

Supplementary file S1

In this supplementary file, we describe how Laasasenaho (1982) taper curves were fitted to the stem diameters by Metsäteho Oy. The taper curve fitting model is applicable to any diameter measurements, but in this work, the model was specifically applied to fit taper curves to diameters that were measured by the harvesters during harvesting operations.

Taper curve model

Laasasenaho (1982) presented a polynomial model for determining a stem's taper curve, f_b , based on the relative diameters of the stem at certain heights. In the taper curve equation, the powers of the polynomial model are the members of the Fibonacci sequence:

$$f_b(x) = \frac{d_l}{d_{0.2h}} = b_1x + b_2x^2 + b_3x^3 + b_4x^5 + b_5x^8 + b_6x^{13} + b_7x^{21} + b_8x^{34} \quad (A1)$$

where $d_{0.2h}$ is the diameter of the stem at a height of 20%, h is the height of the tree, d_l is the diameter at height l from the ground, $x = 1 - l/h$ (i.e., the relative distance from the top of the tree) and $b_1 \dots b_8$ are coefficients.

The values of the coefficients $b_1 \dots b_8$ were determined by Laasasenaho (1982) based on stem measurements from National Forest Inventory tracts for pine, spruce and birch. The coefficients $b_4 \dots b_8$ mostly affect the shape of the stem's butt.

The major advantage in knowing the taper curve is that the diameters at any height of the stem can then be calculated using Eq. (A1). In addition, the volumes between any two heights can be calculated by integrating the taper curve as a solid of revolution.

There are differences in the relative shapes of trees that have the same DBH, but different heights. To take the stem-wise differences into account, Laasasenaho (1982) presented how the taper curve can be adjusted when the tree height, h , and DBH, d , are known. Denoted as $y_i(d, h)$, the difference between the real relative diameter and the modeled relative diameter, f_b , is

$$y_i = \frac{d_i}{\hat{d}_{0.2h}} - f_b(i) \quad (A2)$$

where d_i is the real diameter at relative height i and $\hat{d}_{0.2h}$ is as denoted in Laasasenaho (1982).

The difference y_i is approximated by computing the values of Eq. (A2) at three different relative heights. The relative heights used were 0.1, 0.4, and 0.7. The difference equations, $y_{0.1}$, $y_{0.4}$, and $y_{0.7}$, are given in Laasasenaho (1982), and the values of the differences are constrained to a maximum of 0.1. The cubic correction polynomial,

$f_r(x)$, is determined for each stem to fulfill the difference values, and the corrected taper curve, f_c , is obtained. Then, the value of $d_{0.2h}$ is re-calculated.

This taper curve model can be applied to all stems, regardless of their dimensions, as the stems are modeled using relative height and diameter variables. Each stem is modeled separately, which guarantees a high level of accuracy in the resulting taper curve.

Stem diameter measurements by harvesters

Harvesters measure the diameters and lengths of all individual stems that are fed through the harvester head. For each stem, they measure: 1) the diameter that represents the DBH; and 2) the diameter vector covering the stem's commercial part. These measurements are recorded in HPR files according to the StanForD 2010 standard (Skogforsk 2025). With the three most common harvester brands in Finland, the operator can select whether the middle and top diameters of the logs or also the complete diameter vector are recorded in the HPR files. The diameter representing the DBH is, however, recorded independently of the diameter vector.

In harvester measurements, the diameter representing the DBH is commonly measured at a constant height of 120 cm from the felling cut, instead of 130 cm from the ground. This originates from assuming that the height of the felled tree's stump is 10 cm. (This diameter value is not used, however, when calculating log volumes, for example.)

When the whole diameter vector is stored in the HPR files, the values of the diameter vector are obtained by both measuring and calculating. The established procedure in Finland is to calculate the values of the diameter vector between the felling cut and 130 cm above it by using butt-end functions or tables (Luonnonvarakeskus 2017) instead of direct measurements. The tree species-wise butt-end functions and tables represent the neiloid shape of the stem butt in an averaged manner, despite the irregular shapes of the stem butts that commonly occur. The inputs of the butt-end functions are the harvester-measured diameter value at 130 cm above the felling cut and the tree species. The diameters of the butt are calculated at intervals of 10 cm.

For the upper part of the stem, the diameters are measured up to or slightly above the place where the diameter of the stem meets a certain minimum value (typically a few centimeters, depending on the bucked assortments). For all diameter measurements, the respective heights are measured and appended to the values of the diameter vector.

Where only the mid and top diameters of the logs are available in the HPR files, the separate harvester-measured diameter from 120 cm above the felling cut is also regarded as a diameter value. This is because of the sparse total amount of diameter measurements of the logs for fitting the taper curve model.

Applying Laasasenaho taper curve fitting to harvester-measured stem diameters

Metsäteho has applied the Laasasenaho taper curve model to harvester-measured diameters. The idea was to determine individual taper curves by fitting the taper curve model to the measured diameter values. Because of the form of the model, an algorithm for nonlinear least squares (LS) fitting was needed. In this work, the Levenberg–Marquardt algorithm maximum-likelihood method was used, with constraints. The taper curve fitting was implemented by using the C# programming language.

The taper curves modeling process was divided into four consecutive stages, as detailed below. The initial values and constraints during the fitting are also described. In the following, the DBH or its estimate is denoted as d .

The initial values of the LS fitting are prepared as follows. The initial tree height, h , is determined based on the 13th diameter of the diameter vector, using average stem sizes of Finland and depending on the tree species. For the initial diameter value at a height of 20%, the value of the diameter vector closest in height to $0.2h$ is used. The initial value of the stump height is determined using the stump height model of Laasasenaho (1982), which is based on tree height and DBH.

Constraints are used in each fitting. The tree height can vary within the typical range of Finnish trees. The diameter, $d_{0.2h}$, can vary from 60 to 1,000 mm. The stump height, h_s , can vary within the range $[h_s - d/2, h_s + d/2]$, where d is the initial value of the DBH. However, the stump height has to be at least 0.07 m.

Stage 1: The process begins with the LS fitting of the model (Eq. (A1)) to the measured diameters. Here, the fitted parameters are the tree height, h , diameter, $d_{0.2h}$, and stump height, h_s . After the fitting, the estimate of d is calculated based on the taper curve model.

Stage 2: The correction polynomial is determined using the fitting results of h and d from Stage 1. The coefficients are added to the coefficients b_1 , b_2 , and b_3 of the taper curve. The DBH, d , is calculated using the adjusted taper curve.

Stage 3: At first, the height and stump height constraints are updated based on the current value of d . The LS fitting is performed using the updated coefficients of the taper curve. The fitted parameters are h , $d_{0.2h}$, and h_s , similarly to in Stage 1. After the fitting, the DBH, d , is calculated again.

Stage 4: In the last stages, the parameter $d_{0.2h}$ and the coefficients b_1 , b_2 , and b_3 of the taper curve are estimated. The values of h and h_s are fixed based on the results from Stage 3. The rest of the process depends on the diameter data coverage. If the diameter data of the stem is available for over one-third of the tree height ($h/3$), the values of $d_{0.2h}$, b_1 , and b_2 are obtained from LS fitting. The value of b_3 is then calculated. The monotonicity of the obtained taper curve is examined and, if confirmed, the parameters are accepted. If the taper curve is not monotonic, or the diameter data is available only below $h/3$, the value of $d_{0.2h}$ is fitted and b_1 is calculated. The final,

fitted value of the DBH, d , is then calculated and the stem volume can be integrated using the taper curve function. An example of fitted taper curves for one stem is shown in Fig. A1.

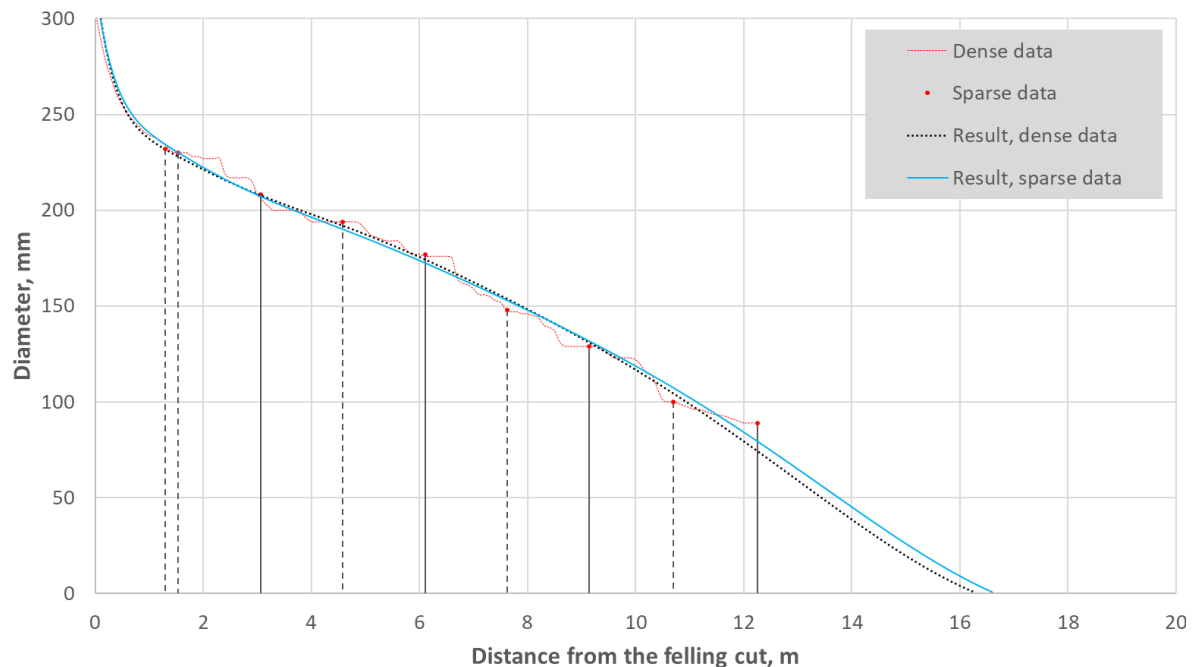


Figure A1. Example of fitted taper curves for one birch stem. The results are shown for both dense and sparse diameter input data. Solid vertical lines indicate the top positions of the logs and dashed lines indicate the position of diameter D130 and the middle positions of the logs.

Validation of the fitting results

Validation of the fitting algorithm was done by comparing the diameters at a height of 120 cm from the felling cut, denoted here as D120. These diameters are recorded directly by the harvester, and the corresponding values are calculated using the resulting taper curve, taking the fitted stump height into account. It should be noted that the DBH (130 cm from the ground) calculated from the taper curve fitting was used in the main study, but the validation of the fitting model was based on D120. The value of D120 resembles the DBH enough to allow validation. A comparison of the D120 diameters was made separately using dense (i.e., 10-cm-interval) and sparse (i.e., log-end and mid-diameter) diameter data.

The data used in the validation was queried from Metsäteho's database of harvester data. The database contains the operative data of Metsäteho's owners (i.e., the largest logging companies in Finland) and covers almost the whole of Finland, excluding the region of Lapland. The validation data for the thinning stands included an even sample with respect to the harvesters manufactured by the three main harvester manufacturers (i.e., John Deere, Komatsu, and Ponsse). The data was also evenly sampled geographically from the available data.

For *pine* and *spruce*, the average difference in the estimate of D120 varied from -0.1 to -1.7 mm, respectively (Table A1). For *birch*, the accuracy of the fit was slightly lower, but still reasonable (Table A1).

Overall, the taper curve fitting can be considered to work well enough for coniferous tree species. For *birch*, and apparently also other deciduous tree species, the diameter information recorded by the harvester is somewhat more heterogeneous for natural reasons, although this can be improved in the future by adjusting the details of the fitting.

Table A1. Accuracy of fitting the taper curve equation to thinning stems for pine, spruce, and birch, with full and sparse diameter data. The diameter recorded by the harvester at 120 cm from the felling cut is denoted as D120. The fitted results of D120 were subtracted stem-wise from the harvester-recorded values. The results are presented in classes where the harvester values of D120 are rounded to the nearest centimeter.

D120		PINE					
Harvester, cm	n	Full diameter data			Sparse diameter data		
		Bias, mm	Median, mm	STD, mm	Bias, mm	Median, mm	STD, mm
8	17 844	-0.9	-0.7	+1.5	-0.2	-0.4	+1.3
9	17 682	-1.8	-1.6	+1.8	-0.2	-0.3	+1.1
10	19 933	-2.2	-1.9	+2.0	-0.2	-0.2	+1.2
11	19 561	-2.2	-1.9	+2.0	-0.2	-0.2	+1.3
12	17 377	-2.2	-1.9	+2.2	-0.2	-0.2	+1.4
13	15 159	-2.1	-1.7	+2.3	-0.1	-0.1	+1.5
14	13 038	-1.8	-1.5	+2.4	+0.0	0.1	+1.8
15	11 912	-1.7	-1.3	+2.7	+0.0	0.1	+2.1
16	10 226	-1.5	-1.2	+2.7	+0.1	0.2	+2.0
17	8 532	-1.3	-0.9	+2.8	+0.2	0.2	+2.2
18	7 142	-1.2	-0.8	+3.0	+0.2	0.2	+2.3
19	5 756	-1.0	-0.5	+3.5	+0.2	0.3	+3.0
20	4 791	-0.7	-0.3	+3.4	+0.3	0.4	+2.7
21	3 793	-0.6	-0.1	+3.8	+0.2	0.4	+3.3
22	2 903	-0.4	0.2	+4.0	+0.3	0.5	+3.9
23	1 891	-0.3	0.3	+4.2	+0.4	0.4	+3.5
24	1 338	-0.4	0.1	+4.2	+0.2	0.2	+3.5
25	872	-0.6	0.1	+5.4	-0.1	0.1	+4.7
26	635	-1.1	-0.3	+7.3	-0.6	-0.3	+7.6
27	367	-1.3	-0.2	+8.4	-0.9	-0.2	+7.3
28	245	-2.1	-1.1	+9.5	-1.1	-0.4	+10.3
29	177	-3.2	-1.1	+12.8	-1.7	-0.6	+11.6
30	125	-2.9	-0.8	+10.8	-2.4	-0.6	+13.6
Total	181 299	-1.7	-1.4	+2.6	-0.1	-0.2	+2.1

D120		SPRUCE	
		Full diameter data	Sparse diameter data

Harvester, cm	n	Bias, mm	Median, mm	STD, mm	Bias, mm	Median, mm	STD, mm
8	8 856	+0.7	0.8	+1.1	-0.3	-0.3	+1.1
9	19 124	+0.1	0.3	+1.7	-0.1	-0.3	+1.2
10	17 866	-0.5	-0.1	+2.2	-0.3	-0.4	+2.1
11	14 368	-0.8	-0.4	+2.5	-0.3	-0.4	+1.8
12	13 311	-0.9	-0.5	+2.6	-0.3	-0.3	+1.4
13	12 039	-1.2	-0.8	+2.8	-0.4	-0.4	+1.6
14	9 752	-1.5	-1.1	+3.0	-0.4	-0.4	+1.6
15	7 487	-1.7	-1.4	+3.1	-0.4	-0.4	+1.6
16	5 566	-1.9	-1.6	+3.3	-0.5	-0.3	+2.0
17	4 343	-2.1	-1.7	+3.5	-0.5	-0.4	+2.1
18	3 423	-2.3	-2.0	+3.6	-0.4	-0.3	+2.2
19	2 715	-2.5	-2.2	+3.7	-0.6	-0.4	+2.6
20	2 062	-2.6	-2.2	+4.2	-0.6	-0.3	+3.1
21	1 625	-2.6	-2.2	+3.9	-0.6	-0.2	+2.8
22	1 254	-2.8	-2.2	+4.7	-0.8	-0.4	+3.9
23	910	-3.2	-2.5	+5.4	-0.9	-0.3	+4.9
24	688	-3.8	-2.5	+6.7	-1.4	-0.5	+6.6
25	511	-3.9	-3.0	+6.0	-1.1	-0.4	+5.9
26	382	-4.7	-3.7	+6.7	-1.6	-0.9	+5.9
27	287	-5.1	-3.2	+9.9	-2.1	-0.8	+9.9
28	208	-4.3	-3.2	+6.2	-1.5	-0.7	+6.1
29	144	-6.6	-3.7	+13.5	-4.6	-1.2	+18.1
30	111	-4.3	-3.0	+10.2	-2.5	-1.2	+10.2
Total	127 032	-1.0	-0.4	+3.0	-0.4	-0.4	+2.1

D120		BIRCH					
Harvester, cm	n	<i>Full diameter data</i>			<i>Sparse diameter data</i>		
		Bias, mm	Median, mm	STD, mm	Bias, mm	Median, mm	STD, mm
8	7 054	+0.4	0.5	+1.4	+0.4	0.1	+1.9
9	7 419	-0.3	-0.1	+2.3	+0.1	0.0	+2.1
10	7 953	-0.3	0.1	+2.5	-0.2	-0.2	+1.9
11	6 548	-0.6	-0.1	+2.9	-0.4	-0.2	+2.1
12	5 642	-0.7	-0.1	+3.2	-0.4	-0.1	+2.2
13	4 543	-0.9	-0.3	+3.3	-0.4	-0.1	+2.1
14	3 448	-1.0	-0.4	+3.6	-0.3	0.0	+2.3
15	2 697	-1.4	-0.7	+4.1	-0.3	-0.1	+2.8
16	1 986	-1.8	-1.0	+4.8	-0.5	-0.1	+3.2
17	1 413	-2.4	-1.6	+4.9	-0.7	-0.3	+3.4
18	1 092	-2.9	-2.1	+4.9	-0.8	-0.6	+3.2
19	731	-3.8	-2.8	+5.6	-1.0	-0.6	+4.1
20	585	-4.5	-3.2	+6.3	-1.6	-0.7	+6.1
21	387	-4.5	-3.4	+6.4	-1.6	-1.1	+4.7

22	245	−6.4	−3.9	+9.6	−2.6	−1.4	+8.8
23	172	−5.6	−3.7	+8.4	−2.3	−0.8	+7.4
24	150	−6.3	−4.3	+10.1	2.8	−1.6	+9.8
Total	52 065	−0.8	−0.1	+3.5	−0.3	−0.1	+2.6

References

Laasasenaho J (1982) Taper curve and volume functions for pine, spruce and birch. PhD thesis, Communicationes Instituti Forestalis Fenniae, University of Helsinki, Helsinki, Finland.

Luonnonvarakeskus (2017) Luonnonvarakeskuksen määräys puutavaran mittaukseen liittyvistä yleisistä muuntoluvuista. Ajantasainen määräys 22.12.2017. In Finnish. Available at: https://www.luke.fi/sites/default/files/2022-04/Luonnonvarakeskuksen-maarays-puutavaran-mittaukseen-liittyvista-yleisista-muuntoluvuista_FI_22122017_0.pdf

Skogforsk (2025) StanForD/StanForD 2010—Standard for forest machine data and communication. Skogforsk, Sweden, 2020. <http://www.skogforsk.se/english/projects/stanford/>