stem curve sets for tally trees are predicted as a function of $D_{1.3}$ using the average stem curve of the population and the stand effects estimated from the sample trees. The reliability of the stem, bark and increment volume estimates are studied using the relative between-stand standard error ($RMSE_b\%$).

The sample tree selection was simulated using random sampling. The stand effects were estimated from the sample trees. Only trees that were not used as sample trees were used as tally trees. The error variances are averages of 50 independent replications. The study was made using model 3.1.3.

$RMSE_b\%$ of the prediction of stem volume with bark for tally trees is independent of the measurement combination of the sample trees if at least $D_{1.3}$ and H are measured (Fig. 5.3). The

Table 5.6. Reliability of the for stem, bark and increment volume predictions with different measurement combinations. Measurements are made from the stem curve with bark and also from bark and increment. The relative statistics are given as percentages and the absolute statistics as dm³. Model 3.1.3 has been used. g is the index of the elementary stem curve.

Measure-ments	g	Absolute errors				Relative errors			
		mean	s_b	s_w	RMSE	mean	s_b	s_w	RMSE
$D_{1.3}$,H $B_{1.3}$	1	1.4	11.6	15.1	19.0	0.40	2.65	5.33	5.96
	2	0.9	10.6	14.5	17.9	0.28	3.48	5.82	6.78
	3	1.0	16.0	15.8	22.4	0.74	8.91	7.08	11.37
	B	0.5	4.8	3.7	6.0	1.20	11.01	8.98	14.21
	I	–0.1	13.5	10.7	17.2	–1.28	23.18	18.43	29.56
$D_{1.3}$,H $B_{1.3}$ $I_{1.3}$	1	1.7	12.0	15.1	19.3	0.50	2.63	5.35	5.97
	2	1.4	11.2	14.5	18.4	0.42	3.28	5.81	6.68
	3	1.1	11.1	13.0	17.1	0.18	4.58	6.26	7.76
	B	0.3	4.4	3.6	5.7	1.07	9.89	9.03	13.39
	I	0.3	3.3	5.5	6.4	0.97	6.64	10.86	12.75
$D_{1.3}$,D_6,H $B_{1.3}$ $I_{1.3}$	1	1.7	8.7	10.5	13.7	0.33	1.71	3.19	3.64
	2	1.5	9.4	10.5	14.1	0.28	2.49	3.71	4.48
	3	1.4	9.4	9.9	13.7	0.22	3.76	4.66	6.00
	B	0.3	4.0	3.5	5.4	0.74	9.77	8.61	13.01
	I	0.1	3.9	5.1	6.5	0.28	7.71	9.67	12.35
$D_{1.3}$,H H_c,$B_{1.3}$ $I_{1.3}$	1	1.7	12.0	15.0	19.2	0.48	2.60	5.32	5.96
	2	1.4	11.3	14.5	18.3	0.40	3.19	5.77	6.61
	3	1.1	11.3	12.9	17.1	0.17	4.46	6.13	7.57
	B	0.3	4.3	3.6	5.7	1.04	9.74	9.07	13.31
	I	0.3	3.3	5.4	6.3	0.95	6.61	10.83	12.71
$D_{1.3}$,D_6,H H_c,$B_{1.3}$ $I_{1.3}$	1	1.8	9.3	10.6	14.2	0.31	1.67	3.20	3.62
	2	1.5	10.0	10.7	14.7	0.24	2.32	3.62	4.31
	3	1.4	9.9	10.1	14.2	0.17	3.52	4.51	5.73
	B	0.3	4.0	3.6	5.4	0.82	9.58	8.75	12.97
	I	0.1	3.9	5.1	6.4	0.27	7.82	9.52	12.30
$D_{1.3}$,H $B_{1.3}$ $I_{1.3}$,I_H	1	1.6	11.3	15.0	18.8	0.52	2.48	5.33	5.90
	2	1.2	10.2	14.5	17.7	0.43	3.12	5.80	6.60
	3	0.4	7.9	12.8	15.1	0.17	3.63	6.15	7.14
	B	0.4	4.4	3.6	5.7	1.14	9.87	9.01	13.37
	I	0.8	3.5	5.6	6.6	1.31	4.76	10.62	11.66
$D_{1.3}$,D_6,H $B_{1.3}$ $I_{1.3}$,I_H	1	1.5	7.4	10.0	12.6	0.30	1.65	3.18	3.60
	2	1.1	8.1	10.2	13.0	0.23	2.37	3.61	4.33
	3	0.5	5.7	9.3	10.9	0.10	2.93	4.43	5.30
	B	0.3	4.0	3.5	5.4	0.84	9.71	8.74	13.06
	I	0.6	3.6	5.3	6.5	0.66	5.32	9.39	10.79
$D_{1.3}$,H H_c,$B_{1.3}$ $I_{1.3}$,I_H	1	1.5	11.3	15.0	18.8	0.50	2.45	5.31	5.89
	2	1.1	10.3	14.4	17.7	0.41	3.02	5.77	6.52
	3	0.4	8.0	12.7	15.0	0.15	3.48	6.04	6.98
	B	0.4	4.4	3.6	5.7	1.11	9.72	9.06	13.28
	I	0.7	3.3	5.5	6.4	1.27	4.67	10.61	11.61
$D_{1.3}$,D_6,H H_c,$B_{1.3}$ $I_{1.3}$,I_H	1	1.5	7.9	10.4	13.1	0.28	1.61	3.19	3.59
	2	1.2	8.6	10.4	13.5	0.19	2.24	3.63	4.26
	3	0.6	6.2	9.4	11.3	0.06	2.67	4.39	5.13
	B	0.3	4.0	3.6	5.4	0.87	9.56	8.72	12.94
	I	0.6	3.5	5.3	6.3	0.66	5.35	9.32	10.74

sample tree measurements have a marginal reducing effect on the RMSE$_b$% of bark volume and volume increment predictions when only stem dimensions with bark are measured. The crown height measurement reduces the RMSE$_b$% of the bark volume prediction from 19 % to 17 % and the volume increment prediction from 30 % to 27 %, assuming one measured sample tree per stand.

One sample tree with the measurement of $B_{1.3}$, in addition to the measurements of stem dimensions with bark, reduces the RMSE$_b$% of the bark volume prediction of tally trees from 19 % to 15 %. The additional measurement of $B_{1.3}$ and $I_{1.3}$ reduces the corresponding RMSE$_b$% of the volume increment prediction from 30 % to 18 %. The $I_{1.3}$ measurement of two trees gives smaller RMSE$_b$% for the volume increment prediction than does the measurement of both $I_{1.3}$ and I_H from one tree.

The RMSE$_b$%:s of the sample tree predictions indicate the asymptotic RMSE$_b$% that can be reached for tally tree prediction when the number of sample trees approaches the total number of trees in the stand (Fig. 5.3). In all cases examined, the first sample tree reduced the RMSEb% by about half of the total decrease that can be reached by increasing the number of sample trees.

5.4 Prediction without measured dimensions with bark

In the preceding discussion the use of the stem curve set model has been examined in situations where one or more dimensions from the stem curve with bark have been known. The model can also be used without measured dimensions with bark, e.g. to predict the stem curve with bark from measurements without bark or to predict the future growth without bark as a function of the dimensions without bark at the beginning of the growth period.

The reliability of volume prediction is presented in Table 5.7 with explanatory variables measured only from the elementary stem curves without bark (g = 2,3). All predictions are unbiased. The main result, compared with the predictions using measured dimensions with bark, is larger standard errors in the predictions of bark volume and volume increment. This can be reasoned because the size, which is defined as the weighted sum of the tree dimensions with bark, is used as an independent variable in the stem curve set model. In the model application the size has to

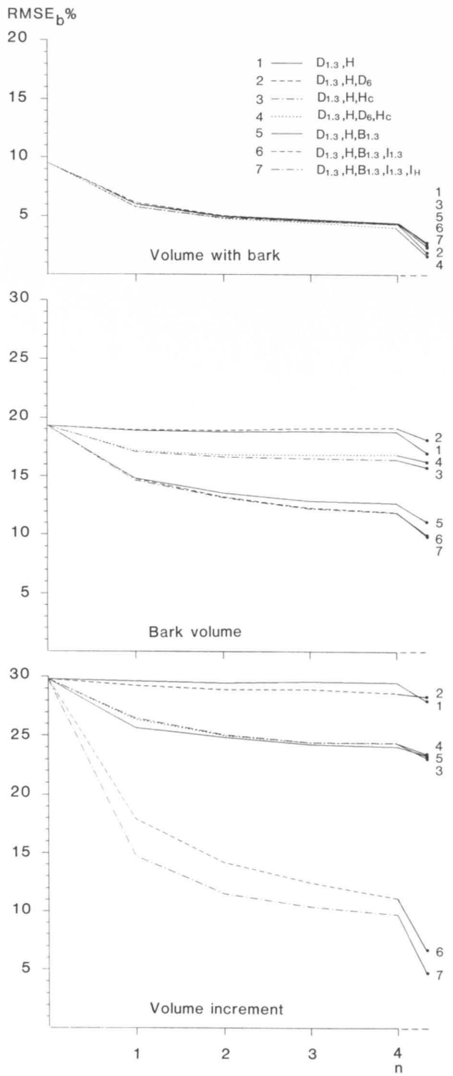

Fig. 5.3. Relative between-stand standard error of tally trees for the stem with bark, bark and increment volume predictions as a function of the number of sample trees with different combinations of sample tree measurements. The relative between-stand standard errors of the volume predictions of sample trees are also indicated by small dots. The model with average size (3.1.3) is used.

Table 5.7. Reliability of the stem, bark and increment volume predictions with different measurement combinations, when no measurements have been made from the stem curve with bark. The relative statistics are given as percentages and absolute errors as dm³. Model 3.1.3 has been used. $D_{1.3b}$ = diameter at breast height without bark in the year t, D_{6b} = diameter at a height of six meters without bark in the year t, $D_{1.3-5}$ and D_{6-5} in the year t–5, respectively, and H_{-5} tree height in the year t–5. g is the index of the elementary stem curve.

Measure-ments	g	Absolute errors s_b	s_w	RMSE	mean	Relative errors s_b	s_w	RMSE	
$D_{1.3b}$	1	1.8	25.9	27.7	37.7	–0.13	12.40	8.79	15.16
	2	2.1	22.5	25.1	33.7	0.06	12.10	9.15	15.13
	3	1.3	26.4	23.0	35.0	0.04	17.63	9.89	20.16
	B	–0.3	6.4	5.0	8.1	–1.32	22.37	13.34	25.98
	I	0.8	16.2	11.7	19.9	0.19	26.16	19.90	32.79
$D_{1.3b}$,H	1	0.1	11.2	16.6	20.0	–0.10	4.84	5.88	7.61
	2	0.3	10.0	14.9	17.9	–0.03	3.99	5.94	7.15
	3	–0.3	19.2	17.0	25.5	–0.08	10.51	7.22	12.72
	B	–0.2	5.5	4.7	7.2	–0.54	19.27	12.82	23.08
	I	0.6	16.4	11.3	19.8	0.21	26.15	18.38	31.88
$D_{1.3b}$,D_{6b},H	1	1.5	9.1	10.8	14.0	0.43	3.84	3.67	5.30
	2	1.3	7.1	9.1	11.5	0.38	1.86	3.50	3.96
	3	1.1	14.0	12.0	18.4	0.69	8.19	6.01	10.13
	B	0.2	5.7	4.7	7.4	0.73	19.92	13.42	23.41
	I	0.3	15.9	10.6	19.1	–0.57	25.34	16.77	30.31
$D_{1.3b}$,H,H_c	1	4.7	10.4	19.3	21.9	0.42	3.73	6.49	7.48
	2	4.5	9.4	17.2	19.6	0.68	3.12	6.51	7.21
	3	2.9	16.0	16.1	22.5	1.88	8.70	7.05	11.17
	B	0.2	5.0	4.7	6.9	–1.12	17.90	13–19	22.18
	I	1.6	12.1	11.9	16.9	–2.00	21.67	23.55	31.95
$D_{1.3-5}$	1	2.1	28.2	30.9	41.8	0.01	11.68	10.60	15.74
	2	2.3	25.4	28.1	37.9	0.17	11.36	10.96	15.75
	3	1.5	19.0	21.2	28.4	0.19	13.19	9.58	16.26
	B	–0.2	6.2	5.3	8.1	–1.04	21.94	14.31	26.12
	I	0.8	17.3	13.1	21.7	0.22	30.24	23.63	38.29
$D_{1.3-5}$,H_{-5}	1	0.4	18.8	20.4	27.7	–0.08	6.45	7.86	10.15
	2	0.7	18.0	18.4	25.7	0.01	6.18	7.99	10.09
	3	–0.2	6.8	13.3	14.9	–0.02	4.00	6.39	7.53
	B	–0.2	5.3	4.9	7.2	–0.69	19.05	13.58	23.34
	I	0.9	17.9	12.6	21.9	0.30	30.49	21.95	37.48
$D_{1.3-5}$,H_{-5},D_{6-5}	1	1.2	20.1	15.9	25.5	0.22	6.70	6.08	9.03
	2	1.2	19.1	14.2	23.7	0.21	6.45	6.00	8.79
	3	0.6	3.6	7.2	8.0	0.29	1.43	3.67	3.94
	B	0.0	5.3	4.7	7.1	0.26	18.87	13.21	22.97
	I	0.6	17.0	11.9	20.7	0.11	26.68	21.34	35.67
$D_{1.3-5}$,H_{-5},H_c	1	0.7	19.5	20.3	28.1	–0.03	6.44	7.84	10.13
	2	0.7	18.1	18.4	25.7	0.03	6.15	8.00	10.08
	3	0.2	7.2	13.1	14.9	0.03	3.70	6.34	7.34
	B	0.0	5.2	4.6	6.9	–0.41	18.78	13.42	23.03
	I	0.5	16.8	12.2	20.7	0.10	29.69	21.50	36.57

be estimated as a function of the measurements. The association between the size and the dimensions of the stem curve with bark is closer than between the size and the dimensions without bark, because dimensions with bark are elements of the measurement function or can be interpolated from it. This produces more reliable estimates of size when dimensions with bark are measured. When the diameter at breast height of one elementary stem curve is known, the averages of the standard errors for size estimates are 0.0396, 0.0472 and 0.0503, corresponding to the measures from elementary curves 1, 2, and 3, respectively.

Formally the stand growth and development can be simulated as a sum of the trees in a stand. This can be done with the stem curve set model by using repeated predictions so that the two outermost stem curves (stem curves with and without bark in the year t) are predicted as a function of the dimensions of the innermost stem curve (stem curve without bark in the year t–5). At each loop the predictions of the previous loop for the stem curve without bark in the year t are located on the innermost stem curve in the year t–5. If the model is inconsistent with regard to change in stem form, a simulation will lead to different tree form development than that which occurred according the cross-sectional data used for parameter estimation. Therefore the inconsistent model without average size (3.1.5) is inconvenient for simulation purposes because it will lead to rapid height development in relation to diameter development. In addition, the model with average size (3.1.3) will lead to inconsistent development of stem form because of its inability to take into account, that the relative size varies with the average size.

5.5 Measurement errors

The measurement errors influence the utility of the measurements. The reliability of the measurements is studied here from the literature assuming the measurements made from standing trees with the following instruments (named according Loetsch et al. 1973) and reading accuracies:

Table 5.8. Effect of measurement errors on the relative volume prediction errors of sample trees with different measurement combinations. Errors are given as percentages. Prediction with measurement errors are denoted by w. Model 3.1.3 has been used.

Measurements		Stem with bark mean	Stem with bark RMSE	Bark mean	Bark RMSE	Increment mean	Increment RMSE
$D_{1.3}$	w	0.12	14.37	–0.68	21.82	0.17	34.65
		0.57	13.38	–0.30	21.50	0.58	34.27
$D_{1.3}$,H	w	0.18	8.06	–0.20	20.67	–0.04	33.52
		0.31	6.06	–0.01	20.07	–0.03	33.36
$D_{1.3}$,H,D_6	w	0.31	6.85	0.14	20.42	–0.83	32.45
		0.34	3.65	0.85	20.71	–1.51	32.64
$D_{1.3}$,H,H_c	w	0.26	7.90	–0.20	19.39	–0.07	29.66
		0.27	6.04	0.14	18.98	–0.29	29.17
$D_{1.3}$,H,$B_{1.3}$	w	0.46	8.03	–0.85	17.20	–0.79	30.30
		0.40	5.96	1.20	14.21	–1.28	29.56
$D_{1.3}$,H,$B_{1.3}$,	w	0.84	8.79	–1.04	16.33	0.78	16.29
$I_{1.3}$		0.50	5.97	1.07	13.39	0.97	12.75
$D_{1.3}$,H,$B_{1.3}$,	w	0.84	9.02	–1.12	17.01	0.78	16.59
$I_{1.3}$,I_H		0.52	5.90	1.14	13.37	1.31	11.66

$D_{1.3}$	caliper	0.1 cm
D_6	the Finnish parabolic caliper for the upper-diameters	1.0 cm
H	Suunto hypsometer	10 cm
$B_{1.3}$	the Swedish bark gauge	0.1 cm
$I_{1.3}$	the Swedish increment borer and annual ring microscope	0.001 cm
I_H	binoculars with built-in scale	10 cm

According to Hyppönen & Roiko-Jokela (1978), the standard deviation for the $D_{1.3}$ measurement is 0.3 cm (1.4 %), for D_6 measurement 0.8 cm (5.3 %) and for H measurement 0.9 m (6.3 %). Päivinen et al. (1992) found that the standard deviation of diameter measurement at breast height is 0.4 cm, at six meter height (D_6) 0.6 cm and the standard deviation of height measurement is 0.22 m. According to Ihalainen (1987), the standard deviation for D_6 measurement is 0.8 cm and this is independent of the tree size. He also found that the standard deviation of height measurement is proportional to the tree height and is about 3.6 %. Standard error for the measurement of the crown height is 0.5 m independently of tree size.

Kujala (1979) and Daamen (1980) both studied the measurement error of bark thickness. Daamen found that the standard deviation of the differences between the original measurements and the control measurements in the Swedish National Forest Inventory were 5.6 mm. Kujala compared the normal bark measurement with one accurate measurement made with a bark measurer constructed on the increment borer. When later measurements were assumed to be the right bark thickness, he found that the standard error of the bark measurement was 4.1 mm.

If the direction of increment boring is not perpendicular to the height axis of the tree, the diameter increment is overestimated. This bias cannot be eliminated when the annual rings are measured. If boring is directed past the midpoint of the tree, the bias can be corrected during measurement of the annual rings. Kujala (1979) found that in the Finnish National Forest Inventory the overestimation of the radial increment was 0.4 %.

Noncircularity causes variation and overestimation of the cross-sectional area when the calculation is based on one or more diameter measurements (Matérn 1956). The reliability of measurements of diameter- and cross-sectional area increment is especially sensitive to noncircularity. Matérn (1961) obtained the empirical result that the standard deviation of diameter increment caused by the noncircularity of the stem is over 5 %. The irregularities caused by noncircularity in tree diameter and diameter increment are omitted here by assuming the cross-sectional area to be circular. The diameter is assumed to be the average of the measured diameters.

The height increment measurement is the most unreliable of the measurements studied here. Many authors have found a clear overestimation of binocular measurements of height increment (Tiihonen 1967, Alalammi 1968 and Päivinen et al. 1992). According to Päivinen et al., the overestimation is 0.10 m (18 %) and the standard deviation is 0.22 m (40 %).

The effect of the measurement errors was studied by generating random errors for the measurements. The measurements were assumed to be unbiased; and the measurement errors were assumed to be normally distributed, mutually uncorrelated and uncorrelated with the random stand and tree effects. The standard deviations used are:

Measurement	s_t cm	s_t %
$D_{1.3}$	0.3	
D_6	0.8	
H		3.6
H_c	0.5	
$B_{1.3}$	0.4	
$I_{1.3}$		5.0
I_H	24	

where the standard deviation of $I_{1.3}$ is assumed without any empirical studies. The measurement errors were taken into account in the prediction of stem curve set by adding the measurement error variances to the diagonal elements of the covariance matrix of the measured random tree effects (Lappi 1986, p. 28–29). The reliability statistics were calculated as an average of 50 independent replications.

The measurement errors studied have no significant effect on the bias of the predictions of the stem, bark and increment volume of the sample trees (Table 5.8). If at least $D_{1.3}$ and H are measured, the measurement errors increase the relative standard error of stem volume prediction with bark by about 2 percentage points. The measurement errors of $B_{1.3}$ and $I_{1.3}$ increase the relative standard errors of the bark volume and the volume increment predictions about 3 percentage points, and the measurement error of I_H increases the standard error of the volume increment prediction by nearly 5 percentage points.

When measurement errors are taken into account, the standard error of the volume increment prediction is higher with I_H measurement than without it.

The measurement errors have only very small effects on the predictions of stem volume with bark of tally trees (Fig 5.4). Calibration of the bark volume and the volume increment predictions is considerably slower as a function of the number of sample trees with measurement errors than without them. According to the results for sample trees, it is not possible to eliminate totally the measurement error effect by increasing the number of sample trees. The measurement errors decrease the relative efficiency of the measurement of I_H compared to the measurement of $I_{1.3}$.

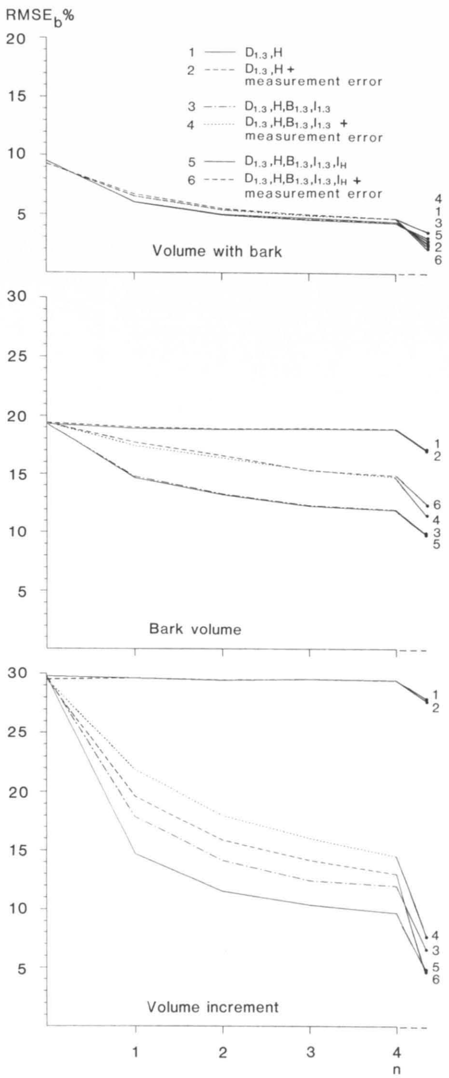

Fig. 5.4. Relative between-stand standard error of tally trees for the stem with bark, bark and increment volume predictions as a function of the number of sample trees with and without measurement errors. The relative between-stand standard errors of the volume predictions of sample trees are also indicated by small dots. The model with average size (3.1.3) is used.

6 Comparison of the stem curve set model with the volume and increment methods

The volume method is the prevailing method used in Finland determining bark volume and volume increment. Here the predictions of the stem curve set model and those of the volume method are compared with each other. The stem curve set model utilizes the covariation of the stand and tree effects between the elementary stem curves. In order to separate the effects of using variance components and using the covariations between the elementary stem curves, three different methods of prediction were compared: 1) normal use of the stem curve set model, 2) use of the stem curve set model without the associations between the elementary stem curves when the stand effects are estimated, and 3) the volume method using volume functions. In all these methods, the bark volume and volume increment are differences between the stem volumes. To obtain reasonable results for the bark volume and volume increment predictions using Methods 2 and 3, the same dimensions have to be known from all the elementary stem curves. This study was made using the following sample tree measurements: $D_{1.3}$, H, $B_{1.3}$, $I_{1.3}$ and I_H. $D_{1.3}$ and H are thus known for all elementary stem curves.

In Method 2, the elementary stem curves were predicted separately using diameters $\mathbf{D}_g$ and heights $\mathbf{H}_g$ of the elementary stem curve g measured in the stand. The estimates of the sample tree sizes and the principal components of the stand effects are denoted by $\hat{s}$ and $\hat{c}$ when they are estimated from the measurements from the stem curve with bark in the year t, by $\hat{s}'$ and $\hat{c}'$ when they are estimated from the measurements from the stem curve without bark in the year t, or by $\hat{s}''$ and $\hat{c}''$ when they are estimated from the measurements from the stem curve without bark in the year t–5. The predictions of the elementary stem curves and the volumes are given in set-up 6.1. In Method 2 the stand structure of the data is utilized using variance components. If no sample trees are measured, the predictions of Method 2 are identical to the predictions for the normal use of the stem curve set model (Method 1).

Set up 6.1

Parameters estimated as a function of the sample trees. Equation 4.1.2			Prediction of the volume of sample tree i. See equation 4.2.3			Prediction of the volume of tally tree i. See equation 4.2.6	
($\hat{\mathbf{s}}$ and $\hat{\mathbf{c}}$)	$\lvert\mathbf{D}_1,\mathbf{H}_1$	->	$\hat{V}_{1i}$	$\lvert\mathbf{D}_{1i},H_{1i},\hat{s}_i,\hat{\mathbf{c}}$	and	$\hat{V}_{1i}$	$\lvert D_{1i},\hat{\mathbf{c}}$
($\hat{\mathbf{s}}'$ and $\hat{\mathbf{c}}'$)	$\lvert\mathbf{D}_2,\mathbf{H}_2$	->	$\hat{V}_{2i}$	$\lvert\mathbf{D}_{2i},H_{2i},\hat{s}_i',\hat{\mathbf{c}}'$	and	$\hat{V}_{2i}$	$\lvert D_{2i},\hat{\mathbf{c}}'$
($\hat{\mathbf{s}}''$ and $\hat{\mathbf{c}}''$)	$\lvert\mathbf{D}_3,\mathbf{H}_3$	->	$\hat{V}_{3i}$	$\lvert\mathbf{D}_{3i},H_{3i},\hat{s}_i'',\hat{\mathbf{c}}''$	and	$\hat{V}_{3i}$	$\lvert D_{3i},\hat{\mathbf{c}}''$

In Method 3 the form of the volume function 61.3 by Laasasenaho (1982) was used:

$$\ln(V) = a_0 + a_1\ln(D_{1.3}) + a_2\ln(H) + a_3\ln(H-1.3) + a_4D_{1.3} + \varepsilon \qquad (6.1)$$

The parameters of the model (6.1) were estimated anew by the ordinary least square method from the primary data set separately for the stem volumes with and without bark in the year t (V_1 and V_2) and without bark in the year t–5 (V_3). The parameter estimates are given in set-up 6.2. This method cannot be used in predicting the tally tree volumes without auxiliary models.

Set-up 6.2

Dependent variable	a_0	a_1	a_2	a_3	a_4	s_t
V_1	–3.44412	1.88812	2.88479	–1.72647	–0.0010275	0.0588
V_2	–3.29529	1.89733	2.21488	–1.08823	–0.0013091	0.0682
V_3	–3.42612	1.90404	2.46212	–1.29223	–0.0023385	0.0752

With all methods the standard errors of the stem volume predictions for the sample trees are nearly the same (Table 6.1). The bark volume and the volume increment predictions of Methods 2 and 3 also gives similar standard errors, but Method 1 gives slightly smaller standard errors for the volume increment predictions and clearly smaller standard errors for the bark volume predictions. When Method 1 is used without measurement of I_H, the standard errors for stem volume and volume increment are nearly the same as using Methods 2 and 3 with the measurement of I_H.

The reliability of the predictions for stem, bark and increment volume of tally trees were studied with $RMSE_b\%$ (Fig. 6.1). Only Methods 1 and 2 were used, since Method 3 requires auxiliary models for tally tree predictions. Because for Methods 2 and 3 the prediction results for sample trees were nearly identical, Method 2 can be used as an approximation for the volume method where volume functions are calibrated with variance components.

In Method 2 the $RMSE_b\%$ of the bark volume prediction increases as a function of the number of sample trees. This is determined by the properties of Method 2. The predictions of Method 2 for tally trees are identical to those obtained with Method 1 if no sample trees are measured. When the number of sample trees increases, the weight of Method 1 decreases in relation to the weight of the sample tree information estimated by Method 2. For sample trees the bark volume predictions of Method 1 as a function of $D_{1.3}$ have nearly the same $RMSE_b\%$ as the prediction of Method 2 as a function of $D_{1.3}$, H, $B_{1.3}$, $I_{1.3}$ and I_H. When the number of sample trees increases, the decreasing sampling error cannot compensate for the increasing weight of the prediction error of Method 2.

The prediction for the stem volume with bark in Method 1 has a slightly smaller RMSEb% than does the prediction in Method 2. With Method 1 the $RMSE_b\%$:s for the bark volume and the volume increment predictions are about half those obtained with Method 2 when the number of sample trees is small. This is also the case, if the height increment is not measured when the stem curve set (Method 1) is used.

Predictions based on the increment method are also compared with those of the stem curve set model. The functions of the increment method are taken from Svensson (1988) and the parameters are estimated from the primary data. Some modifications are done: 1) the form quotient $D_6/D_{1.3}$ is used instead of the form quotients $D_5/D_{1.3}$ and $D_3/D_{1.3}$, 2) the geographical variables are ignored and 3) for each function a model without tree age as independent variable was also estimated. The bark volume functions corresponding the simple bark volume function by Svensson (1988) are

$$B_V = a_0 + a_1 D_{1.3b} + a_2 \ln(D_{1.3b}) + a_3 \ln(T) + \varepsilon \quad (6.2)$$

$$B_V = a_0 + a_1 D_{1.3b} + a_2 \ln(D_{1.3b}) + \varepsilon \quad (6.3)$$

Corresponding the accurate bark volume function they are

$$B_V = a_0 + a_1 D_{1.3b} + a_2 \ln(H) + a_3 B_{1.3} + a_4 \ln(B_{1.3}) + a_5 (D_6 / D_{1.3}) + a_6 \ln(D_6 / D_{1.3}) + a_7 T + a_8 \ln(T) + \varepsilon \quad (6.4)$$

$$B_V = a_0 + a_1 D_{1.3b} + a_2 \ln(H) + a_3 B_{1.3} + a_4 \ln(B_{1.3}) + a_5 (D_6 / D_{1.3}) + a_6 \ln(D_6 / D_{1.3}) + \varepsilon \quad (6.5)$$

and corresponding the volume increment function they are

$$I_V = a_0 + a_1 D_{1.3b} + a_2 \ln(D_{1.3b}) + a_3 \ln(H) + a_4 \ln(I_{1.3}) + a_5 \ln(H_c / H) + a_6 (D_6 / D_{1.3}) + a_7 \ln(D_6 / D_{1.3}) + a_8 T + a_9 \ln(T) + a_{10} \ln(B_{1.3} / D_{1.3}) + \varepsilon \quad (6.6)$$

Table 6.1. Reliability of the stem, bark and increment volume predictions with measured $D_{1.3}$, $B_{1.3}$, $I_{1.3}$, H and I_H using different methods: 1) normal use of the stem curve set model, 2) use of the stem curve set model without the connections between the elementary stem curves when the stand effects are estimated and 3) the volume method using volume functions. The statistics for normal use of the stem curve model when only $D_{1.3}$ is measured (1a) and when $D_{1.3}$, $B_{1.3}$, $I_{1.3}$ an H are measured (1b) are also given. The relative statistics are given as percentages and absolute statistics as dm^3. Model 3.1.3 has been used. g is the index of the elementary stem curve.

			Absolute errors				Relative errors			
g	Method	$\bar{x}$	mean	s_b	s_w	RMSE	mean	s_b	s_w	RMSE
1	1a	235.9	3.7	27.5	27.3	38.7	0.57	10.12	8.76	13.38
	1b		1.7	12.0	15.1	19.3	0.50	2.63	5.35	5.97
	1		1.6	11.3	15.0	18.8	0.52	2.48	5.33	5.90
	2		1.8	14.5	16.7	22.1	0.50	2.74	5.48	6.12
	3		–0.1	14.6	17.0	22.4	–0.43	2.84	5.25	5.97
2	1a	206.9	3.7	26.1	26.0	36.8	0.70	10.95	9.67	14.61
	1b		1.4	11.2	14.5	18.4	0.42	3.28	5.81	6.68
	1		1.2	10.2	14.5	17.7	0.43	3.12	5.80	6.60
	2		0.8	10.7	16.0	19.2	0.11	3.95	6.01	7.19
	3		0.2	9.8	15.8	18.6	–0.40	4.01	5.67	6.93
3	1a	166.9	2.7	23.9	22.8	33.0	0.74	15.04	10.48	18.30
	1b		1.1	11.1	13.0	17.1	0.18	4.58	6.26	7.76
	1		0.4	7.9	12.8	15.1	0.17	3.63	6.15	7.14
	2		0.1	6.8	14.2	15.8	0.11	3.96	6.50	7.60
	3		0.1	6.6	14.2	15.7	–0.31	4.09	6.20	7.42
B	1a	29.1	0.1	5.4	4.4	7.0	–0.30	18.34	11.33	21.50
	1b		0.3	4.4	3.6	5.7	1.07	9.89	9.03	13.39
	1		0.4	4.4	3.6	5.7	1.14	9.87	9.01	13.37
	2		1.1	10.1	7.3	12.5	2.98	17.74	16.00	23.84
	3		–0.3	10.1	7.1	12.3	–0.76	17.33	16.79	24.08
I	1a	40.6	1.0	16.8	11.9	20.5	0.58	27.87	20.07	34.27
	1b		0.3	3.3	5.5	6.4	0.97	6.64	10.86	12.75
	1		0.8	3.5	5.6	6.6	1.31	4.76	10.62	11.66
	2		0.6	5.1	6.2	8.0	0.25	6.83	11.10	13.02
	3		0.1	3.8	6.0	7.0	–0.63	6.39	10.73	12.48

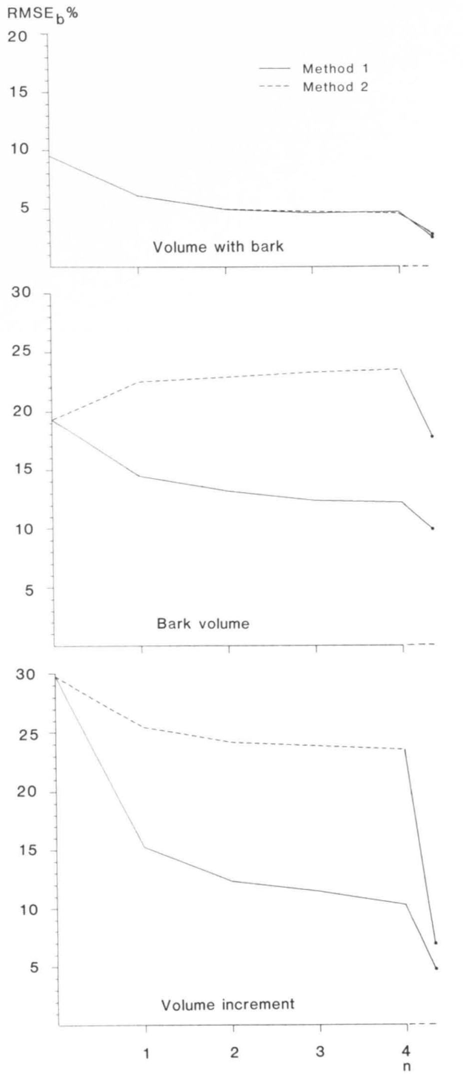

Fig. 6.1 Relative between-stand standard error of tally trees for the stem with bark, bark and increment volume predictions as function of the number of sample trees with normal use of the stem curve set model (Method 1) and use of the stem curve set model without connection between the elementary stem curves (Method 2). The relative between-stand standard errors of the volume predictions of sample trees are also indicated by small dots. The model with average size (3.1.3) is used.

Table 6.2. Reliability of the bark and increment volume predictions using the stem curve set model and the models based on the increment method. Model 3.1.3 is the stem curve set model. a means that the only used tree dimension is diameter at breast height without bark and b that all information of $D_{1.3}$, $B_{1.3}$, $I_{1.3}$, D_6 and H measurements are used.

g	Model	$\bar{x}$	Absolute errors mean	s_b	s_w	RMSE	Relative errors mean	s_b	s_w	RMSE
B	3.1.3a	29.1	–0.3	6.4	5.0	8.1	–1.32	22.37	13.34	25.98
	6.2		–0.2	6.3	5.2	8.2	–0.26	22.01	14.63	27.37
	6.3		–0.2	6.8	5.5	8.7	–0.27	23.21	14.64	27.39
	3.1.3b		0.3	4.0	3.5	5.4	0.74	9.77	8.61	13.01
	6.4		–1.3	4.5	3.7	5.9	–0.61	11.39	9.32	14.70
	6.5		–1.0	4.7	3.7	5.7	–0.52	11.69	9.19	14.85
I	3.1.3b	40.6	0.1	3.9	5.1	6.5	0.28	7.71	9.67	12.35
	6.6		0.3	3.8	4.9	6.2	0.11	6.57	9.64	11.65
	6.7		0.3	4.2	5.3	6.7	–0.05	8.86	9.97	13.31

$$\begin{aligned} I_V &= a_0 + a_1 D_{1.3b} + a_2 \ln(D_{1.3b}) + a_3 \ln(H) \\ &+ a_4 \ln(I_{1.3}) + a_5 \ln(H_c / H) + a_6 (D_6 / D_{1.3}) \\ &+ a_7 \ln(D_6 / D_{1.3}) + a_{10} \ln(B_{1.3} / D_{1.3}) + \varepsilon \end{aligned} \quad (6.7)$$

The tree age in years is denoted by T and the diameters without bark are denoted by the subscript b. The parameter values of the functions are given in appendix C. The bark volume functions 6.2 and 6.4 are used in Svensson's (1988) system to calculate the past bark volume increment as is described earlier (Chapter 1.2, Equation 1.2.1).

It is assumed in the comparisons that the measured sample tree dimensions are $D_{1.3}$, $B_{1.3}$, $I_{1.3}$, D_6, H and, in increment method, also the tree age T (Table 6.2). In contrast to the volume method, the height increment measurement is not needed in the increment method. The predictions based on the simple bark functions (6.2 and 6.3) are compared to predictions of the stem curve model so that the only known dimension is diameter at breast height without bark. Differences between the stem curve set model and the increment method are small. For bark volume, the stem curve set model gives slightly smaller RMSE's in the cases of the simple and the accurate bark volume functions. For volume increment, the function 6.6 with tree age as independent variable gives slightly smaller RMSE's than the stem curve set model, but the function 6.7 without tree age as independent variable gives slightly higher RMSE's.

7 Calibration of the model

7.1 Calibration with external information

A linear model gives the best linear unbiased prediction as a function of the independent variables. For model applications some external variables, which correlate with the dependent variable, are often known. These external variables can be used to improve the reliability of the predictions in two ways: 1) by modelling the fixed parameters as a function of the external variables or 2) by using the conditional prior distributions of the random parameters.

The simplest form of the fixed parameter modelling technique is separate estimation of the parameters in classes of one independent variable. Lappi (1986) estimated the fixed parameters of the regionalized stem curve model separately for eight climatic regions. The fixed variables can also be modelled as a function of continuous external variables. This practice has been used to model the fixed parameters of diameter distributions as a function of stand variables (e.g. Rennolls et al. 1985 and Kilkki & Päivinen 1986).

The conditional prior distributions are utilized in mixed linear model prediction. This prior information can be improved by regressing the random parameters on some external variables. Kilkki & Lappi (1987) and also Korhonen (1991) have calibrated the stem curve model using the expectations and variances of the principal components of the stand effects conditional on some stand variables.

To improve the conditional prior information used in model calibration, unbiased measurements or estimates of stand effects are needed. The random effect estimates are always shrunk towards zero and cannot be used as dependent variables for regression models. To obtain unbiased estimates for the principal components it is necessary to know the delineation vectors exactly, or the potential random error must be independent of the regressors of the random effect model. Here the stand effects are estimated as fixed for each delineation variable separately from the primary data set. The stand effect estimates are unbiased but they include random error caused by the sampling.

The principal components of the stand effects were regressed on some external variables using stepwise regression. The independent variables examined were used to describe the geographical position (longitude, latitude and altitude) and the site quality (temperature sum and forest site type) of the stand, the growing stock (mean diameter, basal area and number of stems per hectare) and also the time since the last thinning. The models are presented in Appendix D. The coefficients of determination (R^2) for the principal components c_{gk} and the standard errors of the models (s_t) are presented in set-up 7.1.

Set-up 7.1

	c_{11}	c_{12}	c_{13}	c_{14}	c_{B1}	c_{B2}	c_{B3}	c_{B4}	c_{I1}	c_{I2}	c_{I3}	c_{I4}
R^2	0.47	0.18	0.09	0.12	0.44	0.23	0.22	0.11	0.38	0.34	0.03	0.11
s_t	1.41	0.13	0.07	0.05	0.35	0.05	0.02	0.02	0.74	0.09	0.03	0.01

The main part of the unexplained variation of the stem curve with bark, the bark curve and the increment curve is in the directions of their first principal components. Because of the quite mechanical formulation of the models for the principal components, it is not reasonable to interpret the meanings of individual independent variables. Still some interdependences are obvious. The variables describing the geographical position and the site quality are the most important for the two first principal components of bark and for the first principal component of increment, the "increment rate" component. Variables describing the growing stock are important for prediction of stem form as it also is for the second principal component of increment, the "slenderness change" component.

The conditional expectations and error variances of the regression models were utilized to predict the stem curve sets. The covariance matrix of the principal components of stand effects was reformed using the variances of the conditional estimates on the diagonal and assuming that the correlations between the different principal components remain unchanged.

When only diameter at breast height with bark

Table 7.1. Reliability of the predictions for stem, bark and increment volumes of sample trees with improved prior information when only diameter at breast height with bark is measured. Method 0 is the control without improved prior information. In methods 1–5 the stand effects are estimated using regression models for the principal components of the stand effects c_{gk} as follows: 1) c_{11}, 2) c_{11}, c_{21}, 3) c_{11}, c_{21}, c_{31}, 4) c_{11}, c_{12}, c_{21}, c_{22}, c_{31}, c_{32} and 5) $c_{1\cdot}$, $c_{2\cdot}$, $c_{3\cdot}$. In methods a–c tree size is used as the random variable as follows: a) mean of s, b) s = f($d_{1.3}$), c) s = f($d_{1.3}$, stand variables). Statistics are in percentages. Model 3.1.3 has been used.

	With bark, t				Without bark, t				Without bark, t–5			
Method	mean	s_b	s_w	s_t	mean	s_b	s_w	s_t	mean	s_b	s_w	s_t
0	0.57	10.12	8.76	13.36	0.70	10.95	9.68	14.59	0.74	15.04	10.49	18.28
1	0.70	8.22	8.75	11.98	0.86	9.23	9.24	13.32	0.98	14.12	10.23	17.39
2	0.70	8.22	8.75	11.98	0.84	9.11	9.65	13.24	0.94	14.41	10.23	17.63
3	0.70	8.22	8.75	11.98	0.94	9.11	9.65	13.24	1.36	12.75	10.19	16.28
4	0.68	8.12	8.80	11.96	0.74	8.92	9.72	13.17	1.21	12.58	10.25	16.19
5	0.74	8.03	8.83	11.91	0.80	8.82	9.75	13.13	0.90	12.54	10.33	16.21
a	0.68	10.70	9.06	13.99	0.94	11.58	9.97	15.25	0.95	15.48	10.87	18.86
b	–0.24	10.50	8.71	13.61	–0.29	11.42	9.64	14.91	0.03	15.29	10.19	18.32
c	–0.49	7.93	8.77	11.80	–0.40	9.00	9.69	13.21	–0.57	13.85	10.33	17.23

	Bark				Increment			
Method	mean	s_b	s_w	s_t	mean	s_b	s_w	s_t
0	–0.30	18.34	11.33	21.50	0.58	27.87	20.07	34.26
1	–0.39	17.67	11.40	20.97	0.53	26.80	20.27	33.52
2	–0.24	16.64	11.57	20.21	0.55	26.11	20.26	32.98
3	–0.24	16.64	11.57	20.21	–0.99	18.30	20.40	27.36
4	0.22	15.14	11.53	19.19	–0.92	18.25	20.41	27.34
5	0.34	15.39	11.52	19.18	0.41	18.25	20.39	27.32
a	–1.13	18.34	11.50	21.59	0.30	27.75	20.01	34.13
b	–0.09	18.39	11.26	21.48	–1.84	28.08	20.61	34.75
c	–1.13	17.16	11.33	20.51	0.24	27.54	20.27	34.11

is measured, the improved prior information clearly decreases the standard error between stands for stem, bark and increment volume predictions, but it has no effect on the standard error within stands (Table 7.1). The first principal component of all elementary stem curves gives almost the total extent of the improvement that can be reached. Only in the bark curve does the second principal component have a significant influence on the standard error of bark volume prediction. If sample trees with measured $D_{1.3}$, H, $B_{1.3}$ and $I_{1.3}$ are available, the improved prior information has no effect on the standard errors of the predictions for stem, bark and increment volumes for tally trees.

Size can also be assumed to be random. When Lappi (1986) used the mean and variance estimated from his primary data to describe the distribution of size, he found only very small differences between the results with fixed and random size. This is due to the close relationship between diameter and size. When one or more diameters are known, they explain most of the size variation and the weight of the prior information is small. The use of random size can be made more efficient by using the conditional distribution as a function of some variables known in the application. Here the conditional expectations and variances are determined as follows:

$$\bar{s} = 2.61327 \qquad var(s) = 0.0873 \qquad (7.1a)$$
$$\hat{s} = 0.237185 + 0.8415733\, d_{1.3} \qquad var(s - \hat{s}) = 0.00207 \qquad (7.1b)$$
$$\hat{s} = f(d_{1.3}, \text{stand variables}) \qquad var(s - \hat{s}) = 0.00123 \qquad (7.1c)$$

Table 7.2. Reliability of the predictions for stem, bark and increment volumes of sample trees with different measurement combinations and as a function of tree size s in the test data set. The relative statistics are given as percentages and absolute statistics as dm^3. RMSE in the primary data is given in parentheses. Model 3.1.3 has been used.

		Absolute errors				Relative errors			
Measurements	g	mean	s_t	RMSE		mean	s_t	RMSE	
s	1	–6.3	18.3	19.3	(10.3)	–1.6	4.8	5.1	(3.5)
	B	–0.0	13.9	13.9	(6.5)	–0.6	20.5	20.5	(19.5)
	I	–11.4	19.2	22.3	(20.5)	–10.2	50.1	51.1	(33.1)
$D_{1.3}$	1	–58.9	132.3	144.8	(38.7)	–14.5	25.3	29.2	(13.4)
	B	–5.0	9.4	10.6	(7.0)	–11.0	19.8	22.7	(21.5)
	I	–13.9	22.9	26.8	(20.5)	–23.8	41.7	48.0	(34.3)
$D_{1.3}$,H	1	–25.2	64.3	69.1	(20.1)	–5.6	10.0	11.5	(6.1)
	B	–2.3	11.0	11.2	(6.4)	–5.1	17.9	18.6	(20.1)
	I	–15.5	26.5	30.7	(20.4)	–23.3	42.0	48.0	(33.4)
$D_{1.3}$,H,D_6	1	1.6	12.3	12.4	(13.0)	–0.3	4.6	4.6	(3.7)
	B	–6.6	12.9	14.5	(6.8)	–9.0	20.1	22.0	(20.7)
	I	–5.0	16.8	17.5	(19.3)	–12.8	38.5	40.6	(32.7)
$D_{1.3}$,H	1	–17.7	46.0	49.3	(19.3)	–4.3	7.9	9.0	(6.0)
$B_{1.3}$	B	–9.2	14.1	16.8	(5.7)	–16.8	18.8	25.2	(13.4)
$I_{1.3}$	I	–1.0	8.8	8.9	(6.4)	0.0	17.3	17.3	(12.8)
$D_{1.3}$,H	1	–18.1	46.6	50.0	(18.8)	–4.0	7.9	8.9	(5.9)
$B_{1.3}$	B	–9.0	13.8	16.5	(5.7)	–16.4	18.7	24.9	(13.4)
$I_{1.3}$,I_H	I	0.9	8.4	8.4	(6.6)	–0.5	14.1	14.1	(11.7)
$D_{1.3}$,H,D_6	1	2.0	12.9	13.1	(13.7)	0.6	4.2	4.2	(3.7)
$B_{1.3}$	B	–7.0	11.1	13.1	(5.4)	–12.5	15.9	20.2	(13.0)
$I_{1.3}$	I	1.4	7.5	7.6	(6.5)	4.5	15.2	15.9	(12.4)
$D_{1.3}$,H,D_6	1	1.3	12.4	12.5	(12.6)	0.6	4.2	4.2	(3.6)
$B_{1.3}$	B	–6.9	10.9	12.9	(5.4)	–12.4	15.7	20.0	(13.4)
$I_{1.3}$,I_H	I	2.8	8.0	8.5	(6.5)	3.8	11.6	12.2	(10.8)

Model 7.1c is presented in Appendix D.

Use of the size conditional to the diameter at breast height with bark (Equation 7.1.b) does not decrease the standard error for prediction of the volume with bark (Table 7.1), since Model 7.1b is already implicitly included in the stem curve set model. The standard error of the prediction for volume with bark can be decreased using the size conditional to variables that are not dimensions included the stem curve set model. Here the standard error decreases two percentage points with Model 7.1c. The improved prior information on size cannot decrease the standard errors of the bark volume and the volume increment predictions.

7.2 Calibration to a set of independent data

Let us study the application of the stem curve set model in the test data set. The test data set obviously represents a different Scots pine population than the primary data set. The predictions for the stem volumes are clearly biased, but the bias decreases as the number of measurements per sample tree increases (Table 7.2). The bark volume predictions are also always clearly biased, even though bark thickness is measured.

Differences between the primary data set and the test data set can be characterized using the principal components of the between-stand effects. Let us call the weighted sums of the stand effects at the knot angles component variables,

Table 7.3. Correlations of the component variables estimated from the test data set. On the diagonal of the correlation matrix of stand effect component variables are their variances*100. Means of the component variables in test data set (mean) are also given.

		Stem curve with bark (g = 1)				Bark curve (g = B)				Increment curve(g = I)			
		Component variable (k)											
		1	2	3	4	1	2	3	4	1	2	3	4
	mean	.101	.056	.064	.034	–.037	–.022	.012	.003	–.022	.017	–.008	.002
g	k												
1	1	4.88											
	2	.63	.40										
	3	.21	.23	.16									
	4	.17	–.16	.–23	.07								
B	1	–.01	–.26	–.11	.24	.27							
	2	–.17	–.15	.06	.19	.58	.02						
	3	.04	–.03	.19	.28	.31	.68	.03					
	4	–.05	.12	.37	.12	–.08	–.01	.12	.01				
I	1	.16	–.03	–.13	–.00	.37	.32	.07	–.16	1.66			
	2	–.09	–.09	.10	–.03	.24	.41	.20	–.04	.47	.21		
	3	–.00	.06	–.24	.27	–.15	–.22	–.02	–.16	–.52	–.34	.06	
	4	–.04	–.01	.14	–.02	.01	.12	.10	.18	–.13	–.16	–.05	.04

when the eigenvectors of the stem curve set model are used for weighting. In the primary data set, the component variables are identical to the principal components and their expectations are zero. The stand effects were calculated conditional to the size as is also the case in the analysis stage. The expectations of the component variables calculated from the test data set clearly deviate from zero (Table 7.3).

The variances and covariances between the component variables in the test data were also examined. Formula A.2.1 in Appendix A.2 was used. In formula A.2.1 the covariances between component variables are functions of the eigenvectors and covariances between the stand effects at knot angles. The variances of the stand effects at knot angles were estimated using the formulas of Searle (1971, p. 474). Reliability of the estimates of the between-stand variances at knot angles was checked by examining the consistency of their proportion of the total variances with neighbor angles (Set-up 7.2).

Set-up 7.2

	u												
g	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0.72	0.78	0.83	0.81	0.76	0.71	0.60	0.64	0.66	0.79	0.82	0.84	0.88
2	0.57	0.61	0.62	0.50	0.33	0.42	0.51	0.30	0.55	0.68	0.58	0.80	
3	0.66	0.65	0.63	0.62	0.68	0.70	0.64	0.66	0.72	0.73	0.81	0.85	0.83

Some estimates for the between stand covariances at knot angles were unreliable in terms of estimated correlation coefficients over 1. Both the fitting constant method and the REML estimation gave the same result. This may be caused by the small size of the data set and by the fact that in each plot there were only 1–5 trees. The stand effect covariances between knot angles were approximated assuming that the between stand correlations are the same as the correlation of total errors.

The variances of the component variables in the test data set are about twice the variances of the stand effect principal components in the primary data set, excluding bark curve, where they are relatively smaller (Table 7.3). This can be deduced by the different measurement accuracy of bark thickness in the data sets. The bark measurement in the primary data was made with a Swedish bark gauge, but the measurement method used to obtain the test data is definitely more accurate. The component variables within each elementary curve have a different character than the principal components in the original model, because in the test data set the component variables are correlated.

The stem curve set model was calibrated to the test data set using the estimated expectations of the component variables. The results for prediction of sample tree volume are given in Table 7.4. When only a few dimensions of a sample tree are measured, the bias is smaller than that for the prediction of uncalibrated model. When the stem is measured more accurately, the bias can also increase with the calibration. The calibration slightly decreases the standard deviation of stem volume, but has no effect on the standard deviations of bark volume and volume increment when only $D_{1.3}$ is measured. The random prediction errors for volume with bark and volume increment of the volume method and the stem curve set model have the same magnitude. For bark volume, the stem curve set model gives smaller standard deviations. The volume prediction for tally trees was not tested in the test data because there were so few measured trees per sample plot.

The inconsistent effect of the sample tree measurements on the bias of the calibrated volume prediction can be seen as being due to the characteristics of the component variables in the test data. If no calibrations are made for the test data set, the bias of the stand effect estimates oscillates as a function of the knot angles (Fig. 7.1). The breast height varies between knot angles 3 and 6 and is, on average slightly over knot angle number 4. The calibration with the averaged component variables diminishes the average bias of the stand effects over all knot angles; but in the case of bark and increment curves can eliminate only a part of the oscillation. Now the effect of each measurement depends on its location on the oscillating bias function; thus an inconveniently located additional measurement can increase the prediction error.

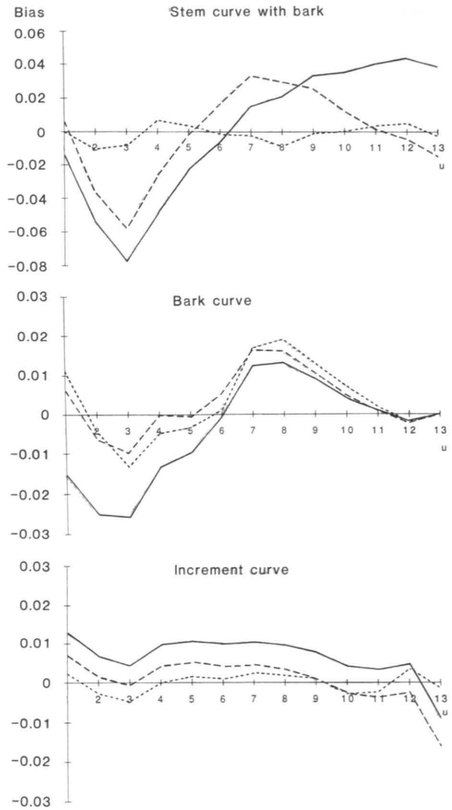

Fig. 7.1. Bias of the component variable estimates in the test data set as function of the knot angles u. The solid line is without calibration, the broken line is calibrated using means of the first principal components of each elementary stem curve, and the dotted line is calibrated using means of the first four principal components of each elementary stem curve. The model with average size (3.1.3) is used.

Table 7.4. Reliability of the stem, bark and increment volume predictions with improved prior information in the test data set. Method 0 is the control without improved prior information. In methods 1–4 the stand effects are estimated using expectations of the component variables c_{gk} estimated from the test data set as follows: 1) $c_{\cdot 1}$, 2) $c_{\cdot 1}$, $c_{\cdot 2}$, 3) $c_{\cdot 1}, c_{\cdot 2}, c_{\cdot 3}$, 4) $c_{\cdot 1}, c_{\cdot 2}, c_{\cdot 3}, c_{\cdot 4}$. The measurements used are denoted as follows: a) $D_{1.3}$, b) $D_{1.3}$ $B_{1.3}$ $I_{1.3}$ and H c) $D_{1.3}$ $B_{1.3}$ $I_{1.3}$ H and I_H. The statistics of the volume method (Method 3 in Chapter 7) are also given for the measurement combination c (M3). Statistics are given as percentages. Model 3.1.3 has been used.

	With bark, t				Without bark, t				Without bark, t–5			
Method	mean	s_b	s_w	s_t	mean	s_b	s_w	s_t	mean	s_b	s_w	s_t
0a	–14.48	21.04	14.49	25.28	–15.05	22.91	16.64	28.03	–12.30	30.18	16.95	34.21
1a	–6.64	19.58	14.08	23.87	–5.22	21.10	15.95	26.20	–3.70	28.36	16.42	32.40
2a	–6.06	18.53	13.51	22.70	–6.63	20.43	15.64	25.48	–4.99	27.72	16.40	31.84
3a	–0.14	17.14	12.62	21.07	–0.31	18.96	14.70	23.76	1.61	26.02	15.66	30.03
4a	0.30	16.87	12.31	20.67	0.17	18.65	14.37	23.32	2.32	25.61	15.50	29.60
0b	–4.28	3.10	7.26	7.88	–2.50	2.28	7.14	7.49	–3.37	4.59	7.89	9.09
1b	–4.51	3.33	7.46	8.14	–2.25	2.32	7.27	7.61	–3.29	4.58	7.97	9.16
2b	–4.23	3.26	7.13	7.82	–3.49	2.97	7.09	7.67	–3.86	5.37	7.93	9.53
3b	2.92	3.64	6.76	7.65	3.44	3.51	6.84	7.66	3.45	5.98	7.66	9.66
4b	1.91	4.27	6.56	7.80	2.21	4.24	6.68	7.88	1.99	6.66	7.57	10.01
M3	–5.80	1.79	7.44	7.65	–1.98	2.65	5.91	6.46	–0.99	0.61	6.53	6.59
0c	–3.98	3.06	7.30	7.90	–2.20	2.42	7.18	7.57	–2.45	2.94	7.86	8.38
1c	–4.15	3.38	7.50	8.20	–1.87	2.57	7.31	7.73	–2.14	3.25	7.96	8.58
2c	–4.14	2.95	7.15	7.72	–3.43	2.70	7.11	7.59	–3.73	2.94	7.88	8.39
3c	2.77	3.46	6.80	7.61	3.15	3.33	6.88	7.62	2.47	3.12	7.55	8.16
4c	1.77	4.04	6.60	7.71	1.93	3.97	6.72	7.78	1.04	3.76	7.45	8.32

	Bark				Increment			
Method	mean	s_b	s_w	s_t	mean	s_b	s_w	s_t
0a	–10.97	12.23	15.66	19.75	–23.67	23.00	35.02	41.70
1a	–15.44	13.73	15.52	20.58	–9.92	22.93	34.13	40.92
2a	–2.62	10.43	15.33	18.45	–12.02	22.66	33.60	40.34
3a	0.84	9.97	15.23	18.12	–7.21	22.98	33.06	40.06
4a	1.04	10.05	15.39	18.30	–7.77	23.07	32.92	40.00
0b	–16.75	12.54	14.14	18.78	0.03	6.32	16.10	17.26
1b	–20.00	13.61	14.69	19.88	0.89	6.53	16.18	17.41
2b	–9.85	8.21	13.29	15.55	–3.07	6.13	15.98	17.08
3b	–0.79	6.76	12.51	14.17	2.04	7.86	16.56	18.28
4b	–0.34	6.48	12.20	13.76	1.49	7.57	16.40	18.01
M3	–32.31	20.03	31.36	39.00	–4.39	7.71	12.67	14.77
0c	–16.33	12.27	14.20	18.65	–0.49	3.48	13.63	14.06
1c	–19.59	13.38	14.75	19.78	–0.06	2.90	13.70	14.00
2c	–9.19	7.96	13.32	15.45	–1.65	4.51	13.52	14.23
3c	0.09	6.84	12.50	14.20	5.72	3.66	14.08	14.54
4c	0.55	6.80	12.21	13.93	5.18	4.89	13.95	14.76

8 Discussion

8.1 Factors affecting the applicability of the stem curve set model in forest inventories

When the direct application of the stem curve set model in forest inventories is discussed, not only the test results from the primary data set, but also A) the representativeness of the primary data set, B) stem form and increment variation not included into the model and C) the model formulation must be taken into account.

A) The fixed and the random parameters are unbiased estimates only for the sampled target population. The primary data set is a subjective collection of the Scots pine cultures at the beginning of the 1970's. The main population of the primary data set is difficult to identify from our present forests. When the stem curve set model is applied to other Scots pine populations, e.g. to the Scots pines of the whole of Finland, it is obvious that the prior information is biased. When the tree stem curve sets are predicted, the predictions shrink to the expectations of the model and the prediction will be biased. The magnitude of the shrinking varies from stand to stand as a function of the quantity and quality of the sample trees.

The test results in the test data set presented in Chapter 7.2 are not directly applicable to the whole of Finland. The test data set used here as the test material is representative for southernmost Finland but because of its marginal geographical location, deviates more from the primary data set, which is a representative material covering the whole country.

The highest measured diameter used in the primary data set is at 70 % height. This can cause inaccuracy to the interpolated stem curves between 70 % height and the stem top. However, this inaccuracy can have only a small effect on the volume prediction errors, because only 6.0 % of the stem volume, 5.5 % of the bark volume and 5.0 % of the volume increment is as average above 70 % height in the primary data set.

The bark-thickness measurement in the primary data set has a random measurement error caused by the measurement method. The measurement error has an effect on the stem curves without bark in the years t and t–5 and on the bark curve, but does not affect the stem curve with bark or the increment curve. If the relative measurement error is independent of the measurement height, it only increases the within-stand variation of the model. If the error is different on the lower stem where the bark is rough and the upper stem where the bark is fine, the eigenvectors of the bark curve can be biased.

The effective estimation of models with random stand effects and their application make opposite demands on the selection of sample trees. To obtain reliable estimates for the within-stand variances, the model estimation requires concentration of the sample trees on plots. In the application the sample trees should be distributed on as many plots as possible, because the first sample tree always has the largest effect on the standard error of tally tree predictions. The conditional prior information in the form of regression models for the stand effects as a function of stand variables can be used to decrease prediction error, but already one sample tree in the plot improves the reliability of the prediction more.

The test data used here as test material will later, when it covers the whole of Finland, be more representative of the whole country than the primary data set used here. In the test data only an average of three sample trees are measured per sample plot. According to the problems found with the test data set (discussed in Chapter 7.2), this seems too few for effective estimation of the variance component. These estimation problems can be avoided by increasing the number of sample trees per plot. The estimation trials made with different numbers of sample plots in the prior data set indicates that the estimation problems also disappears when the number of measured sample plots is increased.

B) Stem form and its changes include variation caused by the noncircularity of stem cross-sections and the variation of the annual growth caused by effects external to the stand. The effects of noncircularity have already been discussed in Chapter 5.5. Usually the annual variation in growth is eliminated by annual growth indices. Thinnings have different effects on the growth in different parts of the stem. In the thinnings, decreasing competition increases diameter growth in the lowest third of the stem; and near the top the diameter increment can even degenerate (Vuokila 1960b). In this study the data were not corrected to the average level, because the effects of climate, thinnings and other exogenous factors are difficult to separate. The five-year growth period used in this study and its place in different calender years reduce the effect of annual growth variation.

C) The model formulation has at least five critical points with regard to its application in forest inventories: 1) the inflexibility of the model in describing differences between stands, 2) the problems caused by the use of size as an independent variable, 3) the use of the concept of stand, 4) the sensitivity of the random part of the model with regard to the data set and 5) fluctuations in the bark curve and in the increment curve predictions. The first three restrictions have already been discussed by Lappi (1986) and Korhonen (1991) with regard to the stem curve model.

C1) The stem curve set model is inflexible in a stand, because the logarithmic scale of the stand effect is only an additive component without any connection to tree size. Lappi (1986) and Korhonen (1991) showed that in the case of the stem curve there exists size-dependent variation between stands. This may also apply to bark and increment curves.

C2) The size as a fixed regressor has effects on the usefulness of the model. The predictions are biased in relation to the measured diameters. When all trees have same probability to be sampled or the sampling probability is proportional to the size, this will not cause significant bias in the predictions for stem, bark, and increment volume. If the sampling is done with changing probabilities in relation to some other tree variables, e.g. in relascope sampling the probability is proportional to the tree basal area, the growing stock volume prediction can be markedly biased, because the model is biased in relation to the sampling probability (Lappi & Bailey 1987).

To solve the problems of bias and inflexibility, Korhonen (1991) proposed the use of diameter at breast height as an independent variable in the stem curve model. This kind of model will produce (also in relascope sampling) unbiased predictions for growing stock volume with respect to measured diameter. If the coefficient of diameter at breast height is used as a random parameter, the stand-wise calibration will be more flexible. The idea of using dimensions measured in standard inventories as independent parameters of the model can be extended to formulate the model in another way, utilizing the covariance structure of random parameters and measurements of random variables (e.g. Lappi 1990).

A theoretical advantage of size as an independent variable is that the model is free of the requirement for fixed measurements. In the stem curve set model this means that prediction can also made if only dimensions without bark in year t or t–5 are made. In the present formulation, where the size is a function of diameters with bark, the predictions without measurement of the elementary stem curve with bark lose information because of the lack of a direct connection between the measured dimensions and tree size. One possibility to eliminate that information loss is to separate sizes for each elementary stem curve defined as a function of its own measurement function. In applications the sizes can then be used as random parameters with known covariance structure.

The allometric theory provides a basis for simple log-linear formation for models of the measurement function when the standard size variable is used as an independent variable. When diameters at fixed heights are used as independent variables, the pure allometric relation between independent variables and the measurement function does not exist, because the fixed height has different biological and geometrical interpretation for trees of different sizes. Theoretically, this will lead to more sophisticated formulation of the model than in the case when size is used as an independent variable. An empirical trial was made by replacing the size by diameter at breast height with bark as the independent variable of the stem curve set model (Equation 3.1.3). The error variances of the elementary stem curve estimates were smaller than the error variances of model 3.1.3 at the knot angles near breast height, but larger on the stem base and on the upper part of the stem. The prediction errors of stem volumes, bark volume and volume increment were the same as those in model 3.1.3.

C3) The concept of stand has here been used as synonymous with sample plot. With larger stands the variances of the within-stand effects will be greater than the estimates made from sample plots. In small sample plots the mean size estimates are inaccurate and they have an increasing effect on the between-stand prediction errors. The plot size of the primary data is quite large and describes better the within-stand variation than do the small sample plots used by Lappi (1986) and Korhonen (1991). When the model is used for prediction, determination of the concept stand or plot is not interesting as such. It is enough if the concept is similar determined in the phases of parameter estimation and stem curve set prediction.

C4) The principal components of the covariance matrix of the stand effects are very important for the stand-wise calibration of the stem curve set model. The analyses in the test data set showed that the principal components estimated from the primary data set do not have the same meaning in the test data set as in the primary data set (Chapter 7.2). This can be partly because the covariance matrix of the stand effects was approximated using the assumption that the correlations between the knot angles are the same as the correlations of the total errors. There are two more reasons which show that the principal components are more stable than the results of Chapter 7.2 indicate. Firstly, the results of the principal component analyses concerning the stem curve with bark of the stem curve set model are identical to the results of Lappi (1986) estimated from an independent set of data. Secondly, the interpretations of the two first principal components of bark curve are consistent with earlier knowledge of the vertical change in Scots pine bark type, and the first principal components of tree and stand effects for the increment curve are consistent with the effect of competition on the relationship between diameter and height increments.

C5) For extremely big trees, the predictions of the bark and increment curves have irregular fluctuations. The use of the seemingly unrelated regressions technique for simultaneously estimating models for all knot angles could decrease the inconstant variation because the residuals of the elements of the delineation vector are correlated (e.g. Harvey 1981, p. 67). The predictions of the model could also be improved by using data sets with larger size variation.

8.2 Concluding remarks

Obviously there exists a large risk of obtaining biased results, if the stem curve set model is applied directly in forest inventories. The main reason for this is the unrepresentativeness of the primary data set and the probable bias if the model is applied to the current Finnish Scots pine population. In spite of the inadequacy of the stem curve set model, the study provides a new approach to the problem of predicting bark and increment on tree and stand level.

Generality of the stem curve set model gives possibilities to critical examination of the efficiency of different sample tree measurements. The generality is obtained by using the tree size as the independent variable so that the model application is independent of fixed tree measurements. In the model, all variation is divided to the fixed, size dependent variation and to the random variation of stand and tree effects. The principal component analysis of the random variation describes the main directions of the between stand and within stand variation in stem form, bark thickness and stem increment.

For special purposes, it is possible to develop simpler or, in some point of view, better models by giving up the generality of the stem curve set model. The volume increment model (Equation 6.7) is an example of a simpler model. It gives the same reliability for sample tree predictions as the stem curve set model, if the only interesting variable is volume increment and the sample tree variables are fixed. An example of a better model for stem curve in terms of stability in extreme cases is the polynomial stem curve model based on the relative form quotient (Laasasenaho 1982). The use of relative heights and diameters gives a stable stem curve which is always logical for sample trees with one measured diameter and height. The RMSE of volume prediction for sample trees is the same using the polynomial stem curve model as when the elementary stem curve with bark of the stem curve set model is used (see Lappi 1986).

The elementary stem curve with bark of the stem curve set model 3.1.3 is formally identical to the stem curve model of Lappi (1986) and the model 3.1.5 with the stem curve model of Korhonen (1991), respectively. The weight vector for size and the model parameters were estimated anew for the stem curve set model from the primary data. Still the differences between the parameters of the stem curve model and the stem curve set model describe the disparity in the data sets used. Prediction errors of the stem curve set model without measured sample trees are smaller in the primary data set than the corresponding errors of the stem curve model of Lappi (1986) in its estimation data. The present results concerning the elementary stem curve with bark and crown height are consistent with those of Lappi (1986) and Korhonen (1991). The sample tree measurements of D_6 and H_c have no or very little influence on the $RMSE_b\%$ of the tally tree volume prediction with a small number of sample trees, if at least $D_{1.3}$ and H are measured from the sample trees. Lappi (1986) showed by theoretical analysis that with a large number of sample trees the measurement combinations with D_6 or H_c give a smaller $RMSE_b$% than do measurement combinations without them.

The application of the stem curve set model for bark volume and volume increment prediction can be interpreted as a combination of the volume method and the increment method. The final predictions are differences between two volumes in the same way as in the volume method. The results of the comparison between the stem curve set model and the volume method shows clearly that the stem curve set model gives significantly more accurate predictions of bark volume and volume increment for tally trees than the volume method, when only few sample trees are measured.

Volume functions using diameter at breast height and tree height as independent variables are normally used with the volume method. Tree height at the beginning of the increment period usually includes measurement error, if the height is measured directly, or prediction error, if it is predicted with an auxiliary model. The effect of omitting the height increment measurement can be studied with the stem curve set model because the sample tree measurements can be chosen freely in the application phase. When the diameter increment at breast height is measured from the sample trees and if the random measurement errors are taken into account, the height increment measurement decreases the standard error of the volume increment only slightly for sample and tally trees. For sample trees, the volume increment prediction using the stem curve set model has a smaller standard error without height increment measurement than using the volume method with measurement of height increment. If we take into account the probable systematic measurement errors of height increment, it is obvious that, using multivariate modelling approach, the volume increment prediction is more reliable without measurement of height increment than using the volume method with measurement of height increment.

In the increment method the volume increment is regressed using generalist least squares estimation directly on some fixed measurements and gives the best linear unbiased prediction (BLUP) of the volume increment. The stem curve set model gives BLUPs for each elementary stem curve conditional to the size. Because of the linear structure of the model, the bark and increment curves are also BLUPs.

The stem curve set model and the increment method (Equations 6.2–6.7) give approximately the same reliability for the predictions of bark volume and volume increment as a function of the same fixed measurements. For tally trees, the same reliability as in the stem curve set model could also be reached using the increment method with random stand parameters. If we ignore the consequences of the use of random stand effects in the stem curve set method, the stem curve set model has at least two advantages over the increment method in increment prediction: 1) the consistency of the stem form and volume predictions at the beginning and at the end of the increment period and 2) free choice of the measured dimensions in the application phase. The standard error of the volume increment prediction with the increment method can be decreased using some stand characteristics as independent variables (Strand & Li 1990) and also stem form variables; e.g. form quotients (Svensson 1988). The stand variables can be taken into account by using the stem curve set model with regression models for the principal components of the stand effect as a function of the independent stand variables (see Chapter 7.1). The measured tree dimensions used in the stem form variables can be utilized directly in the stem curve set model.

The stem curve set model was developed for prediction of past increment. When it is used to stand growth simulation based on repeated predictions, there is a risk for getting biased tree form development. For simulation purposes, the fixed part of the stem curve set model should be formulated a new, at least the interaction between relative size and average size should be included in the model.

References

Alalammi, E. 1968. Puun pituuskasvun arvioimistarkkuudesta ja sen vaikutuksesta kasvun arvioon. [On the reliability of measurement of tree height increment and its effect on the estimate of volume increment. In Finnish.] Mimeograph, University of Helsinki, Department of Forest Mensuration and Management. 43 p. + appendices.

Andersen, S. 1982. Combination of information on the volume level of a stand. Det Forstlige Forsøgsvæsen i Danmark 38(2): 85–92. ISSN 0367-2174.

Assmann, E. 1970. The principles of forest yield study. Pergamon Press, Oxford. 506 p.

Burk, T.E. & Ek, A.R. 1982. Application of empirical Bayes/James-Stein procedures to simultaneous estimation problems in forest inventory. Forest Science 28(4): 753–771. ISSN 0015-749X.

Byrne, J.C. & Reed, D.D. 1986. Complex compatible taper and volume estimation systems for red and Loblolly pine. Forest Science 32(2): 423–443. ISSN 0015-749X.

Cajander, A.K. 1909. Ueber Waldtypen. Acta Forestalia Fennica 1. 175 p. ISSN 0001-5636.

Cajanus, W. 1911. Puun rungon muotoa koskevia tutkimusmetoodeja. Referat: Über zahlenmässige Darstellung der Stammformen der Waldbäume. Suomen metsänhoitoyhdistyksen julkaisuja 27(6–7): 363–370.

Clutter, J.L. 1980. Development of taper functions from variable-top merchantable volume equations. Forest Science 26(1): 117–120. ISSN 0015-749X.

Daamen, W. 1980. Kontrolltaxeringen åren 1973–1977. Resultat från en kontroll av datainsamlingen vid riksskogstaxeringen. Summary: Results from a check on data collected of the National Forest Survey in 1973–1977. Sveriges lantbruksuniversitet, institutionen för skogstaxering, Rapport 27. 135 p. + appendix. ISBN 91-576-0500-9.

Demaerschalk, J.P. 1972. Converting volume equations to compatible taper equations. Forest Science 18(3): 241–245. ISSN 0015-749X.

Fog, D. & Jensen, A. 1953. General volume table for beech in Denmark. Det Forstlige Forsøgsvæsen i Danmark 21: 92–137. ISSN 0367-2174.

Fries, J & Matérn, B. 1966. On the use of multivariate methods for construction of tree taper curves. Skogshögskolan, institutionen för skoglig matematisk statistik, Rapporter och uppsatser 9: 85–117.

Gertner, G.Z. 1984. Localizing a diameter increment model with a sequential Bayesian procedure. Forest Science 30(4): 851–864. ISSN 0015-749X.

Green, J.E. & Strawderman, W.E. 1985. The use of Bayes/Empirical Bayes estimation in individual tree volume equation development. Forest Science 31(4): 975–990. ISSN 0015-749X.

Hagberg, N. 1966. Gödslingsförsök i barrskog. Tillväxtreaktionerna efter gödsling på några av skogshögskolans fasta försöksytor i mellersta och södra Sverige. Summary: Fertilization experiments in Swedish conifer stands. Skogshögskolan, institutionen för skogsproduktionen, Rapporter och uppsatser 11. 146 p.

Harvey, A.C. 1981. The economic analysis of time series. Philip Allan Publishers Limited, Oxford. 384 p. ISBN 0-86003-025-3.

Höjer, A.G. 1903. Tallens och granens tillväxt. [Increment of Scots pine and Norway spruce. In Swedish.] Bihang till Fr. Loven: Om våra barrskogar. Stockholm. p. 106–120.

Holm,S., Hägglund, B. & Mårtensson, A. 1979. En methed för generalisering av Riksskogstaxeringens provträdsdata. Summary: A method for generalization of sample tree data from the Swedish National Forest Inventory. Swedish University of Agricultural Sciences, Department of Forest Survey, Report 25. 79 p + appendices. ISBN 91-576-0182-8.

Hyppönen, M. & Roiko-Jokela P. 1978. Koepuiden mittauksen tarkkuus ja tehokkuus. Summary: On the accuracy and effectivity of measuring sample trees. Folia Forestalia 356. 25 p. ISBN 951-40-0344-6.

Ihalainen, A. 1987. Puukohtaisten mittausten ja arviointien luotettavuudesta valtakunnan metsien 8. inventoinnin pysyvillä koealoilla. [On the reliability of measurements of tree variables on the permanent sample plots in 8. national forest inventory. In Finnish.] Metsäntutkimuslaitoksen tiedonantoja 267. 37 p. ISBN 951-40-0843-X.

Ilvessalo, Y. 1947. Pystypuiden kuutioimistaulukot. Summary: Volume tables for standing trees. Communicationes Instituti Forestalis Fenniae 34(4). 149 p. ISSN 0358-9609.

IMSL library reference manual. Edition 9. 1982. IMSL Inc. Houston, Texas.

Jacobson, O. 1978. Beräkning av volymtillväxten hos enskilda träd. [Calculation of the volume growth of a single tree. In Swedish.] In: Skog för framtid. Betänkande av 1973 års skogsutredning. Bilagor. Statens officiella utredningar 178(7): 200–205. Stockholm. ISSN 0375-250X.

Jonsson, T. 1928. Några nya metoder för beräkning av stamvolym och tillväxt hos stående träd. Summary: Some new methods for calculating volume and increment of standing timber. Sveriges Skogsvårdsföreningens tidskrift 26: 423–527. ISSN 0371-2907.

Kilkki, P. 1979. Outline for a data processing system in forest mensuration. Silva Fennica 13(4): 368–384. ISSN 0037-5330.

— 1983. Sample trees in timber volume estimation. Acta Forestalia Fennica 182. 35 p. ISBN 951-651-057-4.

— & Lappi, J. 1987. Estimation of taper curve using stand variables and sample tree measurements. Scandinavian Journal of Forest Research 2(1): 121–126. ISSN 0282-7581.

— & Päivinen, R. 1986. Weibull function in the estimation of basal area dbh-distribution. Silva Fennica 20(2): 149–156. ISSN 0037-5330.

— , Saramäki, M. & Varmola, M. 1978. A simultaneous equation model to determine taper curve. Silva Fennica 12 (2): 120–125. ISSN 0037-5330.
— & Varmola, M. 1979. A nonlinear simultaneous equation model to determine taper curve. Silva Fennica 13(4): 293–303. ISSN 0037-5330.
— & Varmola, M. 1981. Taper curve models for Scots pine and their applications. Acta Forestalia Fennica 174. 60 p. ISBN 951-651-049-3.
Korhonen, K.T. 1991. Sekamallitekniikalla laadittujen runkokäyrämallien käyttö metsäninventoinnissa. Summary: Using taper curve models based on mixed linear models in forest inventory. Folia Forestalia 774. 27 p. ISBN 951-40-1162-7.
— & Maltamo, M. 1990. Männyn maanpäällisten osien kuivamassat Etelä-Suomessa. [Dry weights of the above ground parts of Scots pine in southern Finland. In Finnish.] Metsäntutkimuslaitoksen tiedonantoja 371. 29 p. + appendix. ISBN 951-40-1137-6
Kozak, A., Munro, D.D. & Smith, J.H.G. 1969. Taper functions and their application in forest inventory. The Forestry Chronicle 45(4): 278–283. ISSN 0015-7576.
Kujala, M. 1979. Mittaustarkkuus valtakunnan metsien inventoinnissa. [Accuracy of the measurements in the national forest inventory. In Finnish.] Mimeograph, The Finnish Forest Research Institute, Department of Forest Resources. 8 p.
— 1980. Runkopuun kuorellisen tilavuuskasvun laskentamenetelmä. Summary: A calculation method for measuring the volume growth over bark of stemwood. Folia Forestalia 441. 8 p. ISBN 951-40-0460-4.
Kuusela, K. 1965. A method for estimating the volume and taper curve of tree stem and for preparing volume functions and tables. Communicationes Instituti Forestalis Fenniae 60(2). 18 p. ISSN 0358-9609.
Laasasenaho, J. 1982. Taper curve and volume functions for pine, spruce and birch. Communicationes Instituti Forestalis Fenniae 108. 74 p. ISBN 951-40-0589-9.
Lahtinen, A. 1988. On the construction of monotony preserving taper curves. Acta Forestalia Fennica 203. 34 p. ISBN 951-651-081-7.
— & Laasasenaho, J. 1979. On the construction of taper curves by using spline functions. Communicationes Instituti Forestalis Fenniae 95(8). 63 p. ISBN 951-40-0427-2.
Lappi, J. 1986. Mixed linear models for analyzing and predicting stem form variation of Scots pine. Communicationes Instituti Forestalis Fenniae 134. 69 p. ISBN 951-40-0731-X.
— 1990. Statistical methods for changing and nonhomogenous forest. IUFRO 16th Word Congress 5–11, August 1990. Proceedings, Division 6. Montreal. p. 115–122.
— 1991. Calibration of height and volume equations with random parameters. Forest Science 37(3): 781–801. ISSN 0015-749X.
— & Bailey, R.L. 1987. Estimation of diameter increment function or other tree relations using angle-count samples. Forest Science 33(3): 725–739. ISSN 0015-749X.
Larson, P.R. 1963. Stem form development of forest trees. Forest Science-Monograph 5. 42 p.
Liu, C.J. & Keister, T.D. 1978. Southern pine stem form defined through principal component analysis. Canadian Journal of Forestry Research 8(2): 188–197. ISSN 0045-5067.
Loetsch, L., Zöhrer, F. & Haller, K.E. 1973. Forest inventory, volume II. BVL Verlagsgesellschaft, München. 496 p. ISBN 3-304-10812-2.
Lönnroth, E. 1925. Die innere Struktur und Entwicklung gleichaltriger naturnormalen Kiefernbestände. Acta Forestalia Fennica 30. 269 p. ISSN 0001-5636.
Matérn, B. 1956. On the geometry of the cross-section of a stem. Meddelanden från statens skogsforskningsinstitut 46(11). 28 p.
— 1961. On the precision of estimates of diameter growth from increment borings. IUFRO 13th Congress in Wien September 1961, Proceedings, 24/8-S/2.
Max, T.A. & Burkhard, H.E. 1976. Segmented polynomial regression applied to taper equations. Forest Science 22(3): 283–289. ISSN 0015-749X.
Mosimann, J.E. 1970. Size allometry: Size and shape variables with characterizations of the lognormal and generalized gamma distributions. Journal of American Statistical Assosiation. 65: 930–945. ISSN 0162-1459.
Nyyssönen, A. 1952. Puiden kasvusta ja sen määrittämisestä harsintamännikössä. Summary: On tree growth and its ascertainment in selectively cut Scots pine stands. Communicationes Instituti Forestalis Fenniae 40(4). 18 p. ISSN 0358-9609.
Östlin, E. 1963. Bark data for pine, spruce, birch etc. Part 1: Bark data for provinces and regions. Part 2: Bark data for site and age classes and for sawlogs and pulpwood. Skogshögskolan, institutionen för skogstaxering, Rapport 5. 245 p.
Päivinen, R. 1987. Metsän inventoinnin suunnittelumalli. Summary: A planning model for forest inventory. University of Joensuu, Publications in Science 11. 179 p. ISSN 0781-0342.
— , Nousiainen, M. & Korhonen, K.T. 1992. Puutunnusten mittaamisen luotettavuus. Summary: Accuracy of certain tree measurements. Folia Forestalia 787. 18 p. ISBN 951-40-1197-X.
Pekkonen, T. 1983. Leimikon puuston tilavuuden arviointi regressioennustinta käyttäen. [Regression estimators in predicting the volume of the stand marked for cutting. In Finnish.] Metsäntutkimuslaitoksen tiedonantoja 86. 63 p. ISSN 0358-4283.
Prodan, M. 1965. Holzmesslehre. J. D. Sauerländer's Verlag, Frankfurt am Main. 644 p.
Rennolls, K., Geary, D.N. & Rollinson, T.J.D. 1985. Characterizing diameter distributions by the use of the Weibull Distribution. Forestry 58(1): 57–66. ISSN 0015-752X.
Roiko-Jokela, P. 1976. Die Schaftformfunktion der Fichte und die Bestimmung der Sortimentsanteile am stehenden Baum. Summary: The function of the stem form for spruces and the proportions of the assortments on the standing tree. Eidgenössische Anstalt für das Forstliche Versuchswesen, Mitteilungen 52(1). 84 p.
Saramäki, J. 1980. Typpilannoituksen vaikutus männyn runkomuotoon. Summary: The effect of nitrogen fertilization on stem form of Scots pine. Communicationes Instituti Forestalis Fenniae 99(4). 46 p. ISBN 951-40-0477-9.
Searle, S.R. 1971. Linear models. John Wiley, New York. 532 p. ISBN 0-471-76950-9.
Shinozaki, K., Yoda, K., Hozumi, K. & Kira, T. 1964. A quantitative analysis of plant form – The pipe model theory. I. Basic analyses. Japanese Journal of Ecology 14(3): 97–104.
Sloboda, B. 1977a. Die Beschreibung der Dynamik der Schaftformfortpflanzung mit Hilfe der Ähnlichkeitsdifferentialgleichung und Affinität. Mitteilungen der Forstlichen Bundesversuchsanstalt Wien 120: 53–60.
— 1977b. Spline und Bitterlich'sche Interpolation bei der Wiedergabe der Baumschaftform. Mitteilungen der Forstlichen Bundesversuchsanstalt Wien 120: 61–70.
Sprent, P. 1972. The mathematics of size and shape. Biometrics 28: 23–37. ISSN 0006-341X.
Spurr, S.H. 1952. Forest inventory. The Ronald Press Company, New York. 475 p.
Sterba, H. 1980. Stem curves – a review of the literature. Forest Products Abstracts 4(4): 69–73. ISSN 0140-4784.
Strand, L. 1968. Formhøydetilvekst hos gran og furu. [Increment in form height of Norway Spruce and Scots pine. In Norwegian.] Tidsskrift for skogsbruk 76: 297–307. ISSN 0040-7178.
— & Li, L. 1990. Determination of volume increment on single trees of Norway spruce. Scandinavian Journal of Forest Research 5(4): 513–523. ISSN 0282-7581.
Svensson, S.V. 1988. Estimation of annual stem volume increment. A method to be applied to sample trees of Scots pine and Norway spruce on temporary plots in the Swedish National Forest Inventory. Swedish University of Agricultural Sciences, Department of Forest Survey, Report 46, 140 p. ISBN 91-576-3556-0.
Tiihonen, P. 1967. Pituuskasvun arvioimisesta. Referat: Über die Abschhätzung des Höhenzuwachses. Metsätaloudellinen aikakauslehti. 77(5–6): 190–191.
Vuokila, Y. 1960a. Männyn runkomuodon kehityksestä. Summary: On the development of stem form of Scots pine. Metsätaloudellinen aikakauslehti 84(9): 306–308.
— 1960b. Männyn kasvusta ja sen vaihtelusta harventaen käsitellyissä ja luonnontilaisissa metsiköissä. Summary: On growth and its variations in thinned and unthinned Scots pine stands. Communicationes Instituti Forestalis Fenniae 52(7). 38 p. ISSN 0358-9609.
— 1965. Functions for variable density yield tables of pine based on temporary sample plots. Communicationes Instituti Forestalis Fenniae 60(4). 86 p. ISSN 0358-9609.
— & Väliaho, H. 1980. Viljeltyjen havumetsiköiden kasvatusmallit. Summary: Growth and yield models for conifer cultures in Finland. Communicationes Instituti Forestalis Fenniae 99(2). 271 p. ISBN 951-40-0452-3.

Total of 76 references

Appendices

A Estimation of the stem curve set model

A.1 Matrices

The stem curve set model consists of a mixed linear model for each element of the delineation vector for stem curve set. The parameters are estimated separately for each of the 40 element of the delineation vector (tree elementary stem curves with 13 knot angles and the crown height: 3 x 13 + 1 = 40). Denote the number of stands by K, the number of trees in stand k by n_k and the total number of trees by N. The matrices of the normal equations (3.2.2) at the angle u for the elementary stem curve g are the following:

The dependent variable, the elements of the delineation vector at the angle u for the elementary stem curve g:

$$\mathbf{y}^T = [d_g(u)_{11}, \ldots, d_g(u)_{Kn_K}], \quad 1xN \tag{A.1.1}$$

The fixed independent variables, size s and average size $\bar{s}$:

$$\mathbf{X} = \begin{bmatrix} 1 & s_{11} & s_{11}^2 & \bar{s}_1 \\ & & \cdot & \\ & & \cdot & \\ & & \cdot & \\ 1 & s_{Kn_K} & s_{Kn_K}^2 & \bar{s}_K \end{bmatrix} \quad \text{rank}(\mathbf{X}) = 4 \tag{A.1.2}$$

Nx4

The fixed parameters:

$$\mathbf{a}^T = [a_{g0}(u), a_{g1}(u), a_{g2}(u), a_{g3}(u)], \quad 1xq \tag{A.1.3}$$

The incidence matrix of the stands:

$$\mathbf{Z} = \begin{bmatrix} 1_{11} & & & 0 \\ \cdot & & & \\ \cdot & & & \\ 1_{1n_1} & & & \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot \\ & & & 1_{K1} \\ & & & \cdot \\ 0 & & & 1_{Kn_K} \end{bmatrix} \quad \text{rank}(\mathbf{Z}) = K, \tag{A.1.4}$$

NxK

The first and the last columns of **X** are linear combinations of **Z**, so rank [**X Z**] = K+2.

The random parameters, the stand effects at the knot angle u for the elementary stem curve g:

$$\mathbf{b}^T = [v_g(u)_1, \ldots, v_g(u)_K] \quad 1xK \tag{A.1.5}$$

The covariance matrix of the random parameters:

$$\mathbf{D} = \text{var}[\mathbf{b}], \quad KxK. \tag{A.1.6}$$

The covariance matrix of random errors:

$$\mathbf{R} = \text{var}[\mathbf{e}], \quad NxN. \tag{A.1.7}$$

The variances var[**b**] and var[**e**] are estimated using the fitting constant method (Henderson's method 3, see Searle 1971).

A.2 Estimation of the covariances between the principal components of the stem curve with bark, the bark curve and the increment curve

Covariances between the principal components of the stand effects of the elementary stem curves are linear combinations of the covariances of the stand effects at the knot angles and the elements of the eigenvectors:

$$\text{cov}(c_{gk}, c_{g'k'}) = \sum_{u=1}^{13} \sum_{u'=1}^{13} [q_{gk}(u) q_{g'k'}(u') \text{cov}(v_g(u), v_{g'}(u'))], \tag{A.2.1}$$

where c = principal component
q = element of an eigenvector
v = random stand effect
k and k' = indices of the principal components
g and g' = indices of the elementary stem curves
u and u' = indices of the knot angles

To estimate the covariances of the principal components between the stem curve with bark, the bark curve and the increment curve, we need the covariances between the stand effects v_1, $v_B = (v_1 - v_2)$ and $v_I = (v_2 - v_3)$:

$$\text{cov}(v_1(u), v_B(u')) = \text{cov}(v_1(u), v_1(u') - \text{cov}(v_1(u), v_2(u')), \tag{A.2.2a}$$

$$\text{cov}(v_1(u), v_I(u')) = \text{cov}(v_1(u), v_2(u')) - \text{cov}(v_1(u), v_3(u')), \tag{A.2.2b}$$

$$\text{cov}(v_B(u), v_B(u')) = \text{cov}(v_1(u), v_1(u')) + \text{cov}(v_2(u), v_2(u')) - \text{cov}(v_1(u), v_2(u')) - \text{cov}(v_2(u), v_1(u')), \tag{A.2.2c}$$

$$\text{cov}((v_B(u), v_I(u')) = \text{cov}(v_1(u), v_2(u')) + \text{cov}(v_2(u), v_3(u')) - \text{cov}(v_1(u), v_3(u')) - \text{cov}(v_2(u), v_2(u')), \tag{A.2.2d}$$

$$\text{cov}(v_I(u), v_I(u')) = \text{cov}(v_2(u), v_2(u')) + \text{cov}(v_3(u), v_3(u')) - \text{cov}(v_2(u), v_3(u')) - \text{cov}(v_3(u), v_2(u')), \tag{A.2.2e}$$

where the covariances between the random effects were estimated using the following formula, which is derived from the formula for the sum of variances of two random variables:

$$\text{cov}[x_g(u), x_{g'}(u')] = 0.5\{\text{var}[x_g(u) + x_{g'}(u')] - \text{var}[x_g(u)] - \text{var}[x_{g'}(u')]\} \tag{A.2.3}$$

where x = v or e
g and g' = indices of the elementary stem curves, g = 1, 2 or 3.
u and u' = indices of the knot angles

B Application of the stem curve set model

B.1 Matrices

Denote the number of measured sample trees in a stand by n, the number of measurements of tree i by m_i and the total number of measurements in the stand by M. The matrices of the normal equations (4.1.10) are:

The dependent variable, values of the auxiliary variable y corresponding to the measurements:

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{y}_n \end{bmatrix}, \text{ where } \mathbf{y}_i^T = \left[y_g(u_{i1}),\ldots,y_g(u_{im_i}) \right] \quad \text{(B.1.1)}$$

Mx1

The independent variables, the fixed parameters estimated in the analysis stage, interpolated from the values at knot angles to the measurement angles:

$$\mathbf{X} = \begin{bmatrix} \mathbf{a}_1 & & \\ & \cdots & \\ & & \mathbf{a}_n \end{bmatrix}, \text{ where } \mathbf{a}_i^T = \left[a_g(u_{i1}),\ldots,a_g(u_{im_i}) \right] \quad \text{(B.1.2)}$$

Mxn

The fixed parameters, sizes of the sample trees:

$$\mathbf{a}^T = \mathbf{s}^T = (s_1,\ldots, s_n), \qquad 1\text{xn} \quad \text{(B.1.3)}$$

The interpolated elements of the eigenvectors for stand effects at the measurement angles:

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}_1 \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{Z}_n \end{bmatrix}, \text{ where} \quad \text{(B.1.4)}$$

Mx(3p+1)

$$\mathbf{Z}_i = \left[\mathbf{Z}_{i1} \quad -\mathbf{Z}_{iB} \quad -\mathbf{Z}_{iI} \quad 1 \right], \text{ where} \quad \text{(B.1.5)}$$

$$\mathbf{Z}_{ig} = \begin{bmatrix} q_{g1}(u_{i1}) & \cdots & q_{gp}(u_{i1}) \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ q_{g1}(u_{im_i}) & \cdots & q_{gp}(u_{im_i}) \end{bmatrix} \quad \text{(B.1.6)}$$

The random parameters to be estimated, the principal components of the stand effects (c_{gk}, g = 1, B, I and k = 1, ..., p):

$$\mathbf{b}^T = \mathbf{c}^T = (c_{11}, \ldots, c_{1p}, c_{B1}, \ldots, c_{Bp}, c_{I1}, \ldots, c_{Ip}, v(h_c)), \qquad 1\text{x}(3p+1) \quad \text{(B.1.7)}$$

The covariance matrix of the random parameters, the covariance matrix of the principal components

$$\mathbf{D} = \begin{bmatrix} var(c_{11}) & & \cdots & & cov(c_{11},v(h_c)) \\ cov(c_{11},c_{12}) & var(c_{12}) & \cdots & & cov(c_{12},v(h_c)) \\ & & \cdot & & \\ & & \cdot & & \\ & & \cdot & & \\ cov(c_{11},c_{14}) & & \cdots & var(c_{14}) & cov(c_{14},v(h_c)) \\ cov(c_{11},v(h_c)) & & \cdots & & var(v(h_c)) \end{bmatrix}, \quad \text{(B.1.8)}$$

(3p+1)x(3p+1)

The random errors:

$$\mathbf{e}^T = \left[e_g(u_{11}),\ldots,e_g(u_{nm_n}) \right], \qquad 1\text{xM} \quad \text{(B.1.9)}$$

The covariance matrix of the random errors:

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_1 & & \\ & \cdots & \\ & & \mathbf{R}_n \end{bmatrix}, \text{ where} \quad \text{(B.1.10)}$$

MxM

$$\mathbf{R}_i = var\left[e_g(u_{i1}),\ldots,e_g(u_{im_i}) \right]. \quad \text{(B.1.11)}$$

B.2 Simultaneous estimation of the stand effects for measured and unmeasured elementary stem curves

The normal equations for the mixed linear model are

$$\begin{bmatrix} \mathbf{X}^T\mathbf{R}^{-1}\mathbf{X} & \mathbf{X}^T\mathbf{R}^{-1}\mathbf{Z} \\ \mathbf{Z}^T\mathbf{R}^{-1}\mathbf{X} & \mathbf{Z}^T\mathbf{R}^{-1}\mathbf{Z}+\mathbf{D}^{-1} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{a}} \\ \hat{\mathbf{b}} \end{bmatrix} = \begin{bmatrix} \mathbf{X}^T\mathbf{R}^{-1}\mathbf{y} \\ \mathbf{Z}^T\mathbf{R}^{-1}\mathbf{y} \end{bmatrix}, \text{ where} \quad \text{(B.2.1)}$$

y = vector of the dependent variable
X = matrix of the fixed independent variables
a = vector of the fixed parameters to be estimated
Z = matrix of the random independent variables
b = vector of the random effects to be estimated
e = vector of random errors
R = covariance matrix of the random errors
D = covariance matrix of the random effects

The covariance matrix of the random parameters can be partitioned into the measured and the unmeasured part, if there is a lack of measurements for some random effects. Denote the random effects of the measured part by $\mathbf{b}_m$ and the random effects of the unmeasured part by $\mathbf{b}_u$. **D** is now partitioned as follows:

$$\mathbf{D} = \begin{bmatrix} \mathbf{D}_{11} & \mathbf{D}_{12} \\ \mathbf{D}_{21} & \mathbf{D}_{22} \end{bmatrix} \quad \text{(B.2.2)}$$

where $\mathbf{D}_{11} = var(\mathbf{b}_m)$, $\mathbf{D}_{22} = var(\mathbf{b}_u)$ and $\mathbf{D}_{12} = cov(\mathbf{b}_m, \mathbf{b}_u)$.

The normal equations for the measured part of the stem curves set are

$$\begin{bmatrix} \mathbf{X}^T\mathbf{R}^{-1}\mathbf{X} & \mathbf{X}^T\mathbf{R}^{-1}\mathbf{Z}_m \\ \mathbf{Z}_m^T\mathbf{R}^{-1}\mathbf{X} & \mathbf{Z}_m^T\mathbf{R}^{-1}\mathbf{Z}_m+\mathbf{D}_{11}^{-1} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{a}} \\ \hat{\mathbf{b}}_m \end{bmatrix} = \begin{bmatrix} \mathbf{X}^T\mathbf{R}^{-1}\mathbf{y} \\ \mathbf{Z}_m^T\mathbf{R}^{-1}\mathbf{y} \end{bmatrix}. \quad \text{(B.2.3)}$$

The estimates for the random effects of the measured part are

$$\hat{\mathbf{b}}_m = (\mathbf{Z}_m^T\mathbf{R}^{-1}\mathbf{Z}_m + \mathbf{D}_{11}^{-1})^{-1}\mathbf{Z}_m^T\mathbf{R}^{-1}(\mathbf{y} - \mathbf{X}\hat{\mathbf{a}}), \qquad \text{(B.2.4)}$$

and for the unmeasured part

$$\hat{\mathbf{b}}_u = \mathbf{D}_{21}\mathbf{D}_{11}^{-1}\hat{\mathbf{b}}_m \text{ . (see Lappi 1991)} \qquad \text{(B.2.5)}$$

Let us now write

$$\mathbf{D}^{-1} = \begin{bmatrix} \mathbf{D}^{11} & \mathbf{D}^{12} \\ \mathbf{D}^{21} & \mathbf{D}^{22} \end{bmatrix} \qquad \text{(B.2.6)}$$

and the following normal equations for the simultaneous estimation of the random effects $\mathbf{b}_m$ and $\mathbf{b}_u$:

$$\begin{bmatrix} \mathbf{X}^T\mathbf{R}^{-1}\mathbf{X} & \mathbf{X}^T\mathbf{R}^{-1}\mathbf{Z}_m & \mathbf{0} \\ \mathbf{Z}_m^T\mathbf{R}^{-1}\mathbf{X} & \mathbf{Z}_m^T\mathbf{R}^{-1}\mathbf{Z}_m + \mathbf{D}^{11} & \mathbf{D}^{12} \\ \mathbf{0} & \mathbf{D}^{21} & \mathbf{D}^{22} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{a}} \\ \hat{\mathbf{b}}_m \\ \hat{\mathbf{b}}_u \end{bmatrix} = \begin{bmatrix} \mathbf{X}^T\mathbf{R}^{-1}\mathbf{y} \\ \mathbf{Z}_m^T\mathbf{R}^{-1}\mathbf{y} \\ \mathbf{0} \end{bmatrix} \qquad \text{(B.2.7)}$$

It can be shown that the normal equations B.2.7 are identical to equations B.2.4 and B.2.5. The estimates for the unmeasured random effects are

$$\begin{aligned} \mathbf{D}^{21}\hat{\mathbf{b}}_m + \mathbf{D}^{22}\hat{\mathbf{b}}_u &= \mathbf{0} \\ \hat{\mathbf{b}}_u &= -\mathbf{D}^{22^{-1}}\mathbf{D}^{21}\hat{\mathbf{b}}_m \end{aligned} \qquad \text{(B.2.8)}$$

By inversion of the partitioned matrix we get

$$\begin{aligned} \hat{\mathbf{b}}_u &= -(\mathbf{D}_{22} - \mathbf{D}_{21}(\mathbf{D}_{11}^{-1} - \mathbf{D}_{12}))(-(\mathbf{D}_{22} - \mathbf{D}_{21}(\mathbf{D}_{11}^{-1}\mathbf{D}_{12}))^{-1}(\mathbf{D}_{21}\mathbf{D}_{11}^{-1})\hat{\mathbf{b}}_m \\ &= \mathbf{D}_{21}\mathbf{D}_{11}^{-1}\hat{\mathbf{b}}_m \end{aligned} \qquad \text{(B.2.9)}$$

which is identical to equation B.2.5.

Let us next solve the measured random effects. From the normal equations (B.2.7) we get

$$\mathbf{Z}_m^T\mathbf{R}^{-1}\mathbf{X}\hat{\mathbf{a}} + (\mathbf{Z}_m^T\mathbf{R}^{-1}\mathbf{Z}_m + \mathbf{D}^{11})\hat{\mathbf{b}}_m - (\mathbf{D}^{12}\mathbf{D}^{22^{-1}}\mathbf{D}^{21})\hat{\mathbf{b}}_m = \mathbf{Z}_m^T\mathbf{R}^{-1}\mathbf{y}\,. \qquad \text{(B.2.10)}$$

Now the term $(\mathbf{D}^{12}\mathbf{D}^{22^{-1}}\mathbf{D}^{21})\hat{\mathbf{b}}_m$ can be written in the form

$$(\mathbf{D}_{11}^{-1}\mathbf{D}_{12})\xi^{-1}\xi\xi^{-1}(\mathbf{D}_{21}\mathbf{D}_{11}^{-1})\hat{\mathbf{b}}_m = (\mathbf{D}^{11} - \mathbf{D}_{11}^{-1})\hat{\mathbf{b}}_m, \qquad \text{(B.2.11)}$$

where $\xi = \mathbf{D}_{22} - \mathbf{D}_{21}(\mathbf{D}_{11}^{-1}\mathbf{D}_{12})$.

Now we get

$$\begin{aligned} \mathbf{Z}_m^T\mathbf{R}^{-1}\mathbf{X}\hat{\mathbf{a}} + (\mathbf{Z}_m^T\mathbf{R}^{-1}\mathbf{Z}_m + \mathbf{D}^{11})\hat{\mathbf{b}}_m - (\mathbf{D}^{11} - \mathbf{D}_{11}^{-1})\hat{\mathbf{b}}_m &= \mathbf{Z}_m^T\mathbf{R}^{-1}\mathbf{y} \\ \mathbf{Z}_m^T\mathbf{R}^{-1}\mathbf{X}\hat{\mathbf{a}} + (\mathbf{Z}_m^T\mathbf{R}^{-1}\mathbf{Z}_m + \mathbf{D}^{11} - \mathbf{D}^{11} + \mathbf{D}_{11}^{-1})\hat{\mathbf{b}}_m &= \mathbf{Z}_m^T\mathbf{R}^{-1}\mathbf{y}' \end{aligned} \qquad \text{(B.2.12)}$$

and the estimator for the measured random parameters is

$$\hat{\mathbf{b}}_m = (\mathbf{Z}_m^T\mathbf{R}^{-1}\mathbf{Z}_m + \mathbf{D}_{11}^{-1})^{-1}\mathbf{Z}_m^T\mathbf{R}^{-1}(\mathbf{y} - \mathbf{X}\hat{\mathbf{a}}) \qquad \text{(B.2.13)}$$

which is same as equation B.2.4.

C Regression models for the increment method

Variable	Model 6.2	6.3	6.4	6.5	6.6	6.7
			Coefficients			
Constant	–4.294	–2.637	–2.879	–2.127	–1.357	–1.734
$D_{1.3b}$	–0.01354	–0.002084			0.01598	0.00623
$\ln(D_{1.3b})$	2.1354	2.136	1.154	1.162	1.145	1.167
$\ln(H)$			0.7790	0.7337	0.5875	0.4067
$B_{1.3}$			–0.07102	0.08947		
$\ln(B_{1.3})$			0.7030	0.7189		
$\ln(I_{1.3})$					0.5491	0.6586
$\ln(H_c/H)$					–0.08207	–0.1157
$D_6/D_{1.3}$			–0.008743	0.03030	0.7052	0.8468
$\ln(D_6/D_{1.3})$			–0.02113	–0.01971	0.07111	0.04642
T			–0.05869		–0.008638	
$\ln(T)$	0.4289		0.2457		–0.05189	
$\ln(B_{1.3}/D_{1.3})$					0.06639	0.02337
s_t	0.2709	0.2709	14.66	14.74	0.1442	0.1650
R	0.894	0.882	0.965	0.965	0.969	0.959

D Regression models for the random parameters

Notation:

OMT	Indicator variable. When the forest site type (Cajander 1909) is Oxalis-Myrtillys type, OMT =1, others OMT = 0.
VT	Indicator variable. When the forest site type is Vaccinium type, VT =1, others VT = 0.
CT	Indicator variable. When the forest site type is Calluna type, CT =1, others CT = 0.
STONE	Stoniness index: 0 = no stoniness, 1 = stoniness
PEAT	Peatland index: 0 = mineral soil, 1 = peatland
y-c	y-coordinate
x-c	x-coordinate
ALT	Altitude, m
TS	Temperature sum, d.d. (+5°C threshold)
D	Breast height diameter of a tree, cm
Dmean	Arithmetic mean diameter of the stand, cm
N	Number of stems, 1/ha
G	Stand basal area, m²/ha
TT	Time from the last thinning, years
s_t	Standard error of the model
R^2	Coefficient of determination

Models for the principal component of the stand effects

Stem curve with bark, c_1

Independent variables	Dependent variables Coefficients and F-values							
	c_{11}		c_{12}		c_{13}		c_{14}	
Constant	–8.551	.9	–0.5707	11.8	–0.2608	8.8	–0.07265	5.8
OMT	–0.1487	16.7	0.02999	6.8			0.01894	13.1
CT	0.0677	3.0						
y-c	–0.0005806	3.4	0.00004767	7.3	0.00003496	7.6		
x-c					0.00003357	2.4		
ALT	–0.001018	7.1						
TS	–0.003374	7.3						
ln(TS)	2.754	4.4						
Dmean			–0.006748	3.7				
ln(Dmean)	–0.5692	31.1	0.09621	2.4				
N	0.0001560	7.9						
ln(N)	–0.4278	21.6					0.009568	5.2
ln(G)	0.5248	45.7	0.02450	2.1				
s_t	0.01408		0.00134		0.00073		0.00052	
R^2	0.4684		0.1843		0.0882		0.1224	

Bark curve, c_B

Independent variables	Dependent variables Coefficients and F-values							
	c_{B1}		c_{B2}		c_{B3}		c_{B4}	
Constant	–0.7876	14.3	2.245	2.8	0.01081	0.7	–0.001743	0.3
OMT	0.08674	30.5						
VT	–0.02183	2.7						
CT	–0.03814	4.5						
PEAT					0.05535	14.6	0.02116	6.6
y-c	0.00006592	4.9						
x-c	0.00015619	9.4			0.00003881	12.0	0.00001478	4.6
ALT			0.00015912	14.6			–0.00003788	7.7
TS					–0.00002123	5.5		
ln(TS)			–0.3935	3.1				
ln(Dmean)			–0.02041	5.9				
N					–0.00000428	4.4		
ln(N)	0.03564	9.8						
G			0.00101762	9.5				
s_t	0.00357		0.00046		0.00019		0.00007	
R^2	0.4425		0.2302		0.2216		0.1130	

Increment curve, c_I

Independent variables	Dependent variables Coefficients and F-values							
	c_{I1}		c_{I2}		c_{I3}		c_{I4}	
Constant	9.729	3.4	–1.056	42.2	–0.01221	4.3	–0.009093	8.5
OMT	0.09895	22.3					0.006928	7.3
STONE							0.009257	9.5
y-c			0.00004108	6.9				
x-c			–0.00004356	3.1	0.00002827	5.1		
ALT	0.0004764	8.2					0.00004557	6.1
TS	0.002037	5.2						
ln(TS)	–1.768	3.8						
Dmean			–0.008541	9.3				
ln(Dmean)			0.2226	21.4				
ln(N)	0.05313	10.1	0.04125	24.1				
TT	–0.004452	6.2	0.001192	3.7				
s_t	0.00741		0.00090		0.00026		0.00012	
R^2	0.3772		0.3372		0.0348		0.1069	

Model for tree size (7.1c)

Independent variable	Coefficient	F-value
Constant	–4.787	26.7
OMT	–0.009343	2.3
VT	0.01078	10.8
CT	0.01556	8.0
TS	–0.0007677	23.5
ln(TS)	0.8760	29.4
D	–0.003188	9.4
ln(D)	0.8238	1164.5
N	0.00003916	24.7
G	–0.004306	9.3
ln(N)	–0.1078	97.5
ln(G)	0.2094	40.2
s_t	.00123	
R^2	0.9861	

E List of symbols

a	= a vector of fixed parameters
$a_{g0}(u), a_{g1}(u), a_{g2}(u), a_{g3}(u)$	= fixed parameters of the stem curve set model, in application treated as variables
B	= double bark thickness (cm)
B_x	= double bark thickness at x meter height (cm)
B_v	= bark volume
b	= a vector of random parameters
B	= covariance matrix of the random stand effects at the knot angles
c_{gk}	= k^{th} principal component of the elementary stem curve g
c	= vector of the first p principal components
D	= stem diameter (cm)
D_x	= diameter with bark in year t at x meter height (cm)
D_{xb}	= diameter without bark in year t at x meter height (cm)
D_{x-5}	= diameter without bark in year t–5 at x meter height (cm)
$d_g(u)_{ki}$	= logarithmic diameter at the angle u of tree i in the stand k for the elementary stem curve g, analyze phase
$d_g(u_{ij})$	= logarithmic diameter at the angle u of tree i corresponding to the measurement j for the elementary stem curve g, application phase
d	= logarithmic delineation vector for stem curve set
D	= covariance matrix of the random parameters (between class effects) in linear model
g	= index of the elementary stem curves: 1 = with bark in year t, 2 = without bark in year t, 3 = without bark in year t–5, B = bark curve and I = increment curve
e	= a vector of random errors
$e_g(u)_{ki}$	= random tree effect of the log-diameter at angle u for tree i in stand k for elementary stem curve g
H	= tree height (m)
H_{-5}	= tree height in year t–5 (m)
h	= ln(H)
H_c	= crown height (m)
h_c	= $\ln(H_c)$
I_x	= diameter increment at x meter height (cm)
I_H	= height increment (m)
i	= index for trees
j	= index for measurements
k	= index for stands
K	= number of stands
n_k	= number of trees in the stand k
N	= total number of trees
m_i	= number of measurements for tree i
M	= total number of measurements in a stand
$\mathbf{q}_{gk}$	= k^{th} eigenvector of the between-stand covariance matrix **B** for the elementary stem curve g
Q	= $(\mathbf{q}_{1\,1}, \ldots, \mathbf{q}_{3\,13})$
$r_g(u_{ij})$	= residual of the stand stem curve for measurement j in tree i
R	= covariance matrix of the random errors (within class effects) in linear model
S	= size in arithmetic scale
s	= size in logarithmic scale
$\dot{s}$	= preliminary estimate of s, used in applications
$\bar{s}$	= average size of trees in the same stand
s_b, s_w, s_t	= empirically estimated between-stand, within-stand and total standard deviations of the estimates
u	= angle in the polar coordinate system, integer values u = 1,...,13 are used for the knot angles.
$v_g(u)_k$	= random stand effect of the log-diameter at angle u in stand k for elementary stem curve g
V	= stem volume
W	= covariance matrix of the tree effects at the knot angles

w = vector of the covariances between the random tree effects and observed dimensions and a dimension to be predicted
X = model matrix for the fixed effects
Z = model matrix for the random effects
y = dependent variable
X^T = transpose of matrix X

Instructions to authors — Ohjeita kirjoittajille

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Manuscripts should be sent to the editors of the Society of Forestry as three full, completely finished copies, including copies of all figures and tables. Original material should not be sent at this stage.

The editor-in-chief will forward the manuscript to referees for examination. The author must take into account any revision suggested by the referees or the editorial board. Revision should be made within a year from the return of the manuscript. If the author finds the suggested changes unacceptable, he can inform the editor-in-chief of his differing opinion, so that the matter may be reconsidered if necessary.

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