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Supplementary file S2

NorFor: a forest sector model of Norway

The optimization problem for the Norwegian forest sector model NorFor is expressed as follows:

$$\underset{AREA_{plot,mic},HARV_{r,per,log},INV_{l,r,per},x_{r,fp,per},D_{r,fp,per},x^{FD}_{fr,ap,per},x^{FS}_{fr,ap,per},C_{per},PRODUCE_{ip,r,m,t},CAPA_{r,ip,m,per},CAPA\_BUILD_{r,ip,m,per},CAPA\_MAIN_{r,ip,m,per},TRANS_{ap,ar,ar2,per},DEBRIS_{log,r,per}}{\text{Max}} Z$$

$$= \sum_{per}[(\sum_{r}\sum_{fp}\int_{0}^{x_{r,fp,per}} D_{r,fp,per}(x_{r,fp,per})dx_{r,fp,per}$$

$$+ \sum_{fr}\sum_{ap}\int_{0}^{x^{FD}_{fr,ap,per}} D^{F}{}_{fr,ap,per}(x^{FD}{}_{fr,ap,per})\,dx^{FD}{}_{fr,ap,per}$$

$$+ \sum_{l}\sum_{r} Inv_{l,r,per} \times am + (((C_{per} - C_{per-1}) - (BaseC_{per}$$

$$- BaseC_{per-1})) \times CP) - \sum_{fr}\sum_{ap}\int_{0}^{x^{FS}_{fr,ap,per}} S^{F}{}_{fr,ap}(x^{FS}{}_{fr,ap,per})dx^{FS}{}_{fr,ap,per}$$

$$- \sum_{r}\sum_{l} FC_{r} \times HARV_{r,log,per}$$

$$- \sum_{r}\sum_{ip}\sum_{m}\sum_{f} Exo_{f} \times Ratio_{ip,m,f} \times PRODUCE_{ip,r,m,per}$$

$$- \sum_{r}\sum_{ip}\sum_{m} IC_{r,ip} \times (Ck \times CAPA_{r,ip,m,per} + Cm \times CAPA_{MAIN}{}_{r,ip,m,per}$$

$$+ CAPA_{BUILD}{}_{r,ip,m,per})$$

$$- \sum_{ar}\sum_{ar2}\sum_{ap} TC_{ar,ar2,ap} \times TRANS_{ap,ar,ar2,per}\ ) / [(1+i)^{(Date(Per)-Today)}]] \quad (1)$$

Subject to:

$\sum_{mic} AREA_{plot,mic} = Plot\_area_{plot}$	$\forall\, plot$	(2)
$HARV_{r,per,log} = (\sum_{plot}\sum_{mic} Forest_{plot,mic,per,log} \times Area_{plot,mic})/5$	$\forall\, r, per, log$	(3)
$HARV_{r,log,per} + x^{FS}_{fr,ap,per} + \sum_{ar2} TRANS_{ap,ar2,ar,per} - \sum_{ar2} TRANS_{ap,ar,ar2,per} - DEBRIS_{p,r,t} + \sum_{m} PRODUCE_{ip,r,m,per} - \sum_{m}\sum_{ap} Ratio_{ip,m,ap} \times PRODUCE_{ip,r,m,per} - x^{FD}_{fr,ap,per} = x_{r,fp,per}$	$\forall\, r, per, ap$	(4)
$CAPA_{r,ip,m,per-1}(1-dr) + CAPA\_MAIN_{r,ip,m,per} + CAPA\_BUILD_{r,ip,m,per} = CAPA_{r,ip,m,per}$	$\forall\, r, ip, m, per$	(5)
$CAPA\_MAIN_{r,ip,m,per} \leq CAPA_{r,ip,m,per-1} \times dr$	$\forall\, r, ip, m, per$	(6)
$PRODUCE_{ip,r,m,per} \leq CAPA_{r,ip,m,per}$	$\forall\, r, ip, m, per$	(7)
$\sum_{plot}\sum_{mic} Forest_{plot,mic,per,Vr\_tot(\text{Age} \geq 160 \text{ years})} \times Area_{plot,mic} < 1$	$\forall\, per$	(8)
$\sum_{plot}\sum_{mic}\sum_{Vr\_Warm} Forest_{plot,mic,per,Vr\_Warm} \times Area_{plot,mic} < 1$	$\forall\, per$	(9)
$\sum_{plot}\sum_{mic}\sum_{Vr\_ROS} Forest_{plot,mic,per,Vr\_ROS} \times Area_{plot,mic} < 250\,000$	$\forall\, per$	(10)

The sets, variables, and parameters that are used in the model are shown in Tables S1, S2, and S3. The equations are explained in Table S4.

Table S1: Sets and their description used in NorFor

Set	Description
per	periods
$plot$	national forest inventory plots
mic	management alternatives
$ar, ar2$	all regions, within and outside Norway
r	domestic regions
fr	foreign regions
ap	all products
log	log products (wood assortment)
fp	final products, i.e., with a demand function in Norwegian regions
cf	Forestry cost factor, i.e., costs of logging (final harvest and thinning) and silviculture
ip	industrial product, i.e., intermediate, and final products from industrial production
f	input factors in industry of input with exogenously determined prices

Table S2: Variables and their descriptions and units used in NorFor

Variable	Description	Unit
Z	welfare	
$AREA_{plot,mic}$	Area in each $plot$ allocated to management alternative mic	ha
$HARV_{r,per,log}$	Harvested wood of assortment log in region r and period per	m^3
$INV_{l,r,per}$	Inventory of log l in region r in period per	m^3
$x_{r,fp,per}$	Demand quantity for final products fp in region r in period per	$m^3, tons, MWh$
$x^{FD}_{fr,ap,per}$	Demand quantity for all products ap in region fr in period per	$m^3, tons, MWh$
$x^{FS}_{fr,ap,per}$	Supply quantity for all products ap in region fr in period per	$m^3, tons, MWh$
C_{per}	Carbon stocks in period per for a scenario with carbon pricing	$tons\ CO_2\ eq$
$PRODUCE_{r,ip,m,per}$	Production of industrial product ip, in region r, in machines type m in period per	$m^3, tons, MWh$
$CAPA_{r,ip,m,per}$	Capacity level in region r, of industrial product ip, of machines m in period per	$m^3, tons, MWh$
$CAPA\_BUILD_{r,ip,m,per}$	New capacity in region r, of industrial product ip, and of machines m in period per	$m^3, tons, MWh$
$CAPA\_MAIN_{r,ip,m,per}$	Maintained capacity in region r, of industrial product ip, of machines m in period per	$m^3, tons, MWh$
$TRANS_{ap,ar,ar2,per}$	Amount of product ap transported from region ar to region $ar2$ in period per	$m^3, tons, MWh$
$DEBRIS_{log,r,per}$	Wood debri of log type log, in region r, in period per	m^3
OLD_Forest_{per}	Area with forest at least 160 years old in period per	hectare

Table S3: Parameters and their descriptions and units used in NorFor

Parameters	Description	Unit
$Plot\_area_{plot}$	Area of each $plot$	ha
$Forest_{plot,mic,per,log}$	Log products of type log harvested from a $plot$ with management mic, in period per	m^3/ha
$Forest_{plot,mic,per,t}$	Age of forest from a $plot$ with management mic,in period per	*years*
$Forest_{plot,mic,per,Vr\_tot}$	Total removed volume from a $plot$ with management mic,in period per	m^3/ha
$Forest_{plot,mic,per,Vr\_Warm}$	Removed volume of warm broadleaves from a $plot$ with management mic,in period per	m^3/ha
$Forest_{plot,mic,per,Vr\_ROS}$	Removed volume of ROS species from a $plot$ with management mic,in period per	m^3/ha
$BaseC_{per}$	Baseline carbon stocks in period per in the reference scenario without carbon pricing	
FC_r	Forestry costs in region r	NOK/m^3
Exo_f	Exogenously defined unit costs in industry for input factor f	
$Ratio_{ip,m,f}$	input-output ratio for producing industrial product ip, in machinery m and of input f	
$Ratio_{ip,m,ap}$	input-output ratio for producing industrial product ip, in machinery m and of input ap	
$IC_{r,ip}$	Costs of building new capacity in region r and for industrial product ip	$NOK/unit$
$TC_{ar,ar2,ap}$	Costs of transporting a product ap from region ar to region $ar2$	$NOK/unit$
$Date_{per}$	Year of period per	
$Today$	Year of first period	
am	Amenity value of forest	
Ck	Costs of keeping capacity as a share of IC	$NOK/unit$
Cm	Costs to maintain capacity as a share of IC	$NOK/unit$
dr	depreciation rate in industry	
CP	carbon price	NOK/tCO_2
i	discount rate	

Table S4: Description of equations used in NorFor

Equation (1)	This equation specifies the objective function in NorFor: to maximize the sum of consumers' and producers' surplus. Equation (1) consists of the terms that represent the area underneath the demand curve for final products in domestic regions, the area underneath the demand curve for all products in foreign regions, economic benefits obtained from the amenity value, the value of the differences of carbon stocks between the beginning and end of each period, the area underneath the supply curve for all products in foreign regions, the harvest costs of woody biomass, the process costs of industrial products, capacity and investment costs, the transport costs of woody biomass from one region to another region, and the last term converts the future value of all terms to an equivalent present value.
Equation (2)	This equation ensures that a management alternative is allocated to the entire area in each plot.
Equation (3)	Yearly harvested volumes must equal the harvested volumes given by the yield tables summed over the selected management alternatives and plots. Divided by 5 as the Forest data are simulated for 5-year periods.

Equation (4)	This equation is the material balance constraint that makes sure that all logs and products that enter a region are either consumed in processing, consumed as final products, exported, or discarded.
Equation (5)	Capacity in industry equals last period's capacity subtracted depreciation, added maintained capacity, and built capacity.
Equation (6)	Capacity depreciation can be reduced or avoided by maintenance.
Equation (7)	Production in industry cannot exceed capacity.
Equation (8)	Biodiversity constraint: do not harvest plot with stand age ≥ 160 years.
Equation (9)	Biodiversity constraint: total harvest of broadleaves tree species (see Table 2 in paper for species) should be maximum 1 m^3/year.
Equation (10)	Biodiversity constraint: total harvest of ungulate tree species (see Table 2 in paper for species) should be maximum 250 000 m^3/year.